UTM
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INSPIRING CREATIVE AND INNOVATIVE MINDS

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Finite Automata

Deterministic Finite Automata (DFA)

- In computer science, we study different types of computer languages, such as Basic, Pascal, and C++.

- We will discuss a type of a language that can be recognized by special types of machines.

Deterministic Finite Automata (DFA)

- A deterministic finite automaton (pl. automata) is a mathematical model of a machine that accepts languages of some alphabet.

Deterministic Finite Automata (DFA)

- Deterministic Finite Automaton is a quintuple
  $M = \{ S, I, q_0, f_s, F \}$
  where,
  S is a finite nonempty set of states
  I is the input alphabet (a finite nonempty set of symbols)
  q_0 is the initial state
  f_s is the state transition function
  F is the set of final states, subset of S.

example

- Let M={ $\{q_0,q_1,q_2\}$, $\{0,1\}$, q_0, f_s, $\{q_2\}$}
  where f_s is defined as follows:

  $f_s(q_0,0)= q_1$, $f_s(q_1,1)= q_2$
  $f_s(q_0,1)= q_0$, $f_s(q_2,0)= q_0$
  $f_s(q_1,0)= q_2$, $f_s(q_2,1)= q_1$

- Note that for M:

  $S=\{q_0,q_1,q_2\}$, $I=\{0,1\}$, $F=\{q_2\}$
  q_0 is the initial state

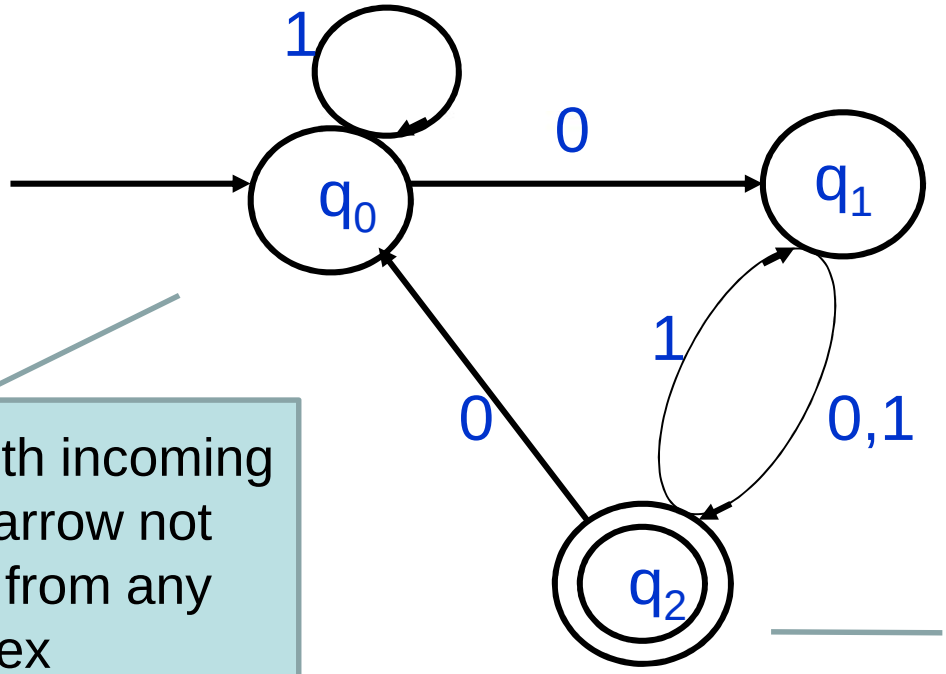
example

- The state transition function of a DFA is often described by means of a table, called a **transition table**.

