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PSDA Assignment 4

1.(a) $n = 1100$, $x = 990$

(i) $\hat{p} = \frac{x}{n}$

$= \frac{990}{1100}$

$= 0.9$

(ii) $1-\alpha = 0.99$

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$Z_{\frac{\alpha}{2}} = 2.575$

$\left(\hat{p} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$

$= \left(0.9 \pm 2.575\sqrt{\frac{0.9 \times 0.10}{1100}}\right)$

$= (0.8767, 0.9233)$ ##

∴ it is 99% confident that the proportion of all the drivers who have engaged careless or aggressive driving during the previous 6 months is between 0.8767 and 0.9233

(b) $n = 16$, $\bar{x} = 18.6$, $s = 9.486$, $1-\alpha = 0.80$, variance unknown

$1-\alpha = 0.80$

$\alpha = 0.20$

$t_{\text{critical value}} = 1.341$

$\left(\bar{x} \pm t_{\text{critical value}}\sqrt{\frac{s^2}{n}}\right)$

$= \left(18.6 \pm 1.341 \times \frac{9.486}{\sqrt{16}}\right)$

$= (15.42, 21.78)$ ##

∴ It is 80% confident that the mean time is lies between 15.42 and 21.78 seconds.

2.(a) $H_0 : p = 0.30$

$H_1 : p > 0.30$

To commit a type I error, the allergist make a mistake of rejecting H_0 when H_0 is actually true.

(b) $\mu = 35$, $n = 20$, $\bar{x} = 33.1$, $s = 4.3$, σ is unknown , $\alpha = 0.05$

(i) $H_0 : \mu = 35$

$H_1 : \mu < 35$

(ii) $df = n-1$

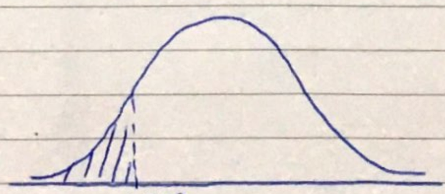
$= 20-1$

$= 19$

$\alpha = 0.05$

$t_{0.05,19} = -1.729$

critical value $= -1.729$



-1.729

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test statistic :

$$t = \frac{\bar{x} - \mu}{\sqrt{\frac{s^2}{n}}}$$

$$= \frac{33.1 - 35}{\frac{4.3}{\sqrt{20}}}$$

$$= -1.976 \#$$

* The test statistic, t is less than the critical value, hence, it lies in the rejection region

Conclusion :

In 0.05 significance level, there is sufficient evidence to conclude that the average time taken is actually less than 35 minutes.

(c) $\sigma = 0.470$, $\alpha = 0.05$

(i) $H_0: \sigma = 0.470$

$H_1: \sigma \neq 0.470$

(ii) $$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(16-1)(0.432)}{(0.470)^2} = 29.335$$

$$\bar{x} = \frac{\sum x}{n} = \frac{58.8}{16} = 3.675$$

$$s^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{6.481}{15} = 0.432$$

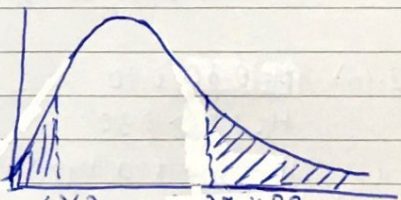
(iii) $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$

$n = 16 - 1$

$= 15$

critical value : $\chi^2_{0.025, 15} = 27.488$

$\chi^2_{1-0.025, 15} = 6.262$



(iv) the test statistic (29.325) is more than the critical value (27.488).

Hence the test statistic is ~~[illegible]~~ lying in the rejection region. ~~[illegible]~~ Reject H_0.

Conclusion :

There is sufficient efficience to conclude that the vitamin supplement appear to affect the variation among birth weights.

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3. $n = 350$, $x = 130$

(a) $1-\alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$Z_{\frac{\alpha}{2}} = 1.960$

$\hat{p} = \frac{x}{n}$

$= \frac{130}{350}$

$= 0.3714$

$$\left(\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$$

$$= \left(0.3714 \pm 1.960 \sqrt{\frac{0.3714(1-0.3714)}{350}} \right)$$

$$= (0.3208,\ 0.422)$$

There is 95% confident that the population proportion of all food shops with special offer is lies between 0.3208 and 0.422.

