SCSI2143: PROBABILITY & STATISTICAL DATA ANALYSIS

CHAPTER 4
(Part 1)

Overview of Probability

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Probability

(Recall from subject SCSI1013 – Discrete Mathematics: Chapter 3 (Part 4 – probability theory)

- Probability – the systematic study of uncertainty.
- Example of activities involve with uncertainty:

Rolling a die

Tossing a coin

Select cards from deck

Chance Experiments

Suppose:

- two six-sided dice is rolled and they both land on sixes.
- a coin is flipped and it lands on heads.

These would be examples of **chance experiments.**

A **chance experiment** is any activity or situation in which there is uncertainty about which of two or more plausible outcomes will result.

Sample Space

Suppose a six-sided dice is rolled. The possible outcomes are that the dice could land with 1 dot up or 2, 3, 4, 5, or 6 dots up.

$$S = \{1, 2, 3, 4, 5, 6\}$$

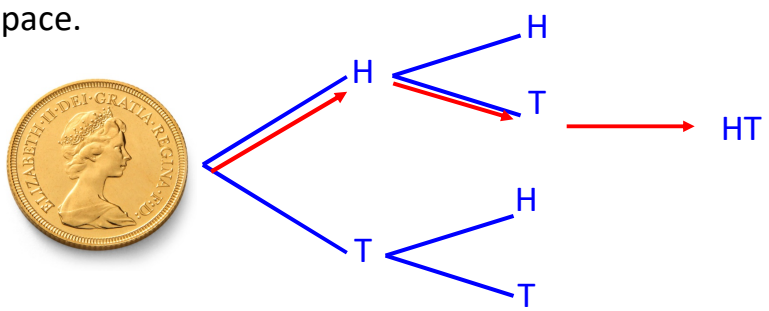
This would be an example of a sample space (S).

The collection of all possible outcomes of a chance experiment is the **sample space** for the experiment.

Suppose two coins are flipped. The sample space would be:

S = {HH, HT, TH, TT} ; H = heads and T = tails

We can also use a tree diagram to represent a sample space.



Event

- Example: The car purchase chance experiment has a sample space:

  S = {MH, FH, MT, FT} ;

  where M=male, F=female, H=hybrid, T= traditional.

- We might focus on a group of outcomes that involve the purchase of a hybrid – the group consisting of (male, hybrid) and (female, hybrid).

- When we combine one or more individual outcomes in a collection, we are creating what is known as **event.**

An **event** is any collection of outcomes from the sample space of a chance experiment.

Complement, Union & Intersection

Let A and B denote two events.

- The event ***not A*** (or **complement of A**) consists of all experimental outcome that are not in event A, denoted by

$$A^c, A', \text{or } \overline{A}$$

- The event ***A or B*** (or **the union of two events**) consists of all experimental outcome that are in at least one of the two events, that is, in A or in B or in both of these, denoted by

$$A \cup B$$

- The event ***A and B*** (or **the intersection of two events**) consists of all experimental outcome that are in both A and B, denoted by

$$A \cap B$$

Example - Complement

Suppose a six-sided dice is rolled. The event that the dice would land on an even number would be

$$E = \{2, 4, 6\}$$

What would the event be that is the dice **NOT** landing on an even number?

$$E^C = \{1, 3, 5\}$$

The **sum** of the probabilities of complementary events equals **ONE**.

Example - Union

Suppose a six-sided dice is rolled. The event that the dice would land on an even number would be

$$E = \{2, 4, 6\}$$

The event that the dice would land on a prime number would be

$$P = \{2, 3, 5\}$$

What would be the event **E or P** happening?

$$E \text{ or } P = \{2, 3, 4, 5, 6\}$$

Example - Intersection

Suppose a six-sided dice is rolled. The event that the dice would land on an even number would be

E = {2, 4, 6}

The event that the dice would land on a prime number would be

P = {2, 3, 5}

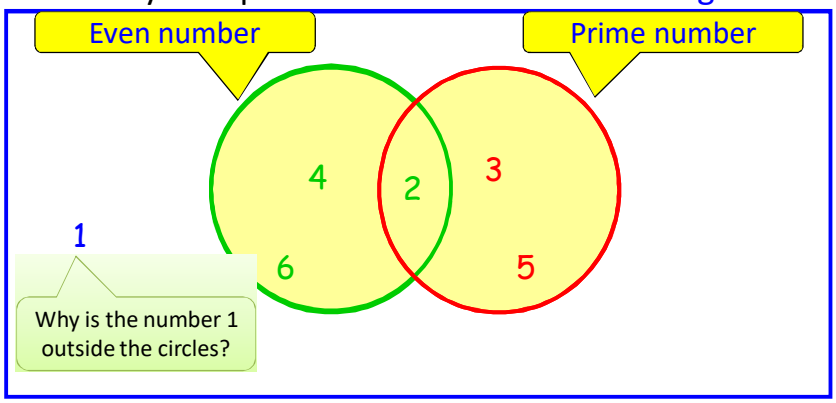
What would be the event **E and P** happening?

E and P = {2}

Venn Diagram

Let's revisit rolling a dice and getting an even or a prime number . . . E or P = {2, 3, 4, 5, 6}

Another way to represent this is with a Venn Diagram.