f_s	0	1
q_0	q_1	q_0
q_1	q_2	q_2
q_2	q_0	q_1

example

- The transition diagram of this DFA is,



Each state represented by a small circle labeled with the state

Initial state with incoming unlabeled arrow not originating from any vertex

Final state with a double circle

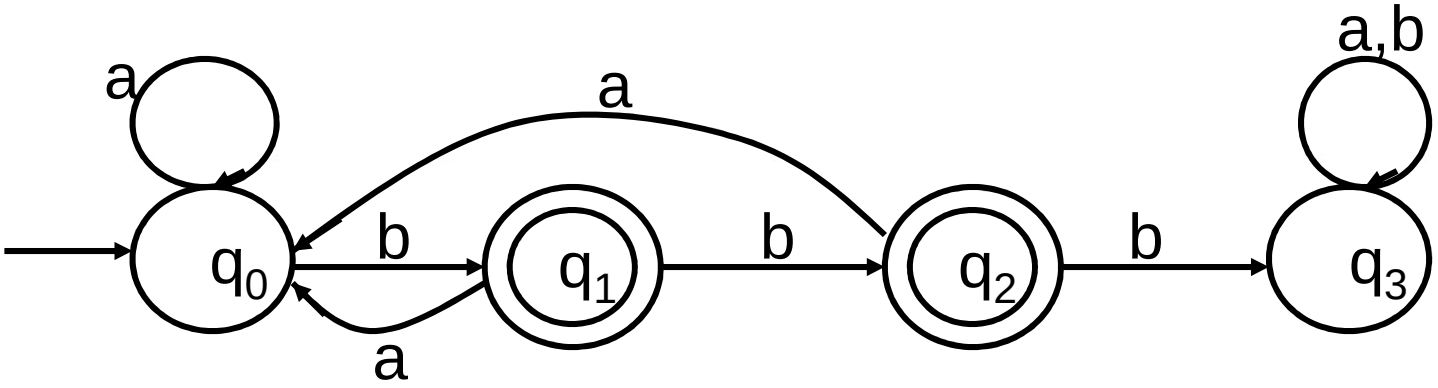
example

Let $M=(\{q_0,q_1,q_2,q_3\}, \{a,b\},q_0, f_s , \{q_1,q_2\})$
where f_s is given by the table

f_s	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

example

- The transition diagram of this DFA is,



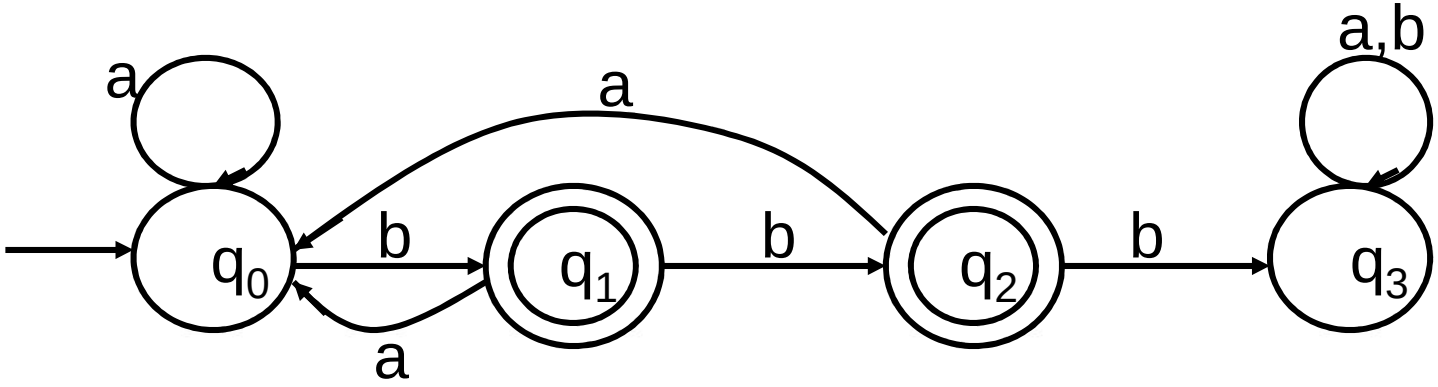
Deterministic Finite Automata (DFA)

- Let M= { S, I, q_0, f_s, F} be a DFA and *w* is an input string,
- *w* is said to be accepted by M if