(b) $\text{margin of error} = Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$$0.04 = 1.960 \sqrt{\frac{0.3714(1-0.3714)}{n}}$$

$n = 560.54$

≈ 561

4(a) $n=6$, $\bar{x} = 58392$, $\sigma = 648$, $\alpha = 0.05$

$H_0: \mu = 58000$

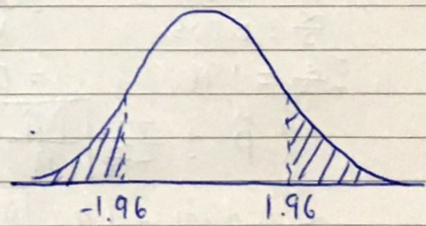
$H_1: \mu \neq 58000$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$Z_{\frac{\alpha}{2}} = 1.96$

critical value : $-Z_{\frac{\alpha}{2}} = -1.96$, $Z_{\frac{\alpha}{2}} = 1.96$



$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{58392 - 58000}{\frac{648}{\sqrt{6}}}$$

$= 1.482$

test statistic: $Z = 1.482$ which is between -1.96 and 1.96. Thus, fail to reject H_0

Conclusion :

There is sufficient evidence to conclude that the true average compressive strength of steel from which this sample came is 58000 psi.

(b) $\sigma^2 = 0.18$, $n = 101$, $S^2 = 0.13$, $\alpha = 0.05$

$H_0 : \sigma^2 = 0.18$

$H_1 : \sigma^2 \neq 0.18$

$\alpha = 0.05$

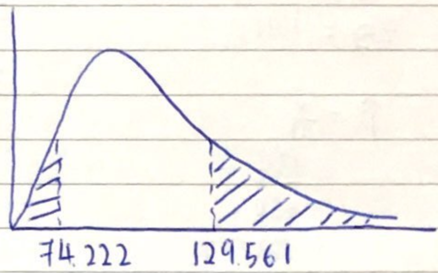
$\frac{\alpha}{2} = 0.025$

$n = 101 - 1$

$= 100$

$\chi^2_{0.025,100} = 129.561$

$\chi^2_{1-0.025,100} = 74.222$



critical value : 74.222 , 129.561

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

$$= \frac{(100)(0.13)}{0.18}$$

$$= 72.22$$

Since $\chi^2 = 72.22$ less than the critical value (74.222), hence, reject H_0

Conclusion :

There are sufficient evidence to conclude that the inspector is not making satisfactory measurements.

5(a) $n = 418$, $X = 280$, $1-\alpha = 0.95$

$1-\alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$Z_{\frac{\alpha}{2}} = 1.96$

$\hat{p} = \frac{x}{n}$

$= \frac{280}{418}$

$= 0.67$

$$\left(\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$$

$$= \left(0.67 \pm 1.96 \sqrt{\frac{0.67 \times (1-0.67)}{418}}\right)$$

$$= (0.625 , 0.715)$$

$\therefore$ It is 95% confident that the population proportion is lies in between 0.625 and 0.715

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(b) $n=126$, $\bar{x}=29.2$, $\sigma=7.5$, $\mu=18.2$, $1-\alpha=0.90$

$1-\alpha=0.90$

$\alpha=0.10$

$\frac{\alpha}{2}=0.05$

$Z_{\frac{\alpha}{2}}=1.645$

$\left(\bar{x} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma^2}{n}}\right)$

$=\left(29.2 \pm 1.645\sqrt{\frac{7.5^2}{126}}\right)$

$=(28.101,\ 30.299)$

$\therefore$ It is 90% confident that the estimator for average blood lead level lies between 28.101 and 30.299

6(a) $n=64$, $\bar{x}=38$, $S=5.8$, $\alpha=0.05$, $\sigma=$ unknown

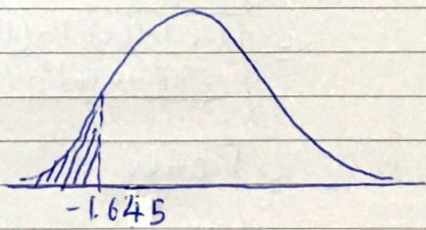
$H_0: \mu=40$

$H_1: \mu<40$

$\alpha=0.05$

since $n \geq 30$,

$Z_{0.05}=1.645$



critical value : $Z=-1.645$.

