

# Two's Complement Multiplication

1

- There are a couple of ways of doing two's complement multiplication by hand.
- Remember that the result can require 2 times as many bits as the original operands.

## 2

- 
- Diagram illustrating the bit-shifting process for multiplying 11011101 by 00000010:
- ```

      00000010
    x 11011101
    -----
      00000010
      00000000
      00000010
      00000010
      00000010
      00000010
      00000010
      00000010
      00000010
      -----
      00000011111010
  
```
- The final result is 00000011111010, where the last four bits (1111010) are highlighted in red.

## #2: " Partial Product Sign Extend Method " for Two's Complement Multiplication

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- Another way is to *sign extend the partial products* to the correct number of bits.
- Example 1:  $(-4) \times 3$

**-4 = 1100**

**3 = 0011**

Sign extend to the correct number of bits → 8

|       |                |
|-------|----------------|
|       | 1100           |
| x     | 0011           |
| <hr/> |                |
|       | 11111100       |
|       | 1111100        |
|       | 000000         |
| <hr/> |                |
|       | 11110100 (-12) |

## #2: "Partial Product Sign Extend Method" for Two's Complement Multiplication

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- Sometimes we do have to make some adjustments (#1).

If (+ve) x (+ve) then  
**OK**

Normal stuff

If (-ve) x (+ve) then  
**Sign extend partial products**

Like the slide before (-4)x3

## #2: " Partial Product Sign Extend Method " for Two's Complement Multiplication

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- Sometimes we do have to make some adjustments (#1).

If (+ve) x (+ve) then  
OK

If (-ve) x (+ve) then

3=0011 ; -4 = 1100

-3=1101; 4 =0100

If (+ve) x (-ve) then  
get additive inverse  
of both

And then

Sign extend partial  
products

Example: 3 x (-4)

1101  
x 0100

00000000

00000000

111101

11110100 (-12)

## #2: " Partial Product Sign Extend Method " for Two's Complement Multiplication

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- Sometimes we do have to make some adjustments (#2).

If (+ve) x (+ve) then  
**OK**

-3=1101

3 = 0011

x

0011

0011

0011

0011

01001 (9)

If (-ve) x (+ve) then  
**Sign extend partial products**

If (-ve) x (-ve) then  
**get additive inverse of both**

**Example: (-3)x(-3)**

## #2: " Partial Product Sign Extend Method " for Two's Complement Multiplication

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- ... some adjustments.

|                                                                                                                                     |                                                                                         |
|-------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| If (+ve) x (+ve) then<br>OK                                                                                                         | If (-ve) x (+ve) then<br>Sign extend partial<br>products                                |
| If (+ve) x (-ve) then<br>get additive inverse<br>of both<br>And then<br>Sign extend partial<br>products<br>Example: $3 \times (-4)$ | If (-ve) x (-ve) then<br>get additive inverse<br>of both<br>Example: $(-3) \times (-3)$ |

# Hardware implementation: multiplication & division

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**REFER TO:  
CHAPTER 3 , PAGE 184**

## COMPUTER ORGANIZATION AND DESIGN

THE HARDWARE/SOFTWARE INTERFACE

FIFTH EDITION

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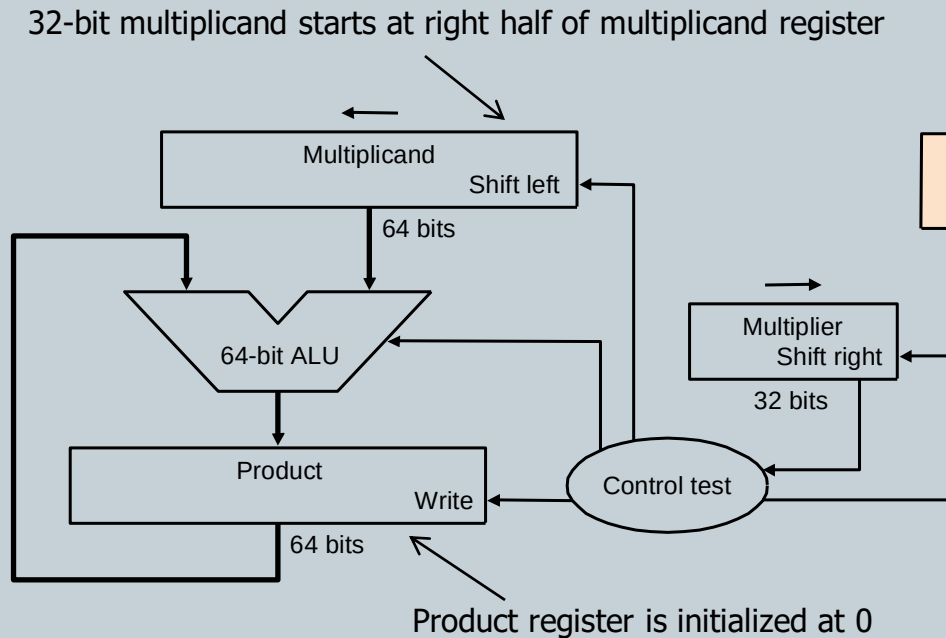
MK  
MOSKOWITZ & KUTNER