$$f_s^*(q_0, w) \in F$$

- f_s^* - extended transition function for M

example

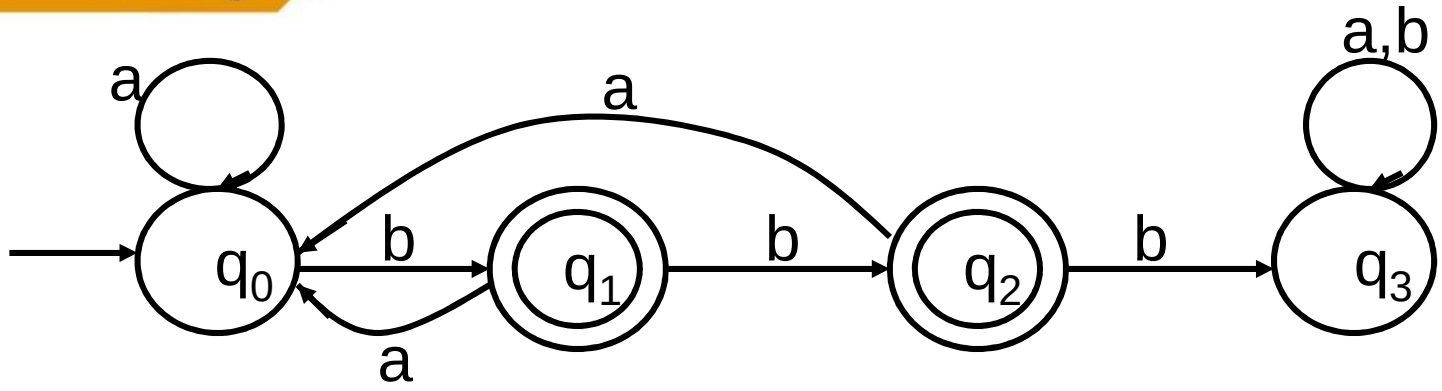


w= abb

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2$$

accepted by M

example

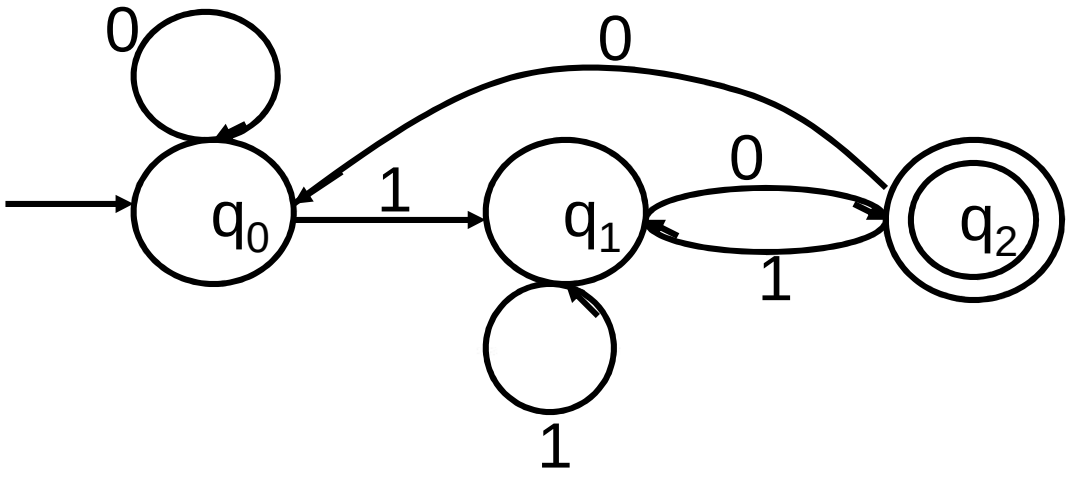


w= abba

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0$$

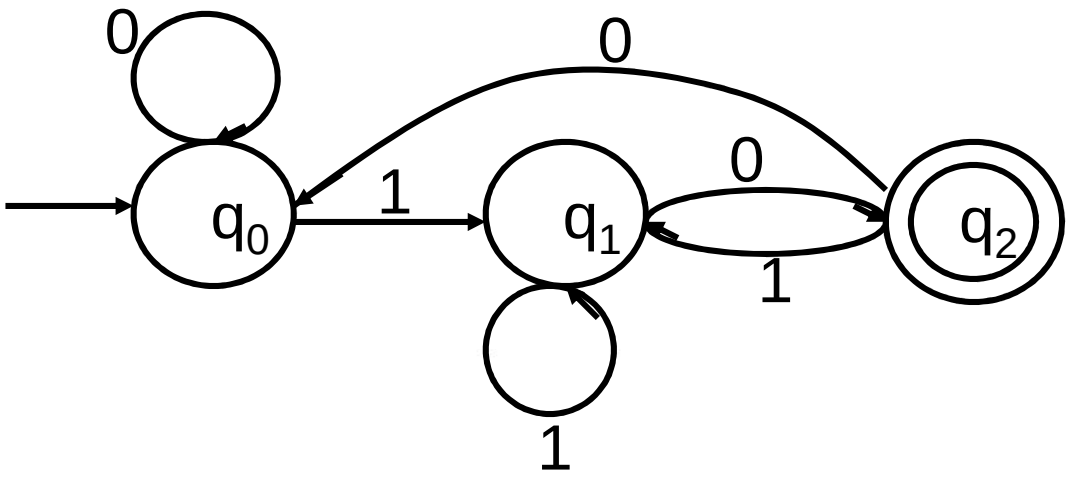
not accepted by M

example



- What are the states of M? q_0, q_1, q_2
- Write the set of input symbols. $I = \{0,1\}$
- Which is the initial state? q_0

example



- Write the set of final states. $F = \{q_2\}$
- Write the transition table for this DFA