$Z=\frac{\bar{x}-\mu}{\sqrt{\frac{S^2}{n}}}$

$=\frac{38-40}{\frac{5.8}{\sqrt{64}}}$

$=-2.759$

Since $Z=-2.759$ less than critical value (-1.645), lies in the rejection region.

Hence, reject H_0

Conclusion:

There is sufficient evidence to conclude that $\mu<40$.

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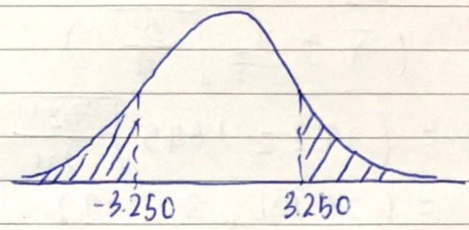
(b) $\mu = 10$, $n = 10$, $\alpha = 0.01$, σ = unknown

$H_0 : \mu = 10$
$H_1 : \mu \neq 10$

$\alpha = 0.01$, $\frac{\alpha}{2} = 0.005$
$df = n-1$
$= 10-1$
$= 9$



critical value : $t_{0.005,9} = 3.250$
$-t_{0.005,9} = -3.250$

$$\bar{x} = \frac{10.2 + 9.7 + 10.1 + 10.3 + 10.1 + 9.8 + 9.9 + 10.4 + 10.3 + 9.8}{10}$$
$$= 10.06$$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$
$$= \sqrt{\frac{(10.2-10.06)^2+(9.7-10.06)^2+(10.1-10.06)^2+(10.3-10.06)^2+(10.1-10.06)^2+(9.8-10.06)^2+(9.9-10.06)^2+(10.4-10.06)^2+(10.3-10.06)^2+(9.8-10.06)^2}{10-1}}$$
$$= \sqrt{\frac{0.544}{9}}$$
$$= 0.246$$

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$
$$= \frac{10.06-10}{\frac{0.246}{\sqrt{10}}}$$
$$= 0.771$$

Since the test statistic value 0.771 lies between the critical vale 3.250 and -3.250
so it fails to reject H_0.

Conclusion :
There is sufficient evidence to support that the average content of container of a particular lubricant is 10 liters.

7(a) confidence interval for population mean (variance unknown)

$= \left(\bar{x} \pm t_{critical\ value} \cdot \frac{s}{\sqrt{n}} \right)$

$\bar{x}$, s and confidence level are the same for both.
If sample size is larger, $\frac{s}{\sqrt{n}}$ become smaller, $t_{critical\ value}$ is based on the degree of freedom and confidence level, thus $t_{critical\ value}$ will also become smaller. So, the confidence interval will become smaller. Thus, the confidence interval based on the larger sample size is the one with a smaller range which is (49.8, 50.6).

range = 50.6 − 49.8 = 0.8

range = 52.7 − 50.3 = 2.4

(b) I will recomend statistic II. ~~Because the standard deviation is the lowest among them. If With the standard deviation~~ In statistic II and statistic III, the true value of population characteristic is at the centre. Thus they are better than statistic I which is far away. Next, by comparing the standard deviation, statistic II has lower standard deviation than statistic III. Lower standard deviation will influence the standard error as well as the estimation error. ~~Th~~ The lower the value of standard deviation, the lower the standard error and estimation error. Thus the best choice is Statistic II with ~~te~~ smaller standard deviation and the ~~ce~~ true value of population characteristic is at the centre of the distribution.

(c) $n = 500$, $1-\alpha = 0.90$, $\hat{p} = 0.15$

$1-\alpha = 0.90$

$\alpha = 0.10$

$\frac{\alpha}{2} = 0.05$

$Z_{\frac{\alpha}{2}} = 1.645$

$\left(\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$

$= \left(0.15 \pm 1.645 \sqrt{\frac{0.15 \times (1-0.15)}{500}} \right)$

$= (0.124, 0.176)$

It is 90% confident that the population proportion lies between 0.124 and 0.176.