Approach to Probability

When the outcomes in a sample space are equally likely, the probability of an event E, denoted by ***P*(*E*)**, is the **ratio** of the number of outcomes favorable to E to the total number of outcomes in the sample space.

$$P(E) = \frac{\text{no. of outcome favorable to E}}{\text{no. of outcome in the sample space}}$$

Examples: flipping a coin, rolling a dice, etc.

Probability - Rules

- Fundamental Properties of Probability

Property 1. Legitimate Values

For any event *E*,

$$0 \leq P(E) \leq 1$$

Property 2. Sample space

If S is the sample space,

$$P(S) = 1$$

Property 3. Addition

If two events E and F are disjoint,

$$P(E \text{ or } F) = P(E) + P(F)$$

Property 4. Complement

For any event E,

$$P(E) + P(\text{not } E) = 1$$

Exercise #1

The student council for a school of science and math has one representative from each of the five academic departments: Biology (B), Chemistry (C), Mathematics (M), Physics (P), and Statistics (S). Two of these students are to be randomly selected for inclusion on a university-wide student committee (by placing five slips of paper in a bowl, mixing and drawing out two of them).

a) What are the 10 possible outcomes?
b) From the description of the selection process, all outcomes are equally likely. What is the probability of each event?
c) What is the probability that one of the committee members is the statistics department representative?
d) What is the probability that both committee members come from laboratory science departments?

Exercise #2

A large department store offers online ordering. When a purchase is made online, the customer can select one of four different delivery options: expedited overnight delivery, expedited second-business-day delivery, standard delivery, or delivery to the nearest store for customer pick-up. Consider the chance experiment that consists of observing the selected delivery option for a randomly selected online purchase.

What are the events that make-up the sample space for this experiment?

Suppose that the probability of an overnight delivery selection is 0.1, the probability of a second-day delivery selection is 0.3, and the probability of a standard-delivery selection is 0.4. Find the following probabilities:

i) The probability that a randomly selected online purchase selects delivery to the nearest store for customer pick-up.
ii) The probability that the customer selects a form of expedited delivery.
iii) The probability that either standard delivery or delivery to the nearest store is selected.

SCSI2143: PROBABILITY & STATISTICAL DATA ANALYSIS

CHAPTER 4
(Part 2)

Random Variables & Probability Distribution

Random Variables

- A numerical variable whose value depends on the outcome of a chance experiment.
- It associates a numerical value with each outcome of a chance experiment.
- Two types of random variables
  1) Discrete
  2) Continuous

Discrete Random Variables

- Discrete – its set of possible values is a collection of isolated points along a number line.

This is typically a "count" of something

- Continuous - its set of possible values includes an entire interval on a number line.

This is typically a "measure" of something

Example

Suppose that a counsellor plans to select a random sample of 50 seniors at a large high school and to as each student in the sample whether he or she plans to attend college after graduation. The process of sampling is a chance experiment. The sample space for this experiment consists of all different possible random samples of size 50 that might result (there is a very large number of these) and for simple random sampling, each of these outcomes is equally likely.

Let *x* is the random variable.

- *x* represent the number of successes in the sample (who plans to attend college after graduation).
- Thus, *x* is discrete random variable (counting).

Example

A point is randomly selected on the surface of a lake that has a maximum depth of 100 feet. Let *y* be the depth of the lake at the random chosen point.

- The possible value of *y* : measurement in feed of the depth of the lake.
- Thus, *y* is continuous random variable.

Example

Consider an experiment in which the type of book, print (P) and digital (D), chosen by each of three successive customers making a purchase from on an online bookstore is noted. Define a **random variable** *x* by

$$x = \text{ number of customers purchasing a book in digital format}$$

The experimental outcome can be abbreviated, e.g., DPD (first and third customer purchase a digital book).

Outcome	PPP	DPP	PDP	PPD	DDP	DPD	PDD	DDD
x value	0	1	1	1	2	2	2	3

There are only 4 possible *x* values: 0, 1, 2 and 3. Thus, *x* is discrete random variable.

Probability Distribution For DISCRETE Random Variables

Definition

- Probability distribution is a model that describes the long-run behaviour of a variable.
- The mathematical definition of a discrete probability distribution is : $p(x)$
- If $x = 2$, we write $p(2)$ in place of $p(x=2)$
- $p(5)$ denotes the probability that $x = 5$.

- It give the possibility associated with each possible x value.
- Each probability is the long-run relative frequency of occurrence of the corresponding x value when the chance experiment is performed a very large number of times.
- Common way to display a probability distribution for discrete random variable: table, histogram, formula.

Example

In a Wolf City, regulations prohibit no more than five dogs or cats per household.

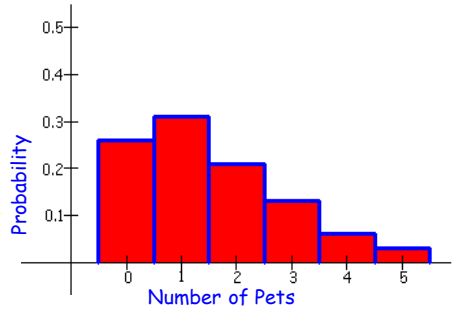
Let, x = the number of dogs and cats in a randomly selected household in Wolf City.

Then, the Department of Animal Control has collected data over the course of several years. They want to estimate the long-run probabilities for the values of x.