example

The transition table, fs

f_s	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_1

example

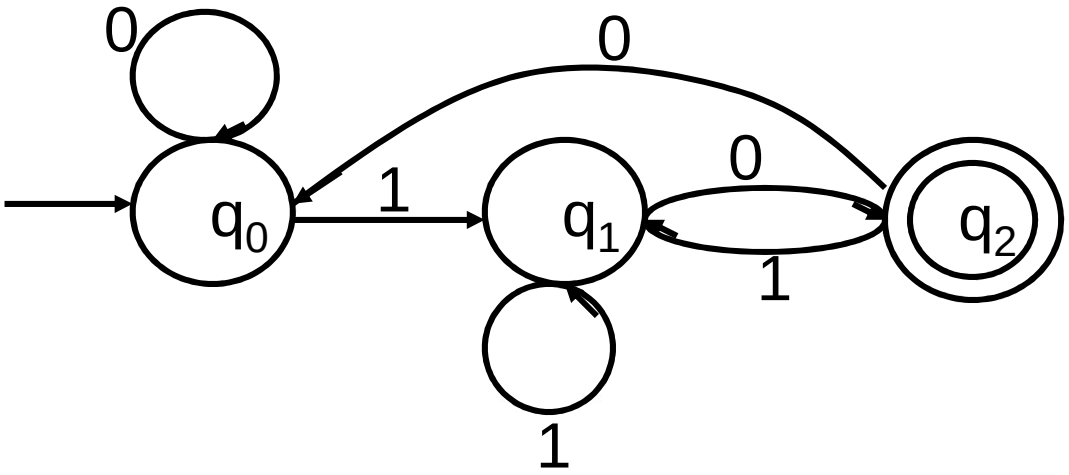
Which of the strings are accepted by M?