# 1st Version of Multiplication Hardware

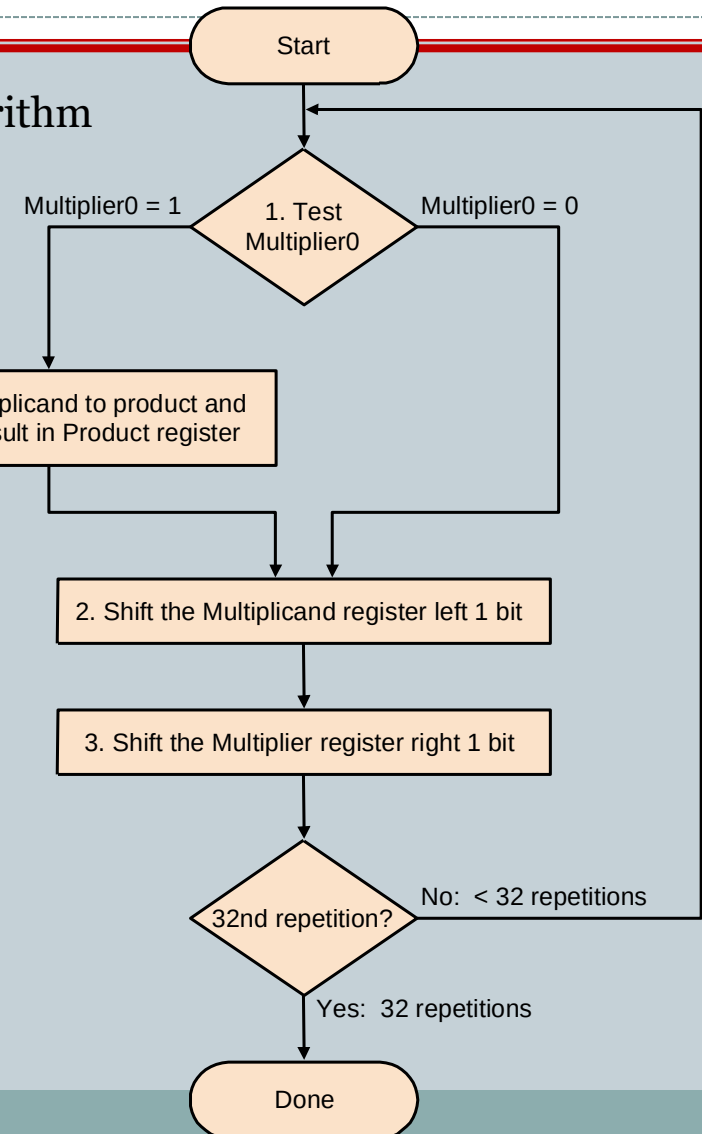
9

## Flows of 1st Version Multiplication



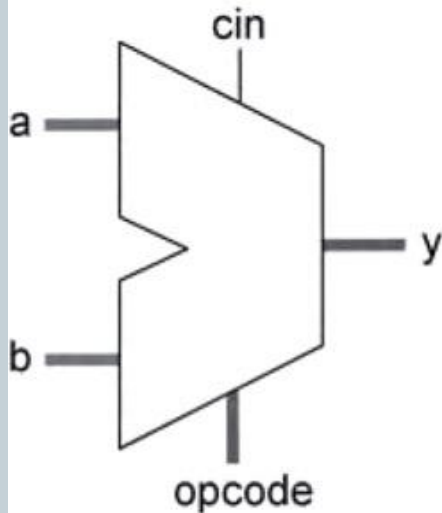
Multiplicand register, product register, ALU are 64-bit wide; multiplier register is 32-bit wide

### Algorithm



# Example of ALU

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| Unit       | Instruction    | Operation     | opcode |
|------------|----------------|---------------|--------|
| Logic      | Transfer a     | $y = a$       | 0000   |
|            | Complement a   | $y = a'$      | 0001   |
|            | Transfer b     | $y = b$       | 0010   |
|            | Complement b   | $y = b'$      | 0011   |
|            | AND            | $y = a.b$     | 0100   |
|            | NAND           | $y = (a.b)'$  | 0101   |
|            | OR             | $y = a+b$     | 0110   |
|            | NOR            | $y = (a+b)'$  | 0111   |
| Arithmetic | Increment a    | $y = a+1$     | 1000   |
|            | Increment b    | $y = b+1$     | 1001   |
|            | Add a and b    | $y = a+b$     | 1010   |
|            | Sub b from a   | $y = a-b$     | 1011   |
|            | Sub a from b   | $y = -a+b$    | 1100   |
|            | Add negative   | $y = -a-b$    | 1101   |
|            | Add with 1     | $y = a+b+1$   | 1110   |
|            | Add with carry | $y = a+b+cin$ | 1111   |