- The results as follows:

x	0	1	2	3	4	5
$p(x)$	0.26	0.31	0.21	0.13	0.06	0.03

This is called a discrete probability distribution. It can also be displayed in a histogram with the probability on the vertical axis.



Properties of Discrete Probability Distributions

1) For every possible x value,

$$0 \leq p(x) \leq 1.$$

2) $$\sum_{\text{all } x \text{ values}} p(x) = 1$$

Example

Refer to previous example (Dogs and Cats) .

x	0	1	2	3	4	5
$p(x)$	0.26	0.31	0.21	0.13	0.06	0.03

What is the probability that a randomly selected household in Wolf City has <u>at most 2 pets</u>?

$p(x \leq 2) =$ 0.26 + 0.31 + 0.21 = 0.78

x	0	1	2	3	4	5
$p(x)$	0.26	0.31	0.21	0.13	0.06	0.03

What is the probability that a randomly selected household in Wolf City has <u>less than 2 pets</u>?

$p(x < 2) =$ 0.26 + 0.31 = 0.57

x	0	1	2	3	4	5
$p(x)$	0.26	0.31	0.21	0.13	0.06	0.03

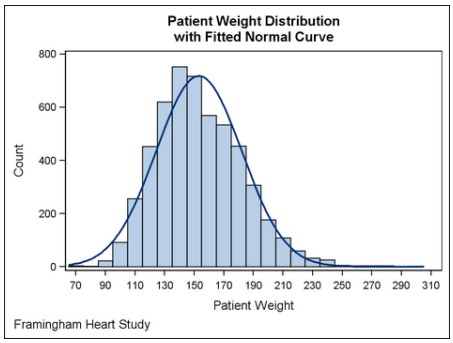
What is the probability that a randomly selected household in Wolf City has <u>more than 1 but no more than 4 pets</u>?

$p(1 < x \leq 4) =$ 0.21 + 0.13 + 0.06 = 0.40

Probability Distribution For CONTINUOUS Variables

Definition : Continuous Random Variable

- A continuous random variables is a random variable where the data can take infinitely many values.
- A continuous random variable "*x*" takes all values in a given interval of numbers.

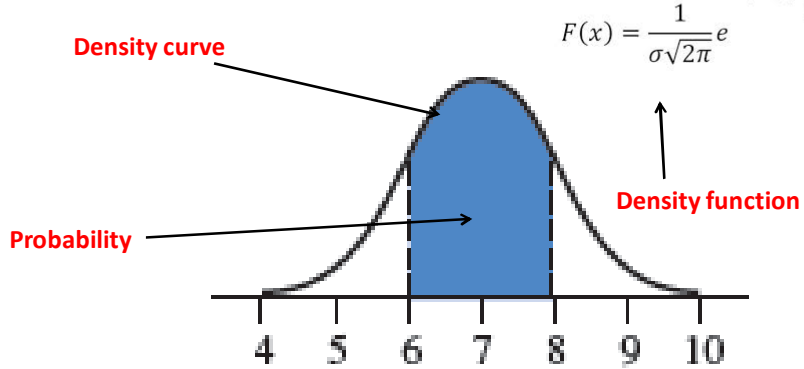


Definition : Probability Distribution for Continuous Random Variable

- It is specified by a curve called a density curve.
- The function that describes this curve is denoted by $f(x)$ and is called the density function.
- The probability of observing a value in a particular interval is the area under the curve and above the given interval.

Normal Probability Density Function

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$



Properties of Continuous Probability Distributions

1. $f(x) \geq 0$ (the curve cannot dip below the horizontal axis).

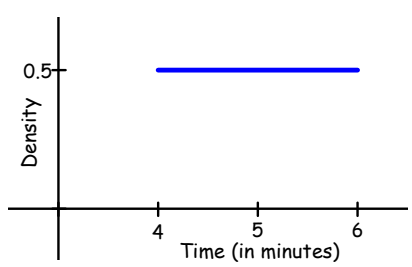
2. The total area under the density curve equals one.

Example

Suppose x is a continuous random variable defined as the amount of time (in minutes) taken by a clerk to process a certain type of application form. Suppose x has a probability distribution with density function:

$$f(x) = \begin{cases} .5 & 4 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

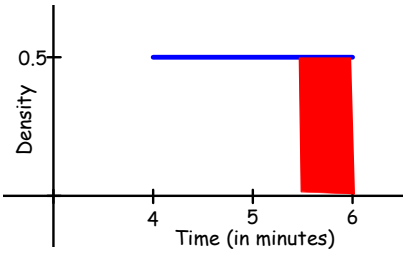
The following is the graph of $f(x)$, the density curve:



Example (cont.)

What is the probability that it takes more than 5.5 minutes to process the application form?

$p(x > 5.5) =$ 0.5(0.5) = 0.25



Other Density Curves

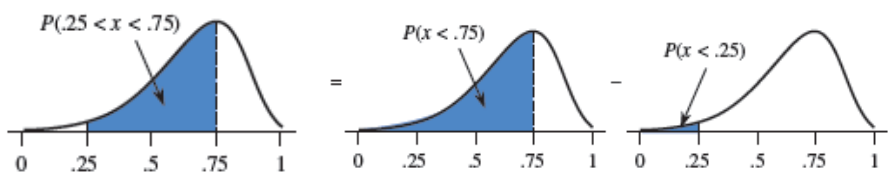
Some density curves resemble the one below. Integral calculus is used to find the area under the these curves.