0111010, 00111, 111010,

0100, 1110

example

0111010

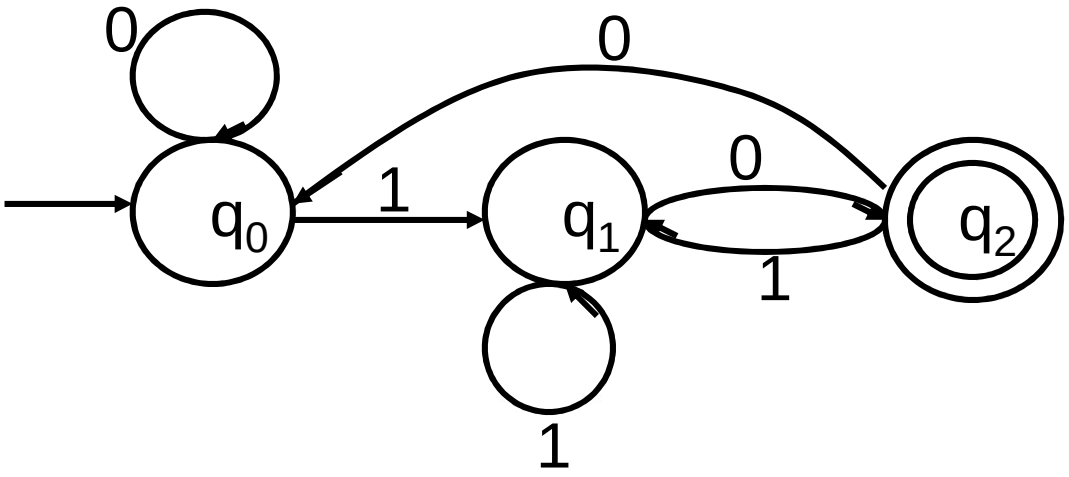


$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

example

00111

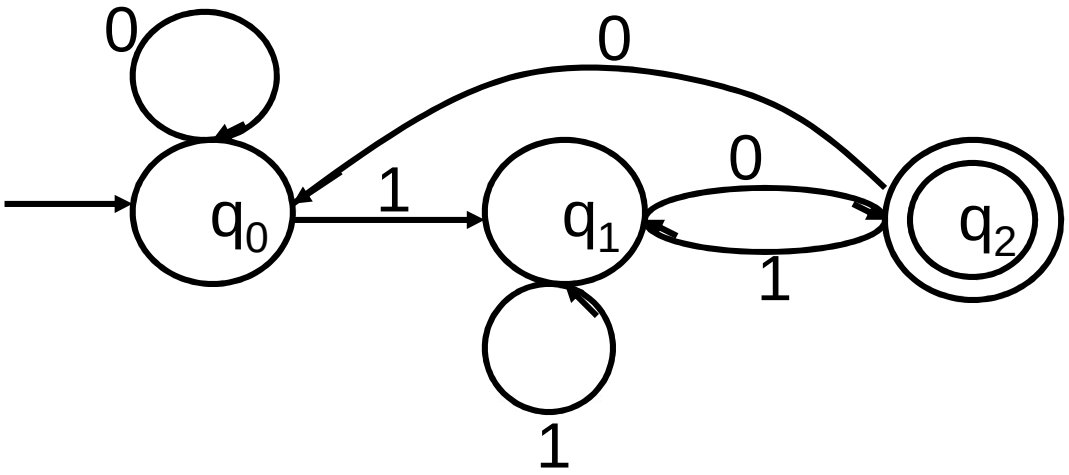


$$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1$$

not accepted by M

example

111010

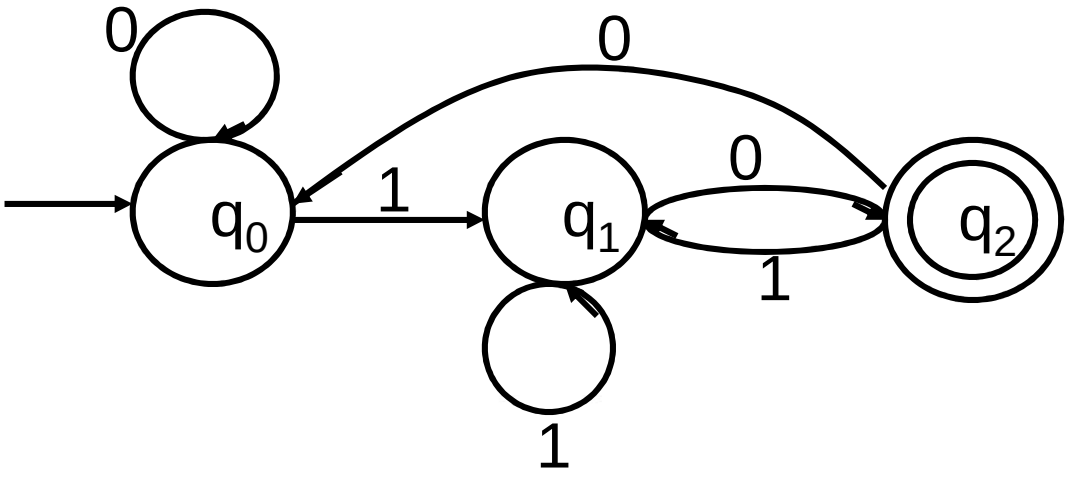


$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

example

0100

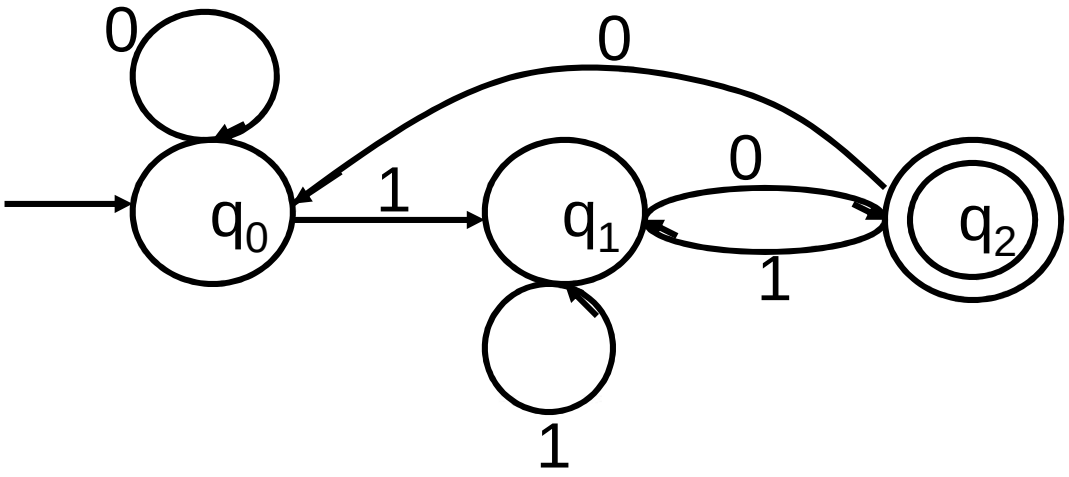


$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} q_0$$

not accepted by M

example

1110



$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

example

Construct a state transition diagram of a DFA that accepts on {a,b} that contain an even number of a's and an odd number of b's.

Example of accepted strings:

aab, baa, baaabba

example

4 states,

q_0	even num. of a's & even num. of b's.
q_1	even num. of a's & odd num. of b's.
q_2	odd num. of a's & odd num. of b's.
q_3	odd num. of a's & even num. of b's.

