

PSDA (Sec 02) Exercise

Question 1

(a) $X_h = 85, X_l = 44$

$R = X_h - X_l = 85 - 44 = 41$

$$i = \frac{R}{1 + 3.322 \log n} = \frac{41}{1 + 3.322 \log 32} = 6.83$$

Trial and error:

$i = 5, h = \frac{R}{i} = \frac{41}{5} \approx 9$

$i = 7, h = \frac{R}{i} = \frac{41}{7} \approx 6$

$i = 9, h = \frac{R}{i} = \frac{41}{9} \approx 5$

$\therefore i = 7$, number of cells = 6

$MP_i = \frac{(43.5 + 50.5)}{2} = 47$

Relative Frequency:

$C_1 = \frac{6}{32} = 0.19$

$C_2 = \frac{10}{32} = 0.31$

$C_3 = \frac{2}{32} = 0.06$

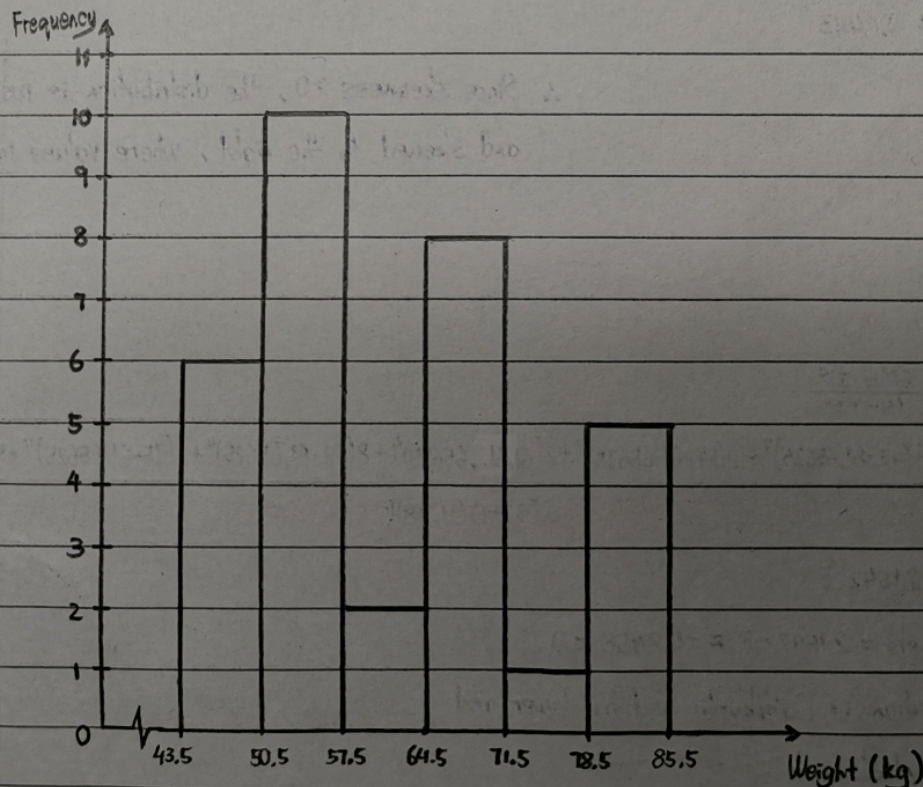
$C_4 = \frac{8}{32} = 0.25$

$C_5 = \frac{1}{32} = 0.03$

$C_6 = \frac{5}{32} = 0.16$

Class Interval	Class Boundaries	Class Midpoints	Frequency	Relative Frequency	Cumulative Frequency
44 - 50	43.5 - 50.5	47	6	0.19	6
51 - 57	50.5 - 57.5	54	10	0.31	16
58 - 64	57.5 - 64.5	61	2	0.06	18
65 - 71	64.5 - 71.5	68	8	0.25	26
72 - 78	71.5 - 78.5	75	1	0.03	27
79 - 85	78.5 - 85.5	82	5	0.16	32

(b)



(c) i) Mean:

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$= \frac{6(47) + 10(54) + 2(61) + 8(68) + 1(75) + 5(82)}{32}$$

$$\bar{x} = 61.65625 \text{ kg}$$

ii) Median:

$$\frac{N}{2} = \frac{32}{2} = 16, \therefore \text{median class} = (51-57) \text{ kg}$$

$$L = 50.5, C_{fp} = 6, f_{med} = 10, w = 7$$

$$\therefore \text{Median} = L + \frac{\frac{N}{2} - C_{fp}}{f_{med}} (w)$$

$$= 50.5 + \frac{\frac{32}{2} - 6}{10} (7)$$

$$\text{Median} = 60.3 \text{ kg}$$

iii) Mode:

$$\text{Modal class} = (51-57) \text{ kg}$$

$$l = 50.5, h = 7, f_1 = 10, f_0 = 6, f_2 = 2$$

$$\therefore \text{Mode} = l + h \times \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)}$$

$$= 50.5 + 7 \left[\frac{10 - 6}{2(10) - 6 - 2} \right]$$

$$\text{Mode} = 52.83 \text{ kg}$$

(d)