We will use tables (with the values already calculated).

The probability that a continuous random variable x lies between a lower limit a and an upper limit b is

$p(a < x < b)$ = (cumulative area to the left of b) – (cumulative area to the left of a)

$$p(a < x < b) = p(x < b) - p(x < a)$$



- Other continuous distribution

We will use tables for certain distribution (with the values already calculated).

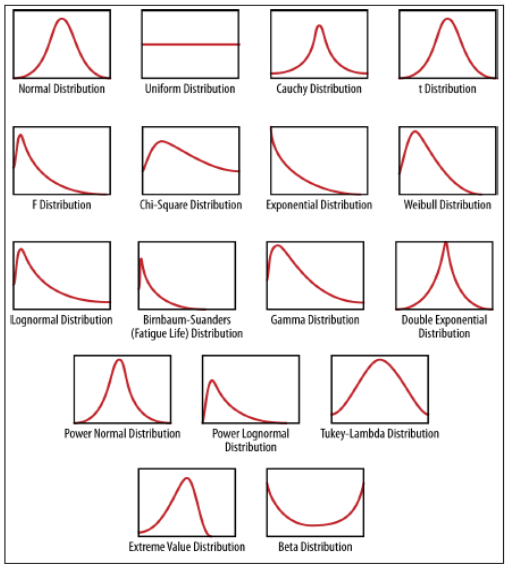


Figure 2-1. A bunch of continuous density functions (aka probability distributions)

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CHAPTER 4
(Part 3)

Discrete & Continuous Probability Distribution

Special Distributions

- Discrete Distribution:
  - Binomial
  - Geometric
- Continuous Distribution:
  - Normal

Discrete Distribution

Binomial Distribution

Suppose we decide to record the gender of the next 25 newborns at a particular hospital.

What is the chance that at least 15 are female?

What is the chance that between 10 and 15 are female?

Out of the 25 newborns, how many can we expect to be female?

Binomial Distribution (cont.)

Properties of a Binomial Experiment:

1. There are a fixed number of trials.
2. Each trial results in one of two mutually exclusive outcomes. (success/failure).
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials.

The binomial random variable *x* is defined as

x = the number of successes observed when a binomial experiment is performed.

Binomial Distribution (cont.)

Binomial Probability function:

Let

n = number of independent trials in a binomial experiment

p = constant probability that any trial results in a success

$$p(x) = P(x \text{ success among } n \text{ trials})$$

$$= \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \quad , x = 0, 1, 2, .., n$$

$$or, \quad p(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad , x = 0, 1, 2, .., n$$

$$or, \quad p(x) = {}_nC_x p^x (1-p)^{n-x} \quad , x = 0, 1, 2, .., n$$

Binomial Distribution (cont.)

Formulas for mean and standard deviation of a binomial distribution:

$$\text{mean: } \mu_x = \sum_x x p(x)$$

$$\text{variance: } \sigma_x^2 = \sum_x (x - \mu_x)^2 p(x)$$

or,

$$\text{mean: } \mu_x = np$$

$$\text{std. dev: } \sigma_x = \sqrt{np(1-p)}$$

Example

A coin is tossed four times. Find the mean and variance of the number of heads that will be obtained? Given the distribution as follows.

No of head, *x*	0	1	2	3	4
Probability	1/16	4/16	6/16	4/16	1/16

Solution (1):

$$\mu_x = \sum_x x p(x)$$
$$= 0(1/16) + 1(4/16) + .. + 4(1/16)$$
$$= 2$$

$$\sigma_x^2 = \sum_x (x - \mu_x)^2 p(x)$$
$$= ((0-2)^2(1/16)) + ((1-2)^2(4/16)) + .. + ((4-2)^2(1/16))$$
$$= 1$$

Solution (2):

$$n = 4, \; p = \frac{1}{2}, \; q = 1 - p = \frac{1}{2}$$

$$\mu = np = 4\left(\frac{1}{2}\right) = 2$$

$$\sigma_x^2 = npq = 4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1$$

Example

The probability that a patient fails to recover from a particular operation is 0.1. Suppose that eight (8) patients having this operation are selected at random. Answer the following questions:

i) What is the probability that at most one patient will not recover?

ii) What is the probability that at least two (2) but no more than three (3) patients will not recover?

iii) What is the probability that all patients will not recover?

iv) What is the expected number of patients that will not recover?

Example (Solution)

i) What is the probability that at most one patient will not recover?

$$p(x \le 1) = p(x=0) + p(x=1)$$

$$p(x=0) = 8C0(0.10)^0(1-0.10)^{8-0} = 0.4305$$

$$p(x=1) = 8C1(0.10)^1(1-0.10)^{8-1} = 0.3826$$

$$\therefore 0.4305 + 0.3826 = 0.8131$$

ii) What is the probability that at least two (2) but no more than three (3) patients will not recover?

$$p(2 \le x \le 3) = p(x=2) + p(x=3)$$

$$p(x=2) = 8C2(0.10)^2(1-0.10)^{8-2} = 0.1488$$

$$p(x=3) = 8C3(0.10)^3(1-0.10)^{8-3} = 0.03307$$

$$\therefore 0.1488 + 0.03307 = 0.18187$$

iii) What is the probability that all patients will not recover?

$$p(x=8) = 8C8(0.10)^8(1-0.10)^{8-8} = 0.00000001$$

iv) What is the expected number of patients that will not recover?

$$np = 8(0.1) = 0.8 \approx 1 \text{ patient}$$

Exercise #3

Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

a) Why this is Binomial distribution?

b) What is the probability that of 6 randomly selected patients, 4 will recover?