$S = \{q_0, q_1, q_2, q_3\}$

example

set of states, $S = \{q_0, q_1, q_2, q_3\}$

set of input symbols, $I = \{a, b\}$

initial state, q_0

final state, q_1

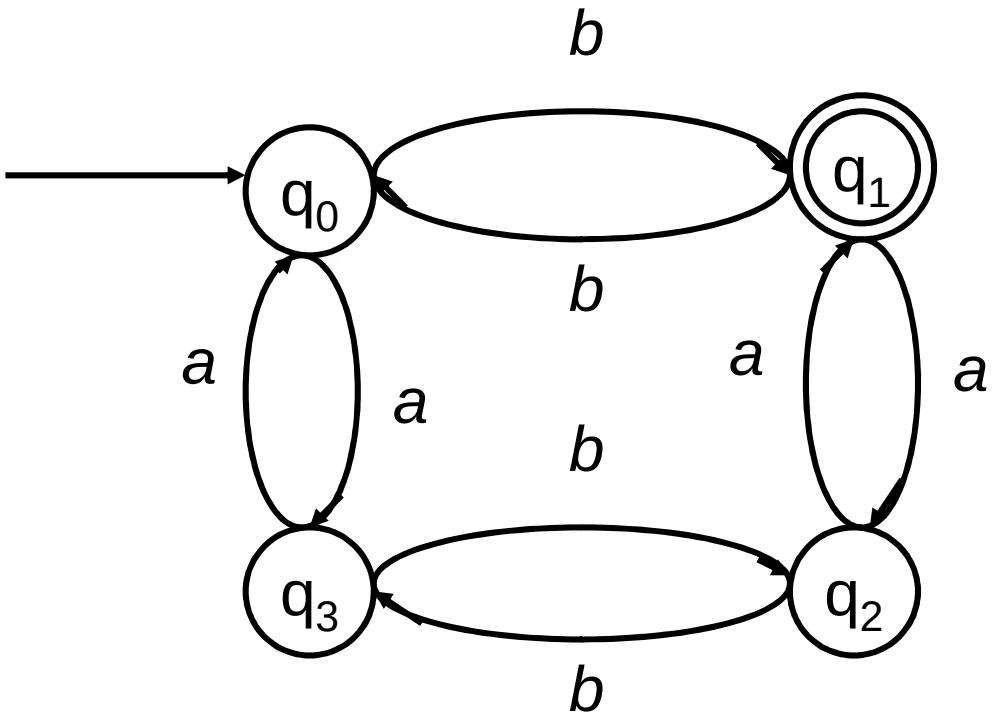
example

State transition function

f_s	a	b
q_0	q_3	q_1
q_1	q_2	q_0
q_2	q_1	q_3
q_3	q_0	q_2

example

State transition diagram



exercise

Let M=(S, I, q_0, f_s, F) be the DFA such that S={q_0,q_1,q_2}, I={a,b}, F={q_2}, q_0 =initial state, and f_s is given by,

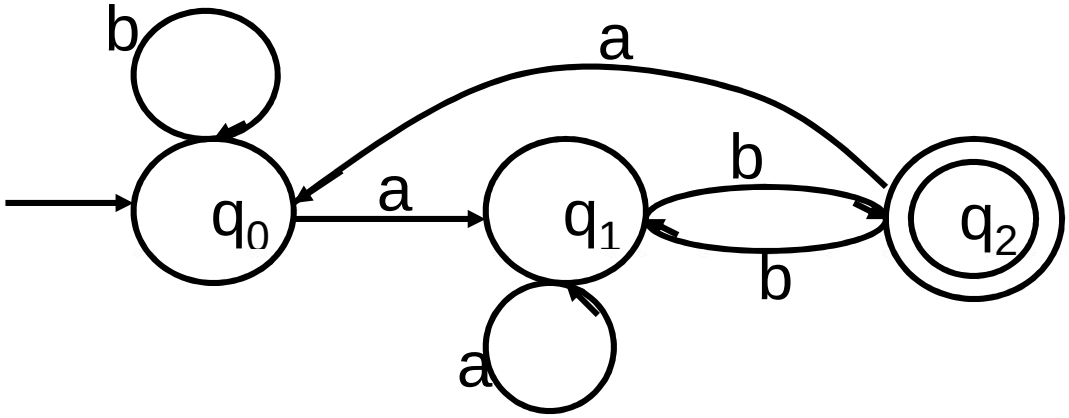
f_s	a	b
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

Draw the state diagram of M.

Which of the strings
abaa, bbbabb, bbbaa dan
bababa
are accepted by M?

exercise

The transition diagram of M is,



Construct the transition table of M.

Which of the strings

baba, baab, abab dan abaab

are accepted by M?

exercise

- Construct a state transition diagram of a DFA M with the input set {0,1} such that M accepts only the string 101.

Exercise

- Construct a deterministic finite automaton (DFA) that accepts the set of all bit strings that contain string ‘0101’.

Exercise

- Construct a deterministic finite automaton (DFA) that accepts all string over {a,b} that contain ab and end in bbb

Finite State Machines (FSM)

- Automata with input as well as output.
- Every state has an input and corresponding to the input the state also has an output.
- These types of automata are commonly called **finite state machines**.

