

PSDA Exercise 8/3

Question 5

(a) $P_{\text{blowout}} = 25\% = 0.25, n=15$

i) $P(3 \leq X \leq 6)$

$= P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= {}^{15}C_3(0.25)^3(0.75)^{12} + {}^{15}C_4(0.25)^4(0.75)^{11} + {}^{15}C_5(0.25)^5(0.75)^{10} + {}^{15}C_6(0.25)^6(0.75)^9$

$= 0.7073$

ii) $P(X < 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= {}^{15}C_0(0.25)^0(0.75)^{15} + {}^{15}C_1(0.25)^1(0.75)^{14}$

$+ {}^{15}C_2(0.25)^2(0.75)^{13} + {}^{15}C_3(0.25)^3(0.75)^{12}$

$P(X < 4) \approx 0.4613$

iii) $P(X > 4) = 1 - P(X < 4) - P(X=4)$

$= 1 - 0.4613 - {}^{15}C_4(0.25)^4(0.75)^{11}$

$P(X > 4) \approx 0.3135$

(b) i) $P_{\text{repair}} = 0.10$

$P(X=8) = (1-0.10)^7(0.10)$

$= 0.0478$

ii) $P_{\text{fine}} = 1 - 0.10 = 0.90$

$P(X=12) = (1-0.90)^{11}(0.90)$

$\approx 9 \times 10^{-12}$

Question 6

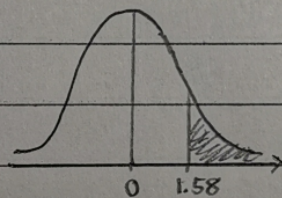
(a) $\sigma = 3.8, \mu = 24, X = 60 \times \frac{1}{2} = 30$

$P(X \geq 30) = P(Z \geq \frac{30-24}{3.8})$

$= P(Z \geq 1.58)$

$= 0.5 - 0.4429$

$P(X \geq 30) \approx 0.0571$



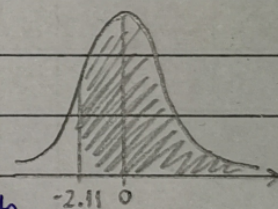
(b) 9.01 am $\Rightarrow$ late, $X = 0901 - 0845 = 16$

$P(X \geq 16) = P(Z \geq \frac{16-24}{3.8})$

$= P(Z \geq -2.11)$

$= 0.5 + 0.4826$

$P(X \geq 16) \approx 0.9826 = 98.26\% \text{ late}$



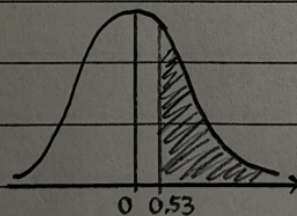
(c) 9.01 am $\Rightarrow$ missed coffee, $X = 0901 - 0835 = 26$

$P(X \geq 26) = P(Z \geq \frac{26-24}{3.8})$

$= P(Z \geq 0.53)$

$= 0.5 - 0.2019$

$P(X \geq 26) \approx 0.2981$



(d) 15% = 0.15 = P(unusual)

$P(Z \geq \frac{X-\mu}{\sigma}) = 0.15$

$P(Z \geq 0.39) \approx 0.15$

$\frac{X-\mu}{\sigma} = 0.39, \mu = 24, \sigma = 3.8$

$\frac{X-24}{3.8} = 0.39$

$\therefore X = 25.48 \text{ min}$

More than 25.48 min is

unusual journey time.

(e) $n=3$

$p = P(X \geq 30) \approx 0.0571$

$q = 1 - p = 1 - 0.0571 = 0.9429$

$\therefore P(X=2) = {}^3C_2(0.0571)^2(0.9429)^1 \approx 9.22 \times 10^{-3}$