# Example of Multiplication – 4 bits

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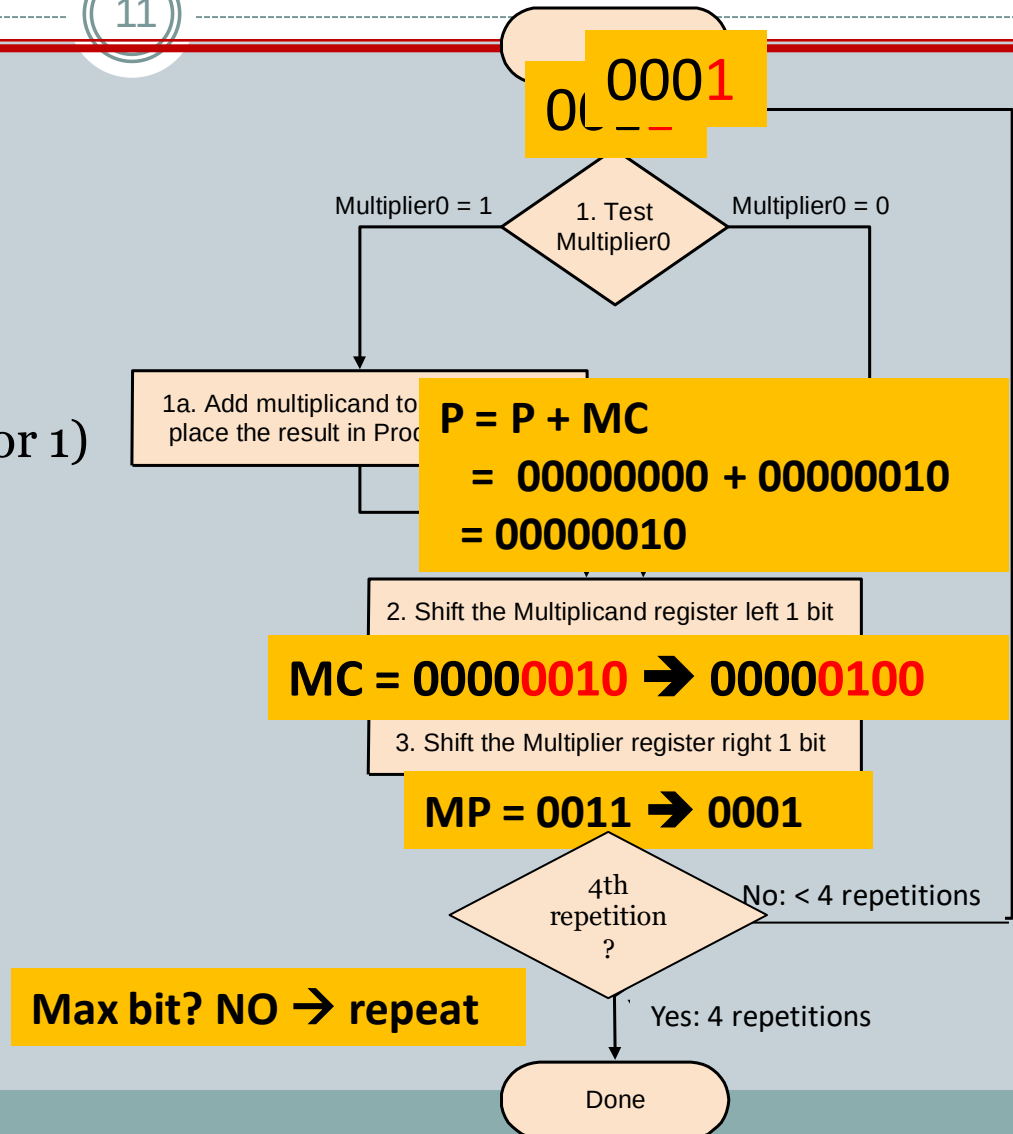
- Example :  $2 \times 3 = ?$

Multiplicand  
(MC)

Multiplier  
(MP)

Product  
(P)

- $2 \times 3 \rightarrow 0010 \times 0011$
- Steps:
  - 1a – test LSB for multiplier (0 or 1)
    - If 1 then  $P = P + MC$
    - If 0 then no operation
  - 2 – shift MC left
  - 3 – shift MP right
  - All bits done?
    - If still < max bit, repeat
    - If = max bit, stop