Finite State Machines (FSM)

- A finite state machine is a sextuple,
  $M= \{ S, I, O, q_0, f_s, f_o\}$
  where,
  S is a finite nonempty set of states
  I is the input alphabet
  O is the output alphabet
  q_0 is the initial state
  f_s is the state transition function
  f_0 is the output function.

example

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM

- where,

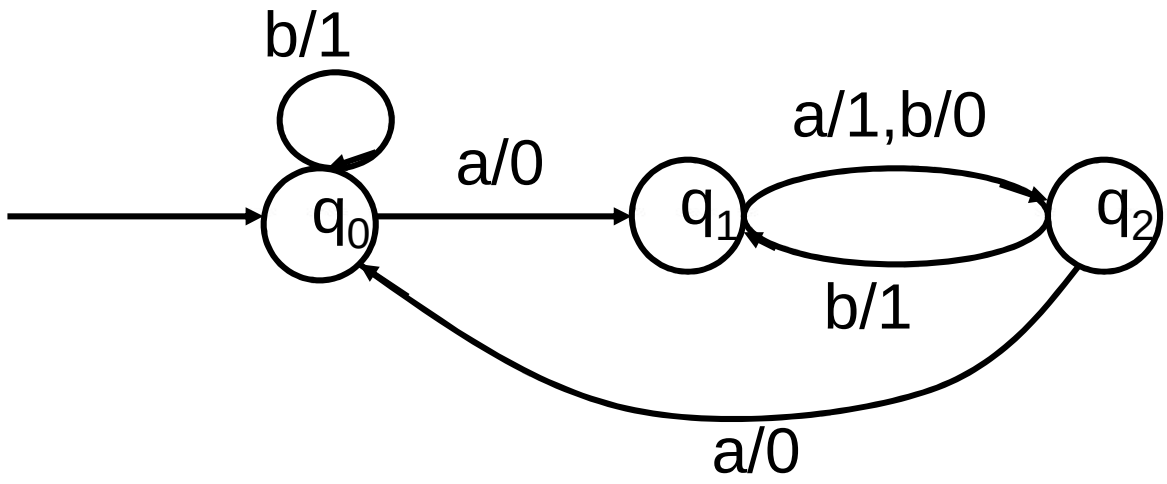
  S = {q_0,q_1,q_2},
  I = {a,b},
  O={0,1},
  q_0= initial state,