$$\text{Variance, } s^2 = \frac{\sum f(x - \bar{x})^2}{N - 1}$$

$$= \frac{4(47 - 61.65625)^2 + 10(54 - 61.65625)^2 + 2(61 - 61.65625)^2 + 8(68 - 61.65625)^2 + 1(75 - 61.65625)^2 + 5(82 - 61.65625)^2}{32 - 1}$$

$$s^2 = 129.5357$$

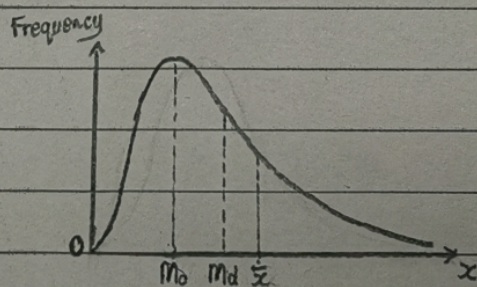
$$\therefore \text{Standard deviation, } s = \sqrt{s^2} = \sqrt{129.5357} = 11.38$$

(e)

$$\text{Skewness} = \frac{\sum f(x_i - \bar{x})^3}{(N - 1) s^3}$$

$$= \frac{4(47 - 61.65625)^3 + 10(54 - 61.65625)^3 + 2(61 - 61.65625)^3 + 8(68 - 61.65625)^3 + 1(75 - 61.65625)^3 + 5(82 - 61.65625)^3}{(32 - 1)(11.38)^3}$$

$$\approx 0.6443$$



$\therefore$ Since skewness > 0 , the distribution is positively skewed and skewed to the right, where values $\text{mode} < \text{median} < \text{mean}$.

(f)

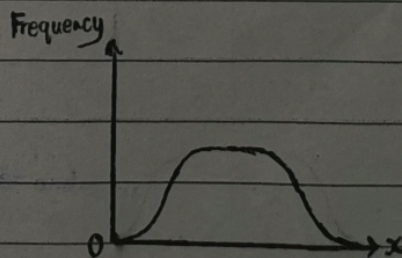
$$\text{Kurtosis} = \frac{\sum f(x_i - \bar{x})^4}{(N - 1) s^4}$$

$$= \frac{4(47 - 61.65625)^4 + 10(54 - 61.65625)^4 + 2(61 - 61.65625)^4 + 8(68 - 61.65625)^4 + 1(75 - 61.65625)^4 + 5(82 - 61.65625)^4}{(32 - 1)(11.38)^4}$$

$$\approx 2.1542$$

$$\text{Excess kurtosis} = 2.1542 - 3 \approx -0.8458 < 0$$

$\therefore$ The distribution is platykurtic and has lower and broader central peak.



Question 2

Arranging data:

1.01, 1.45, 1.54, 1.60, 1.69, 2.03, 2.51, 2.67, 2.69, 2.91, 2.95, 3.41, 3.53, 3.61, 3.68, 4.01, 4.18, 4.99, 5.08, 5.10

(a) i) $N=20, P=25$

$$i = 20 \times \frac{25}{100} = 5$$

$$\therefore Q_1 = \frac{(5^{th} + 6^{th})}{2} = \frac{1.69 + 2.03}{2} = 1.86$$

 $N=20, P=50$

$$i = 20 \times \frac{50}{100} = 10$$

$$\therefore Q_2 = \frac{(10^{th} + 11^{th})}{2} = \frac{2.91 + 2.95}{2} = 2.93$$

 $N=20, P=75$

$$i = 20 \times \frac{75}{100} = 15$$

$$\therefore Q_3 = \frac{(15^{th} + 16^{th})}{2} = \frac{3.68 + 4.01}{2} = 3.845$$

ii) $P=85$

$$i = 20 \times \frac{85}{100} = 17$$

$$\therefore P_{85} = \frac{(17^{th} + 18^{th})}{2} = \frac{4.18 + 4.99}{2} = 4.585$$

iii) $Q_1 = 1.86, Q_3 = 3.845$

$$\therefore IQR = Q_3 - Q_1$$

$$= 3.845 - 1.86 = 1.985$$

iv) Range, $R = X_h - X_l$

$$= 5.10 - 1.01$$

$$= 4.09$$

(b) Minimum = 1.01 Lower Limit Upper Limit

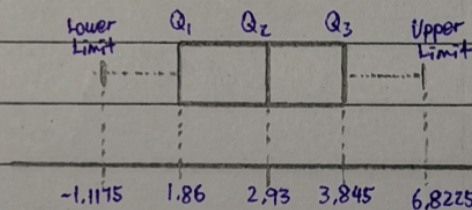
$$Q_1 = 1.86 \quad = Q_1 - 1.5(IQR) \quad = Q_3 + 1.5(IQR)$$

$$Q_2 = 2.93 \quad = 1.86 - 1.5(1.985) \quad = 3.845 + 1.5(1.985)$$

$$Q_3 = 3.845 \quad = -1.1175 \quad = 6.8225$$

$$\text{Maximum} = 5.10$$

$$IQR = 1.985$$



(c) The value 4.99 added is not an outlier. The value is in between lower limit and upper limit.

(d) Rod Glass Diameter:

Stem	Leaf
1	01 45 54 60 69
2	03 51 67 69 91 95
3	41 53 61 68
4	01 18 99
5	08 10

Key: 1|01 means 1.01