2 x 3 (0010 x 0011)

| Iteration | Step                | Multiplier (MP) | Multiplicand (MC) | Product (P) |
|-----------|---------------------|-----------------|-------------------|-------------|
| 0         | Initial value       | 0011            | 0000 0010         | 0000 0000   |
| 1         | 1a:1 → P = P + MC   |                 |                   | 0000 0010   |
|           | 2: Shift MC left    |                 | 0000 0100         |             |
|           | 3: Shift MP right   | 0001            |                   |             |
| 2         | 1a:1 → P = P + MC   |                 |                   | 0000 0110   |
|           | 2: Shift MC left    |                 | 0000 1000         |             |
|           | 3: Shift MP right   | 0000            |                   |             |
| 3         | 1a:0 → no operation |                 |                   |             |
|           | 2: Shift MC left    |                 | 0001 0000         |             |
|           | 3: Shift MP right   | 0000            |                   |             |
| 4         | 1a:0 → no operation |                 |                   |             |
|           | 2: Shift MC left    |                 | 0010 0000         |             |
|           | 3: Shift MP right   | 0000            |                   |             |

Try with 5 x 4

| Iteration | Example : 5 x 4     | Multiplier (MP) 4 | Multiplicand (MC) 5 | Product (P) |
|-----------|---------------------|-------------------|---------------------|-------------|
| 0         | Initial value       | 0100              | 0000 0101           | 0000 0000   |
| 1         | 1a:0 → no operation |                   |                     |             |
|           | 2: Shift MC left    |                   | 0000 1010           |             |
|           | 3: Shift MP right   | 0010              |                     |             |
| 2         | 1a:0 → no operation |                   |                     |             |
|           | 2: Shift MC left    |                   | 0001 0100           |             |
|           | 3: Shift MP right   | 0001              |                     |             |
| 3         | 1a:1 → P = P + MC   |                   |                     | 0001 0100   |
|           | 2: Shift MC left    |                   | 0010 1000           | 20          |
|           | 3: Shift MP right   | 0000              |                     |             |
| 4         | 1a:0 → no operation |                   |                     |             |
|           | 2: Shift MC left    |                   | 0101 000            |             |
|           | 3: Shift MP right   | 0000              |                     |             |

Next try with 2 x (-3)

Example :  $2 \times (-3) \rightarrow$  get additive inverse of both

| Iteration | Step                            | (MP) <sup>1</sup> 3 | (MC) <sup>1</sup> -2 | Product (P) |
|-----------|---------------------------------|---------------------|----------------------|-------------|
| 0         | Initial value                   | 0011                | 1111 1110            | 0000 0000   |
| 1         | 1a:1 $\rightarrow P = P + MC$   |                     |                      | 1111 1110   |
|           | 2: Shift MC left                |                     | 1111 1100            |             |
|           | 3: Shift MP right               | 0001                |                      |             |
| 2         | 1a:1 $\rightarrow P = P + MC$   |                     |                      | 1111 1010   |
|           | 2: Shift MC left                |                     | 1111 1000            | -6          |
|           | 3: Shift MP right               | 0000                |                      |             |
| 3         | 1a:0 $\rightarrow$ no operation |                     |                      |             |
|           | 2: Shift MC left                |                     | 1111 0000            |             |
|           | 3: Shift MP right               | 0000                |                      |             |
| 4         | 1a:0 $\rightarrow$ no operation |                     |                      |             |
|           | 2: Shift MC left                |                     | 1110 0000            |             |
|           | 3: Shift MP right               | 0000                |                      |             |

# Binary Division



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|         |                     |  |                        |           |
|---------|---------------------|--|------------------------|-----------|
|         |                     |  | 1001 <sub>ten</sub>    | Quotient  |
| Divisor | 1000 <sub>ten</sub> |  | 1001010 <sub>ten</sub> | Dividend  |
|         |                     |  | -1000                  |           |
|         |                     |  | 10                     |           |
|         |                     |  | 101                    |           |
|         |                     |  | 1010                   |           |
|         |                     |  | -1000                  |           |
|         |                     |  | 10 <sub>ten</sub>      | Remainder |

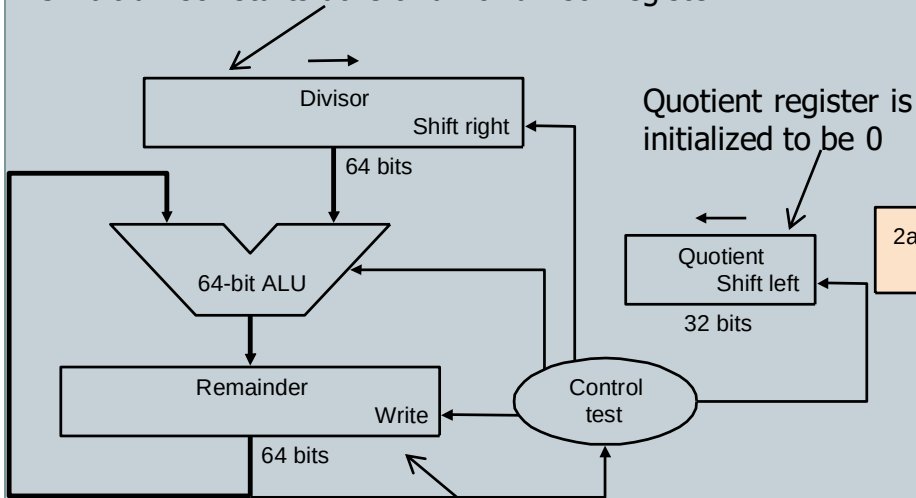
$$\text{Dividend} = \text{Quotient} \times \text{Divisor} + \text{Remainder}$$

