

PSDA (Assignment 2)

1) (a) $n=18, p=90\%=0.9, x=18$

$$p(x=18) = {}^{18}C_{18} (0.9)^{18} (1-0.9)^{18-18}$$

$$= {}^{18}C_{18} (0.9)^{18} (0.1)^0$$

$$\therefore p(x=18) = 0.1501$$

(b) $n=20, p=0.2, q=1-p=1-0.2=0.8$

i) Mean, $\bar{x} = np = 20(0.2) = 4$

ii) Variance $= npq = 20(0.2)(0.8) = 3.2$

(c) $p(\text{hit target}) = 95\% = 0.95$

$$p(\text{miss}) = 1 - 0.95 = 0.05$$

$$p(x=15) = (1-0.05)^{15-1} (0.05)$$

$$= (0.95)^{14} (0.05)$$

$$\therefore p(x=15) = 0.0244$$

2) Total patients, $n = 60 + 150 + 300 + 580 + 678 + 1288 + 1378 = 4434$

(a) $P(50 \leq x \leq 59)$

$$= \frac{580}{4434}$$

$$= 0.1308$$

(b) $P(x < 50)$

$$= \frac{60 + 150 + 300}{4434}$$

$$= \frac{510}{4434}$$

$$= 0.1150$$

(c) $P(40 \leq x \leq 69)$

$$= \frac{300 + 580 + 678}{4434}$$

$$= \frac{1558}{4434}$$

$$= 0.3514$$

(d) $P(x \geq 70)$

$$= \frac{1288 + 1378}{4434}$$

$$= \frac{2666}{4434}$$

$$= 0.6013$$

3) $n=10, p=\frac{3}{5}=0.6$

(a) $P(X=6) = {}^{10}C_6 (0.6)^6 (1-0.6)^{10-6}$

$$= {}^{10}C_6 (0.6)^6 (0.4)^4$$

$$\therefore P(X=6) = 0.2508$$

(b) $P(X < 9) = 1 - P(X=9) - P(X=10)$

$$P(X=9) = {}^{10}C_9 (0.6)^9 (0.4)^1 = 0.0403$$

$$P(X=10) = {}^{10}C_{10} (0.6)^{10} (0.4)^0 = 0.0060$$

$$\therefore P(X < 9) = 1 - 0.0403 - 0.0060$$

$$P(X < 9) = 0.9537$$

(c) Geometric distribution used.

$$P(X=4) = (1-0.6)^{4-1} (0.6)$$

$$= (0.4)^3 (0.6)$$

$$\therefore P(X=4) = 0.0384$$

4) (a) $\mu=25, \sigma=6$

i) $P(X < 10) = P\left(z < \frac{x-\mu}{\sigma}\right)$

$$= P\left(z < \frac{10-25}{6}\right)$$

$$= P(z < -2.5)$$

$$= P(z > 2.5)$$

$$P(X < 10) = 0.5 - 0.4938$$

$$\therefore P(X < 10) = 0.0062 \approx 0.62\%$$

ii) $P(16 < X < 31)$

$$= P\left(\frac{16-25}{6} < z < \frac{31-25}{6}\right)$$

$$= P(-1.5 < z < 1)$$

$$= 0.4332 + 0.3413$$

$$\therefore P(16 < X < 31) = 0.7745 \approx 77.45\%$$

iii) $P(X > x) = 33\% = 0.33$

$$0.5 - 0.33 = 0.17$$

For area 0.17, $z = 0.44$

$$P(z > 0.44) = 0.33$$

$$\frac{x-\mu}{\sigma} = \frac{x-25}{6} = 0.44$$

$$\therefore \text{Highak's score, } x = 0.44(6) + 25 = 27.64$$

(b) i) Density

$$l = (20-14)h$$

$$h = \frac{1}{6}$$

$$f(x) = \begin{cases} \frac{1}{6}, & 14 < x < 20 \\ 0, & \text{otherwise} \end{cases}$$

ii) $P(X \geq 15) = \text{Area}$

$$= \frac{1}{6} (20-15)$$

$$= \frac{5}{6}$$

$$\therefore P(X \geq 15) = 0.8333$$

- 5) (a) a. x is a binomial random variable, each trial results in one of two mutually exclusive outcomes, which are laptops or desktop models.

b. $n=12, p=60\%=0.6$

$$P(X=4) = {}^{12}C_4 (0.6)^4 (1-0.6)^{12-4} \\ = {}^{12}C_4 (0.6)^4 (0.4)^8$$

$$\therefore P(X=4) = 0.0420$$

c. $P(4 \leq X \leq 7) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$

From b, $P(X=4) = 0.0420$

$$P(X=5) = {}^{12}C_5 (0.6)^5 (0.4)^7 = 0.1009$$

$$P(X=6) = {}^{12}C_6 (0.6)^6 (0.4)^6 = 0.1766$$

$$P(X=7) = {}^{12}C_7 (0.6)^7 (0.4)^5 = 0.2270$$

$$\therefore P(4 \leq X \leq 7) = 0.0420 + 0.1009 + 0.1766 + 0.2270 \\ = 0.5465$$

d. $n=12, p=40\%=0.4$

$$P(X=2) = {}^{12}C_2 (0.4)^2 (1-0.4)^{12-2} \\ = {}^{12}C_2 (0.4)^2 (0.6)^{10}$$

$$\therefore P(X=2) = 0.0639$$

- (b) a. x represents geometry random variable, each trial results in one of two possible outcomes, in error or no error.

b. $x=6, p=3\%=0.03$

$$P(x=6) = (1-0.03)^{6-1} (0.03) \\ = (0.97)^5 (0.03)$$

$$\therefore P(x=6) = 0.0258$$

c. $P(x \leq 5) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$$P(x=1) = (0.97)^0 (0.03) = 0.0300$$

$$P(x=2) = (0.97)^1 (0.03) = 0.0291$$

$$P(x=3) = (0.97)^2 (0.03) = 0.0282$$

$$P(x=4) = (0.97)^3 (0.03) = 0.0274$$

$$P(x=5) = (0.97)^4 (0.03) = 0.0266$$

$$\therefore P(x \leq 5) = 0.03 + 0.0291 + 0.0282 + 0.0274 + 0.0266$$

$$P(x \leq 5) = 0.1413$$

6) (a) $\mu=20, \sigma=2, x=24$

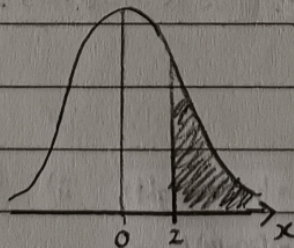
$$P(X \geq 24) = P\left(z \geq \frac{X-\mu}{\sigma}\right)$$

$$= P\left(z \geq \frac{24-20}{2}\right)$$

$$= P(z \geq 2)$$

$$= 0.5 - 0.4772$$

$$\therefore P(X \geq 24) = 0.0228$$



(b) $P(25 \leq X \leq 27)$

$$= P\left(\frac{25-20}{2} \leq z \leq \frac{27-20}{2}\right)$$

$$= P(2.5 \leq z \leq 3.5)$$

$$= 0.4998 - 0.4938$$

$$\therefore P(25 \leq X \leq 27) = 0.0060$$