Geometric Distribution

Properties of Geometric Distributions:

1. The trial is independent.
2. Each trial can result in one of two possible outcomes, success and failure.
3. The probability of success is the same for all trials.

A geometric random variable x is defined as

x = number of trials UNTIL the FIRST success is observed (including the success trial).

Geometric Distribution (cont.)

Geometric distribution is in a sense **INFINITE.** They're asking you what is the probability **UNTIL** the first success happens.

Example:

Malik throws a dice until he gets a '4'.

What is the probability of him getting the first '4' in the 6th throw? How to calculate the probability?

Geometric Distribution (cont.)

Geometric Probability Distribution function:

If *x* is a random variable with probability of success = *p* for each trial, then

$$p(x) = (1-p)^{x-1} p, \quad x = 1, 2, 3, \ldots.$$

Geometric Distribution (cont.)

Formulas for mean and standard deviation of a geometric distribution:

$$\text{mean: } \mu_x = \frac{1}{p}$$

$$\text{std. dev: } \sigma_x = \sqrt{\frac{(1-p)}{p^2}}$$

Example

A product produced by a machine has a 3% defective rate. What is the probability that the first defective occurs in the fifth(5th) item inspected?

Solution:

$$p(x=5) = (1-0.03)^{5-1}(0.03)$$
$$= 0.027$$

Exercise #4

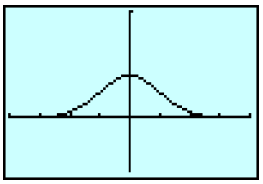
It is claimed that 15% of the ducks in a particular region have patent schistosome infection. Suppose that seven ducks are selected at random. Let X equals the number of ducks that are infected.

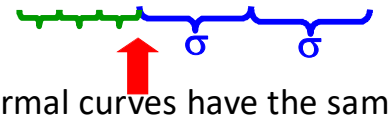
a) Assuming independence trials, how is X distributed?
b) Find the probability that:
   i) At most two ducks are infected
   ii) One duck is not infected
c) A new drug is proposed to treat the schistosome infection. Therefore, the infected duck is randomly selected. What is the probability that the first duck infected is found at the fifth trial?

Continuous Distribution

Continous Distribution

- Symmetrical bell-shaped (unimodal) density curve defined by *m* and *s*
- Area under the curve equals 1.
- Probability of observing a value in a particular interval is calculated by finding the area under the curve.
- As *s* increases, the curve flattens & spreads out.
- As *s* decreases, the curve gets taller and thinner.



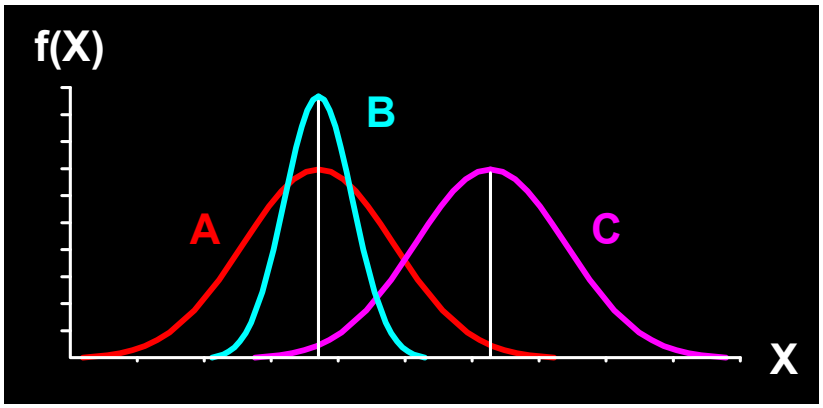


Do these two normal curves have the same mean? If so, what is it? YES

Which normal curve has a standard deviation of 3? B

Which normal curve has a standard deviation of 1? A

Effect of Varying Parameters (μ & σ)



Notation

x is N(μ,σ)

The random variable *x* has a normal distribution (N) with mean, μ and standard deviation, σ.

Example:

x is N(40,1)

x is N(10,5)

x is N(50,3)

Probability Density Function

Normal Distribution

Parameters: μ and σ

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}$$

$$for\ -\infty < x < \infty$$

x = value of random variable (-∞ < x < ∞)

∞ = Population Standard Deviation

π = 3.14159

e = 2.71828

∞ = Mean of Random Variable x

Normal Curve

Notice that the normal curve is <u>curving downwards</u> from the center (mean) to points that are <u>one standard deviation</u> on either side of the mean. At those points, the normal curve begins to <u>turn upward</u>.