# 1st Version of Division Hardware

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**Divisor starts at left half  
of divisor register**

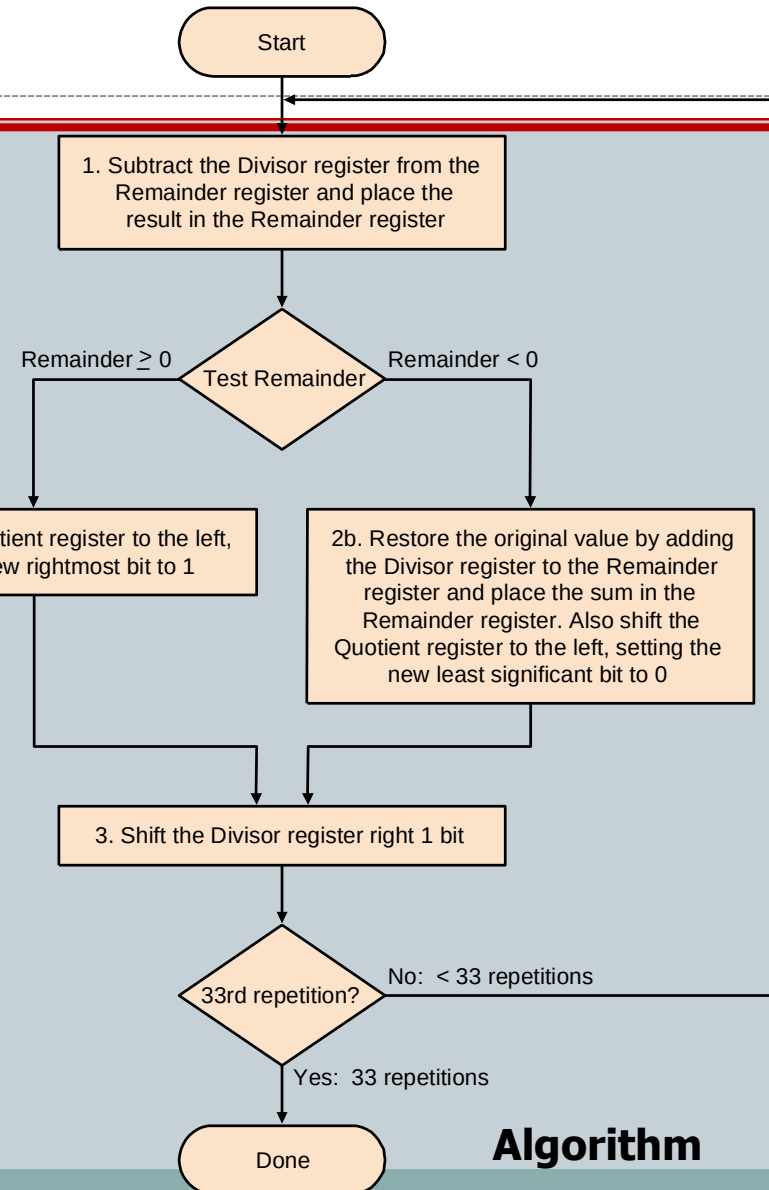
32-bit divisor starts at left half of divisor register



**Remainder initialized with dividend at right**

**Divisor register, remainder register, ALU are  
64-bit wide; quotient register is 32-bit wide**

## Flows of 1st Version Division



**Algorithm**



# Example #1

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$$7 \div 2$$

# Example of Division – 4 bits

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• Example :  $7 / 2 = ?$

Dividend  
(DD)

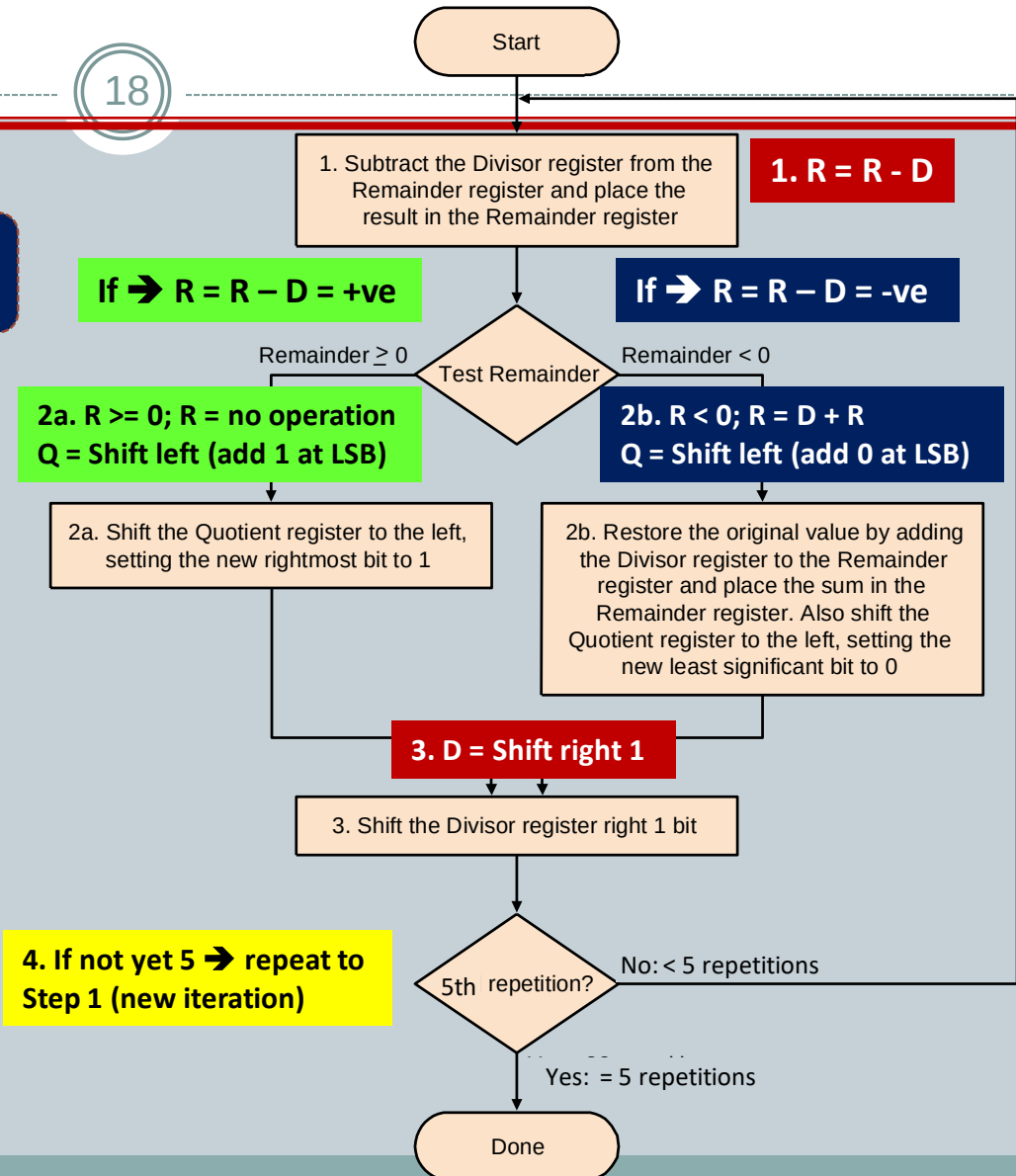
Divisor  
(D)

Quotient  
(Q)

•  $7 / 2 \rightarrow 0111 / 0010$

Steps:

- 1 – Remainder (R) = R – D
- 2 – test new **R** ( **$\geq 0$  or  $< 0$** )
- 2a - If  **$\geq 0$**  then
  - R = no operation;
  - Q = Shift left (add 1 at LSB)
- 2b - If  **$< 0$**  then
  - R = D + R
  - Q = Shift left (add 0 at LSB)
- 3 – shift D right
- All bits done?
  - If still  **$< (\text{max bit} + 1)$** , repeat
  - If  **$= (\text{max bit} + 1)$** , stop