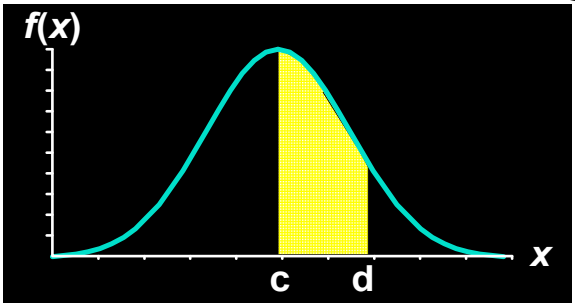
Standard Normal Distribution

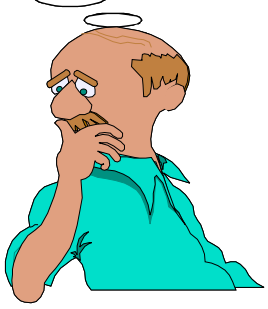
- Is a normal distribution with $\mu = 0$ and $\sigma = 1$
- It is customary to use the letter *z* to represent a variable whose distribution is described by the <u>standard normal curve</u> (or *z* curve).

Normal Distribution Probability

Probability is area under curve!

$$P(c \le x \le d) = \int_c^d f(x)\,dx ?$$

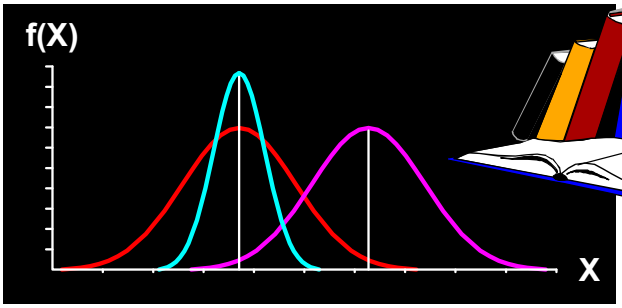




Infinite Number of Distribution Tables

Normal distributions differ by mean & standard deviation.

Each distribution would require its own table.

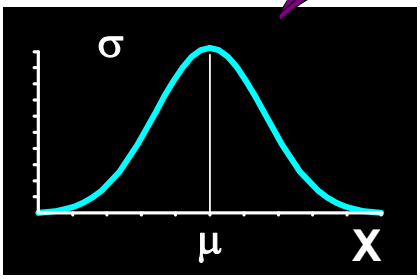


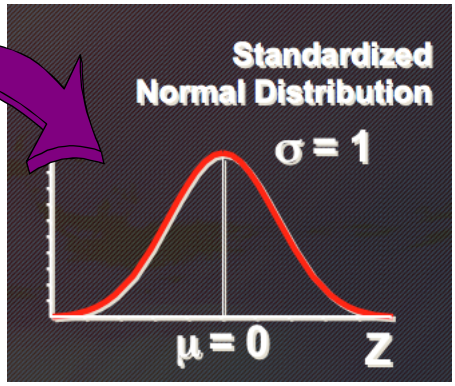
That's an *infinite* number!

Standardize the Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$

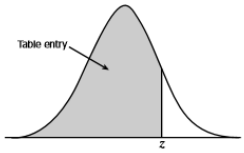
Normal Distribution





One table!

There are 2 type of *Z* distribution tables:

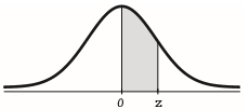


Full *Z* table

The probability area is calculated from the very left area

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141



Half *Z* table

The probability area is calculated from the center area

TABLE A-2 Standard Normal (z) Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141

For full *Z* distribution tables, there are two different tables representing the area:

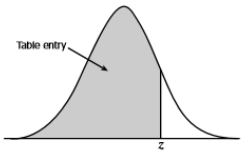


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141

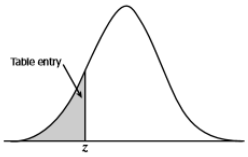


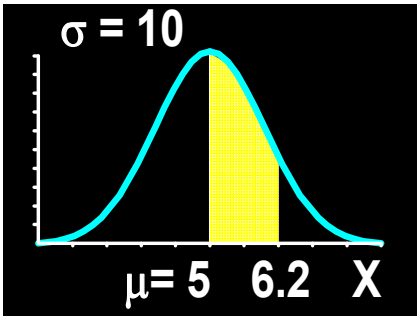
Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005

Example

What is the probability of P(X=6.2), given that the mean is 5 and std. deviation is 10?.

Normal Distribution

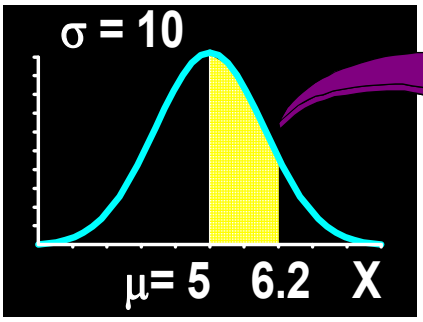


Example -Solution

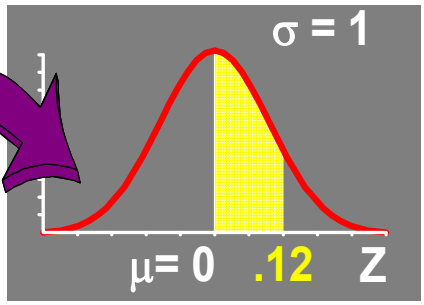
Step 1: Find the Z value.

$$Z = \frac{X-\mu}{\sigma} = \frac{6.2-5}{10} = 0.12$$

Normal Distribution

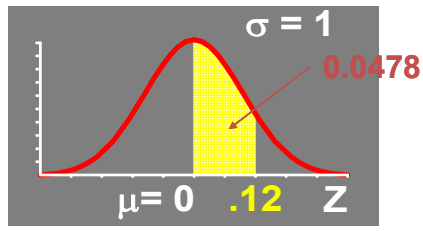


Standardize Normal Distribution



Note: Shaded area exaggerated.