| Iteration | Example:<br>7 ÷ 2<br>(0111 ÷ 0010) | Step                 | Quotient (Q)                   | Divisor (D)                                     | Remainder (R)                               |
|-----------|------------------------------------|----------------------|--------------------------------|-------------------------------------------------|---------------------------------------------|
|           |                                    |                      | Quotient initialized with zero | Divisor starts at left half of divisor register | Reminder initialized with dividend at right |
| 0         | Initial value                      |                      | 0000                           | 0010 0000                                       | 0000 0111                                   |
| 1         | 1. R = R - D                       | R = R - D = R + (-D) |                                | Negate 0010 0000<br>→ 1110 0000                 | 1110 0111                                   |
|           | 2b. R < 0; R = D + R               |                      |                                |                                                 | 0000 0111                                   |
|           | Q = Shift left (add 0 at LSB)      |                      |                                |                                                 | 1110 0000 +<br>-----                        |
|           | 3. D = Shift right                 |                      |                                |                                                 | 1110 0111                                   |
| 2         | 1. R = R - D                       |                      |                                |                                                 |                                             |
|           | 2b. R < 0; R = D + R               |                      |                                |                                                 |                                             |
|           | Q = Shift left (add 0 at LSB)      |                      |                                |                                                 |                                             |
|           | 3. D = Shift right                 |                      |                                |                                                 |                                             |
|           |                                    |                      |                                |                                                 |                                             |
|           |                                    |                      |                                |                                                 |                                             |
|           |                                    |                      |                                |                                                 |                                             |
|           |                                    |                      |                                |                                                 |                                             |

| Iteration | Example:<br>7 ÷ 2<br>(0111 ÷ 0010) | Step                                                                                                              | Quotient<br>(Q) | Divisor<br>(D)                  | Remainder<br>(R)         |
|-----------|------------------------------------|-------------------------------------------------------------------------------------------------------------------|-----------------|---------------------------------|--------------------------|
| 0         |                                    | Initial value                                                                                                     | 0000            | 0010 0000                       | 0000 0111                |
| 1         |                                    | 1. R = R - D                                                                                                      |                 |                                 | 1110 0111                |
|           |                                    | 2b. <b>R &lt; 0</b> ; R = D + R  |                 |                                 | 0000 0111                |
|           |                                    | Q = Shift left (add 0 at LSB)                                                                                     | 0000            |                                 | 0010 0000<br>1110 0111 + |
|           |                                    | 3. D = Shift right                                                                                                |                 | 0001 0000                       | -----                    |
| 2         |                                    | 1. R = R - D                                                                                                      |                 |                                 | 0000 0111                |
|           |                                    | $R = R - D = R + (-D)$                                                                                            |                 | Negate 0001 0000<br>→ 1111 0000 | 0000 0111<br>1111 0000 + |
|           |                                    | 2b. <b>R &lt; 0</b> ; R = D + R                                                                                   |                 |                                 | -----                    |
|           |                                    | Q = Shift left (add 0 at LSB)                                                                                     |                 |                                 | 1111 0111                |
|           |                                    | 3. D = Shift right                                                                                                |                 |                                 |                          |
|           |                                    |                                                                                                                   |                 |                                 |                          |
|           |                                    |                                                                                                                   |                 |                                 |                          |
|           |                                    |                                                                                                                   |                 |                                 |                          |

| Iteration | Example:<br>7 ÷ 2<br>(0111 ÷ 0010) | Step                                                                                                              | Quotient<br>(Q) | Divisor<br>(D) | Remainder<br>(R)                  |
|-----------|------------------------------------|-------------------------------------------------------------------------------------------------------------------|-----------------|----------------|-----------------------------------|
| 0         |                                    | Initial value                                                                                                     | 0000            | 0010 0000      | 0000 0111                         |
| 1         |                                    | 1. R = R - D                                                                                                      |                 |                | 1110 0111                         |
|           |                                    | 2b. <b>R &lt; 0</b> ; R = D + R                                                                                   |                 |                | 0000 0111                         |
|           |                                    | Q = Shift left (add 0 at LSB)                                                                                     | 0000            |                |                                   |
|           |                                    | 3. D = Shift right                                                                                                |                 | 0001 0000      |                                   |
| 2         |                                    | 1. R = R - D                                                                                                      |                 |                | 1111 0111                         |
|           |                                    | 2b. <b>R &lt; 0</b> ; R = D + R  |                 |                | 0000 0111                         |
|           |                                    | Q = Shift left (add 0 at LSB)                                                                                     | 0000            |                | 0001 0000<br>1111 0111 +<br>----- |
|           |                                    | 3. D = Shift right                                                                                                |                 | 0000 1000      | 0000 0111                         |
|           |                                    |                                                                                                                   |                 |                |                                   |
|           |                                    |                                                                                                                   |                 |                |                                   |
|           |                                    |                                                                                                                   |                 |                |                                   |
|           |                                    |                                                                                                                   |                 |                |                                   |

| Iteration | Example:<br>$7 \div 2$<br>(0111 $\div$ 0010) | Step | Quotient<br>(Q) | Divisor<br>(D) | Remainder<br>(R) |
|-----------|----------------------------------------------|------|-----------------|----------------|------------------|
| 0         | Initial value                                |      | 0000            | 0010 0000      | 0000 0111        |
| 1         | 1. R = R - D                                 |      |                 |                | 1110 0111        |
|           | 2b. <b>R &lt; 0</b> ; R = D + R              |      |                 |                | 0000 0111        |
|           | Q = Shift left (add 0 at LSB)                | 0000 |                 |                |                  |
|           | 3. D = Shift right                           |      |                 | 0001 0000      |                  |
| 2         | 1. R = R - D                                 |      |                 |                | 1111 0111        |
|           | 2b. <b>R &lt; 0</b> ; R = D + R              |      |                 |                | 0000 0111        |
|           | Q = Shift left (add 0 at LSB)                | 0000 |                 |                |                  |
|           | 3. D = Shift right                           |      |                 | 0000 1000      |                  |
|           |                                              |      |                 |                |                  |
|           |                                              |      |                 |                |                  |
|           |                                              |      |                 |                |                  |
|           |                                              |      |                 |                |                  |

| Iteration | Step                                        | Quotient (Q)  | Divisor (D) | Remainder (R)              |
|-----------|---------------------------------------------|---------------|-------------|----------------------------|
|           |                                             | 0000          | 0000 1000   |                            |
| 3         | 1. $R = R - D$                              |               |             | <b>1</b> 111 1111          |
|           | 2b. <b>R</b> < <b>o</b> ; $R = D + R$       |               |             | 0000 0111                  |
|           | Q = Shift left (add 0 at LSB)               | 0000          |             |                            |
|           | 3. D = Shift right                          |               | 0000 0100   |                            |
| 4         | 1. $R = R - D$                              |               |             | <b>0</b> 000 0011          |
|           | 2a. <b>R</b> >= <b>o</b> ; R = no operation |               |             |                            |
|           | Q = Shift left (add 1 at LSB)               | 0001          |             |                            |
|           | 3. D =                                      |               | 0000 0010   |                            |
| 5         | 1. R = <b>7/2 = 3 remainder 1</b>           |               |             | <b>0</b> 000 0001 <b>1</b> |
|           | 2a. <b>R</b> >= <b>o</b> ; R = no operation |               |             |                            |
|           | Q = Shift left (add 1 at LSB)               | 0011 <b>3</b> |             |                            |
|           | 3. D = Shift right                          |               | 0000 0001   |                            |

# Example #2

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$$6 \div 3$$



| Iteration | Example:<br>$6 \div 3$ | Step                          | Quotient<br>(Q) | Divisor<br>(D) | Remainder<br>(R) |
|-----------|------------------------|-------------------------------|-----------------|----------------|------------------|
| 0         |                        | Initial value                 | 0000            | 0011 0000      | 0000 0110        |
| 1         |                        | 1. $R = R - D$                |                 |                | 1101 0110        |
|           |                        | 2b. $R < 0$ ; $R = D + R$     |                 |                | 0000 0110        |
|           |                        | Q = Shift left (add 0 at LSB) | 0000            |                |                  |
|           |                        | 3. D = Shift right            |                 | 0001 1000      |                  |
| 2         |                        | 1. $R = R - D$                |                 |                | 1110 1110        |
|           |                        | 2b. $R < 0$ ; $R = D + R$     |                 |                | 0000 0110        |
|           |                        | Q = Shift left (add 0 at LSB) | 0000            |                |                  |
|           |                        | 3. D = Shift right            |                 | 0000 1100      |                  |
| 3         |                        | 1. $R = R - D$                |                 |                | 1111 1010        |
|           |                        | 2b. $R < 0$ ; $R = D + R$     |                 |                | 0000 0110        |
|           |                        | Q = Shift left (add 0 at LSB) | 0000            |                |                  |
|           |                        | 3. D = Shift right            |                 | 0000 0110      |                  |

| Iteration | Example:<br>$6 \div 3$            | Step | Quotient<br>(Q) | Divisor<br>(D) | Remainder<br>(R) |
|-----------|-----------------------------------|------|-----------------|----------------|------------------|
| 3         | 1. $R = R - D$                    |      |                 |                | 1111 1010        |
|           | 2b. $R < 0$ ; $R = D + R$         |      |                 |                | 0000 0110        |
|           | Q = Shift left (add 0 at LSB)     | 0000 |                 |                |                  |
|           | 3. D = Shift right                |      |                 | 0000 0110      |                  |
| 4         | 1. $R = R - D$                    |      |                 |                | 0000 0000        |
|           | 2b. $R \geq 0$ ; R = no operation |      |                 |                |                  |
|           | Q = Shift left (add 1 at LSB)     | 0001 |                 |                |                  |
|           | 3. D = Shift right                |      |                 | 0000 0011      |                  |
| 5         | 1. $R = R - D$                    |      |                 |                | 1111 1101        |
|           | 2b. $R < 0$ ; $R = D + R$         |      |                 |                | 0000 0000        |
|           | Q = Shift left (add 0 at LSB)     | 0010 |                 |                | 0                |
|           | 3. D = Shift right                | 2    |                 | 0000 0001      |                  |

**Example:  $6 \div 3 = 2$**