Step 2: Refer to standard Normal Dist. Table to locate the Z value.



Standardized Normal Probability Table (Portion)

Full *Z* Table

z	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700

Half *Z* Table

z	.00	.01	.02	.03
0.0	.0000	.0040	.0080	.0120
0.1	.0398	.0438	.0478	.0517
0.2	.0793	.0832	.0871	.0910
0.3	.1179	.1217	.1255	.1293
0.4	.1554	.1591	.1628	.1664

P(Z = 0.12) = 0.5478 - 0.5 = 0.0478

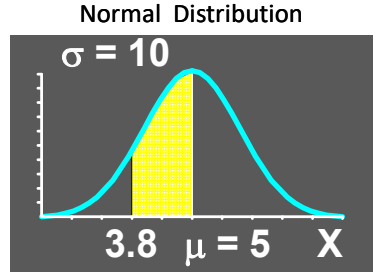
Example: Find $P(3.8 \le X \le 5)$

$$Z = \frac{X-\mu}{\sigma} = \frac{3.8-5}{10} = -.12$$

Full *Z* Table

z	.00	.01	.02	.03
−3.4	.0003	.0003	.0003	.0003
−3.3	.0005	.0005	.0005	.0004
...	...	...	...	...
−0.2	.4207	.4168	.4129	.4090
−0.1	.4602	.4562	.4522	.4483
−0.0	.5000	.4960	.4920	.4880

Normal Distribution

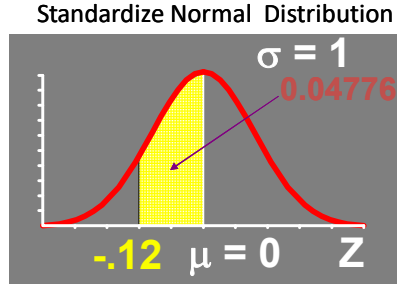


P(Z=0.12) = 0.5 – 0.4522
= 0.0478

Half *Z* Table

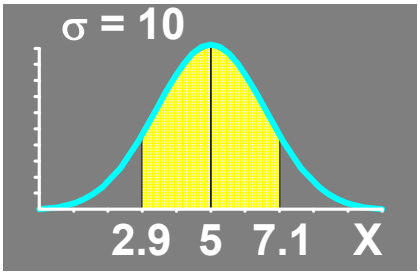
z	.00	.01	.02	.03
0.0	.0000	.0040	.0080	.0120
0.1	.0398	.0438	.0478	.0517
0.2	.0793	.0832	.0871	.0910
0.3	.1179	.1217	.1255	.1293
0.4	.1554	.1591	.1628	.1664

Standardize Normal Distribution



Example: Find $P(2.9 \le X \le 7.1)$

Normal Distribution



$$Z = \frac{X-\mu}{\sigma} = \frac{2.9-5}{10} = -.21$$

$$Z = \frac{X-\mu}{\sigma} = \frac{7.1-5}{10} = .21$$

Example $P(2.9 \le X \le 7.1)$

$$Z = \frac{X-\mu}{\sigma} = \frac{2.9-5}{10} = -0.21$$

Full *Z* Table

z	.00	.01	.02	.03
−3.4	.0003	.0003	.0003	.0003
−3.3	.0005	.0005	.0005	.0004
...	...	...	...	...
−0.2	.4207	.4168	.4129	.4090
−0.1	.4602	.4562	.4522	.4483
−0.0	.5000	.4960	.4920	.4880

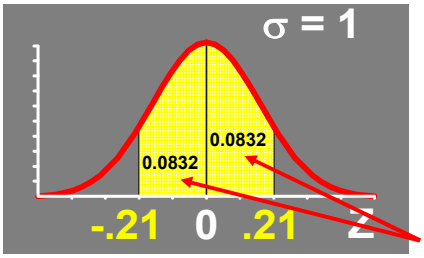
P(Z = - 0.21) = 0.5 – 0.4168 = 0.0832

$$Z = \frac{X-\mu}{\sigma} = \frac{7.1-5}{10} = 0.21$$

Full *Z* Table

z	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700

P(Z = 0.21) = 0.5832 – 0.5= 0.0832



Half *Z* Table

z	.00	.01	.02	.03
0.0	.0000	.0040	.0080	.0120
0.1	.0398	.0438	.0478	.0517
0.2	.0793	.0832	.0871	.0910
0.3	.1179	.1217	.1255	.1293
0.4	.1554	.1591	.1628	.1664

0.0832 + 0.0832 = 0.1664

Example: Find P(*X* ≥ 8)

$$Z = \frac{X-\mu}{\sigma} = \frac{8-5}{10} = .30$$

Full *Z* Table

z	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700

Half *Z* Table

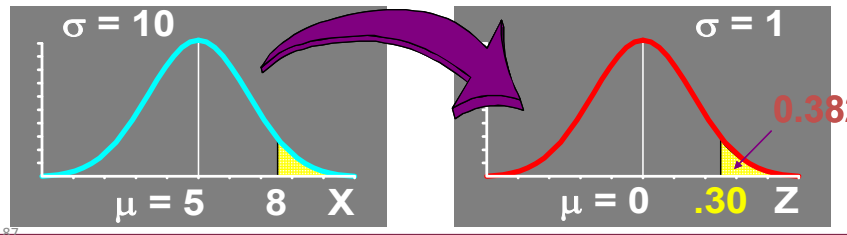
z	.00	.01	.02	.03
0.0	.0000	.0040	.0080	.0120
0.1	.0398	.0438	.0478	.0517
0.2	.0793	.0832	.0871	.0910
0.3	.1179	.1217	.1255	.1293
0.4	.1554	.1591	.1628	.1664

P(Z = 0.30) = 1 - 0.6179 = 0.3821

P(Z = 0.30) = 0.5 - 0.1179 = 0.3821

Normal Distribution

Standardize Normal Distribution



Example: Find P(7.1 ≤ *X* ≤ 8)

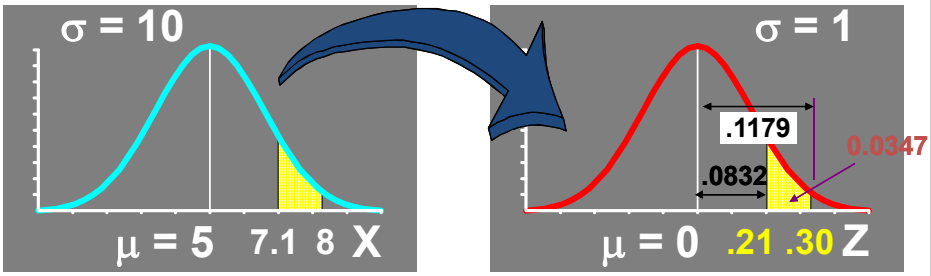
$$Z = \frac{X-\mu}{\sigma} = \frac{7.1-5}{10} = 0.21$$ P(Z = 0.21) = 0.0832

$$Z = \frac{X-\mu}{\sigma} = \frac{8-5}{10} = 0.30$$ P(Z = 0.30) = 0.1179

P(0.21 ≤ *Z* ≤ 0.30) = 0.1179 – 0.0832 = 0.0347

Normal Distribution

Standardize Normal Distribution

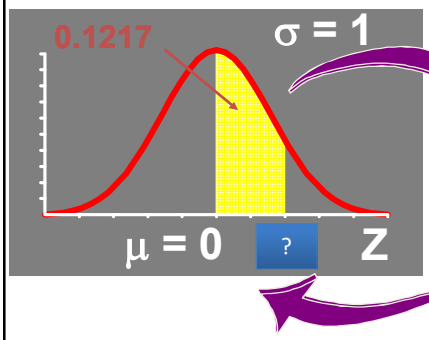


Finding Z Values for Known Probabilities

If Z greater than mean, what is Z given that P(Z) = 0.1217?

P(Z=?) = a - 0.5 = 0.1217 => 0.1217 + 0.5 = 0.6217 **Full *Z* Table**

Standardize Normal Distribution



z	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700

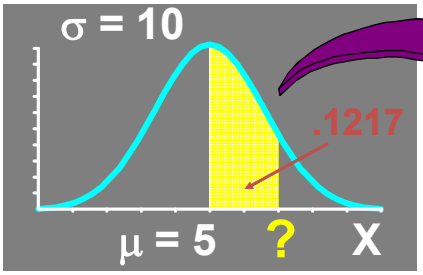
Half *Z* Table

z	.00	.01	.02	.03
0.0	.0000	.0040	.0080	.0120
0.1	.0398	.0438	.0478	.0517
0.2	.0793	.0832	.0871	.0910
0.3	.1179	.1217	.1255	.1293
0.4	.1554	.1591	.1628	.1664

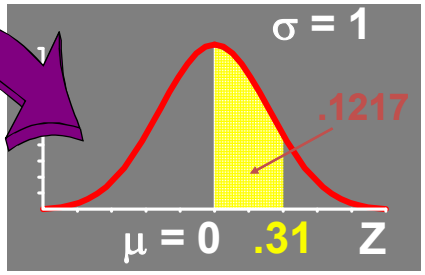
Note:
If Z less than mean, than Z value has negative sign.

Finding *X* Values for Known Probabilities

Normal Distribution



Standardize Normal Distribution



$$X = \mu + Z \cdot \sigma = 5 + (.31)(10) = 8.1$$

Exercise #5

1. X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

a) $P(X \leq 40)$

b) $P(X \geq 21)$

c) $P(18 \leq X \geq 35)$

2. A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hour and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hour?

Exercise #6

A lecturer commutes daily from his suburban home to his midtown university. The average time for a one-way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume that the distribution of trip times to be normally distributed.

a) What is the probability that a trip will take at least ½ hours?

b) If the office open at 9.00 A.M and the lecturer leaves his house at 8.45 A.M daily, what percentage of the time is he late for work?

c) If he leaves the house at 8.35 A.M and coffee is served at the office from 8.50 A.M until 9.00 A.M. what is the probability that he missed coffee?

d) Due to unforeseen incident, the journey takes longer than usual. If 15% is considered as unusual journey time, what is the length of unusual journey time.

e) Using Binomial distribution, find the probability that 2 of the next 3 trips will take at least ½ hour.