example

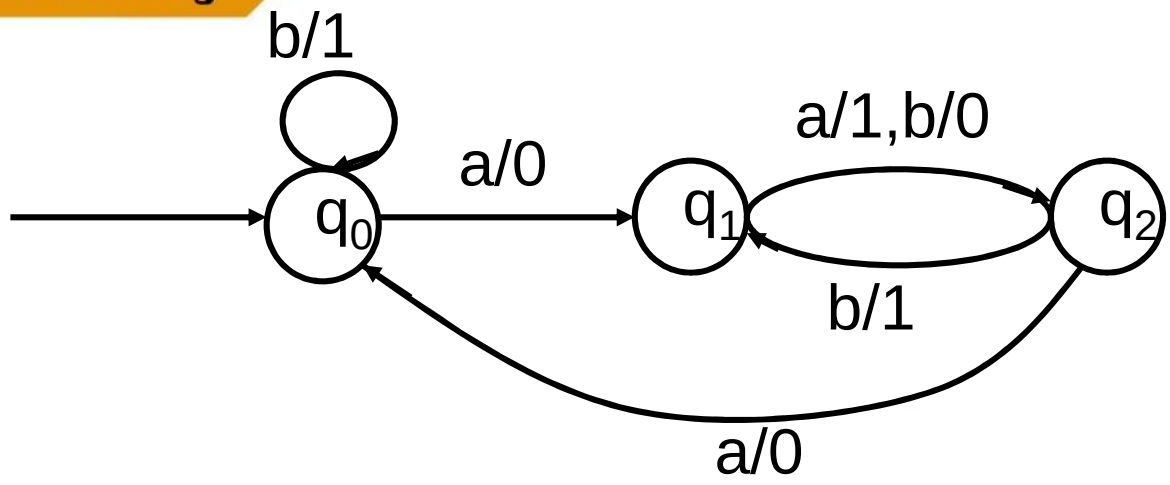
f_s and f_0

	f_s		f_o	
	a	b	a	b
q_0	q_1	q_0	0	1
q_1	q_2	q_2	1	0
q_2	q_0	q_1	0	1

example



example



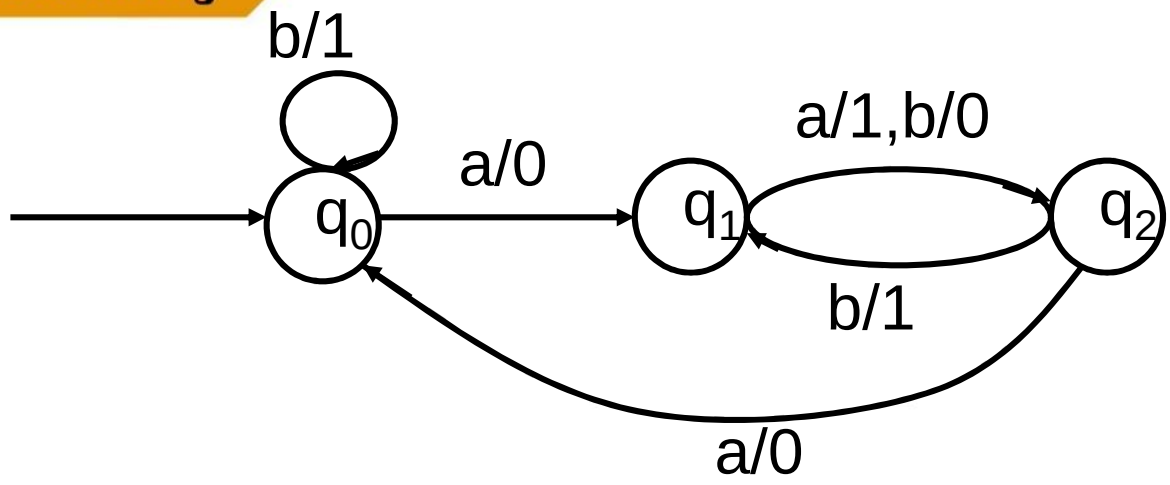
Input string: bbab

$$q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$$

1 1 0 0

Output string: 1100 Output: 0

example



Input string: bababaa

$$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2 \xrightarrow[0]{a} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[1]{a} q_2$$

Output string: 1000101 Output: 1

example

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM

- where,

  S ={q_0,q_1,q_2 ,q_3},

  I ={a,b},

  O ={0,1},

  q_0 = initial state,

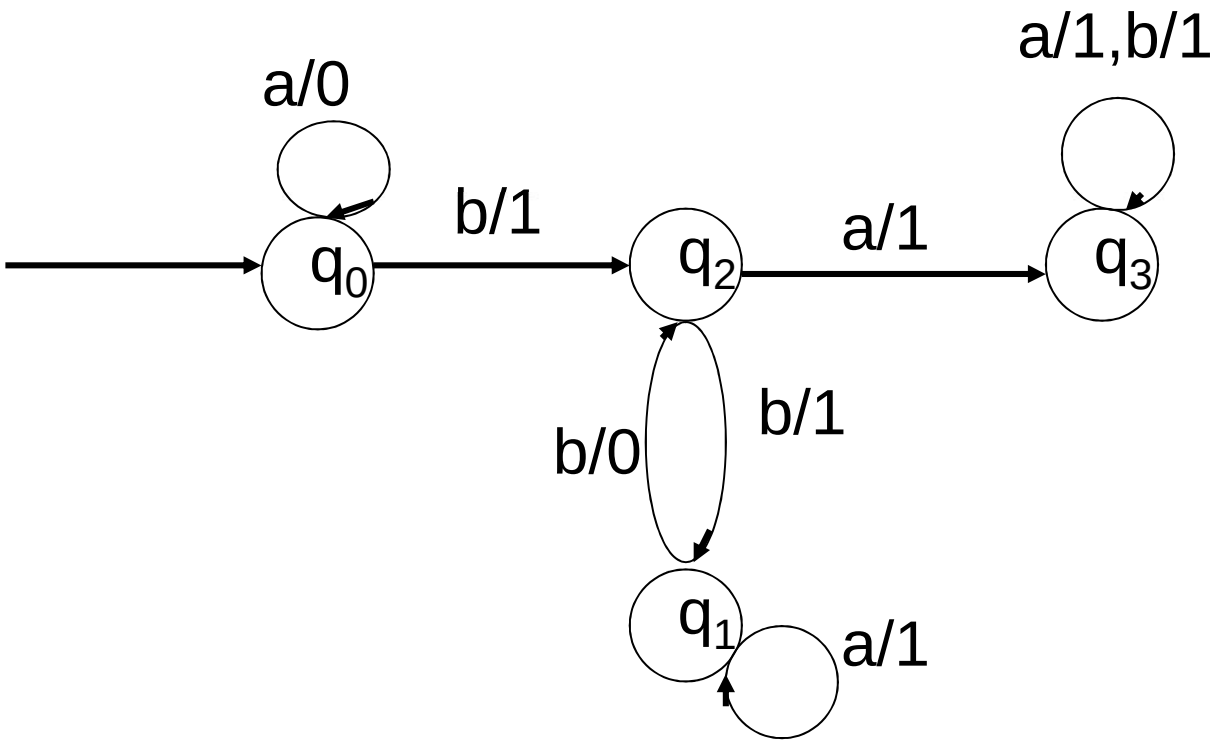
example

- f_s and f_0

	f_s		f_o	
	a	b	a	b
q_0	q_0	q_2	0	1
q_1	q_1	q_2	1	0
q_2	q_3	q_1	1	1
q_3	q_3	q_3	1	1

example

- Draw the transition diagram of M.



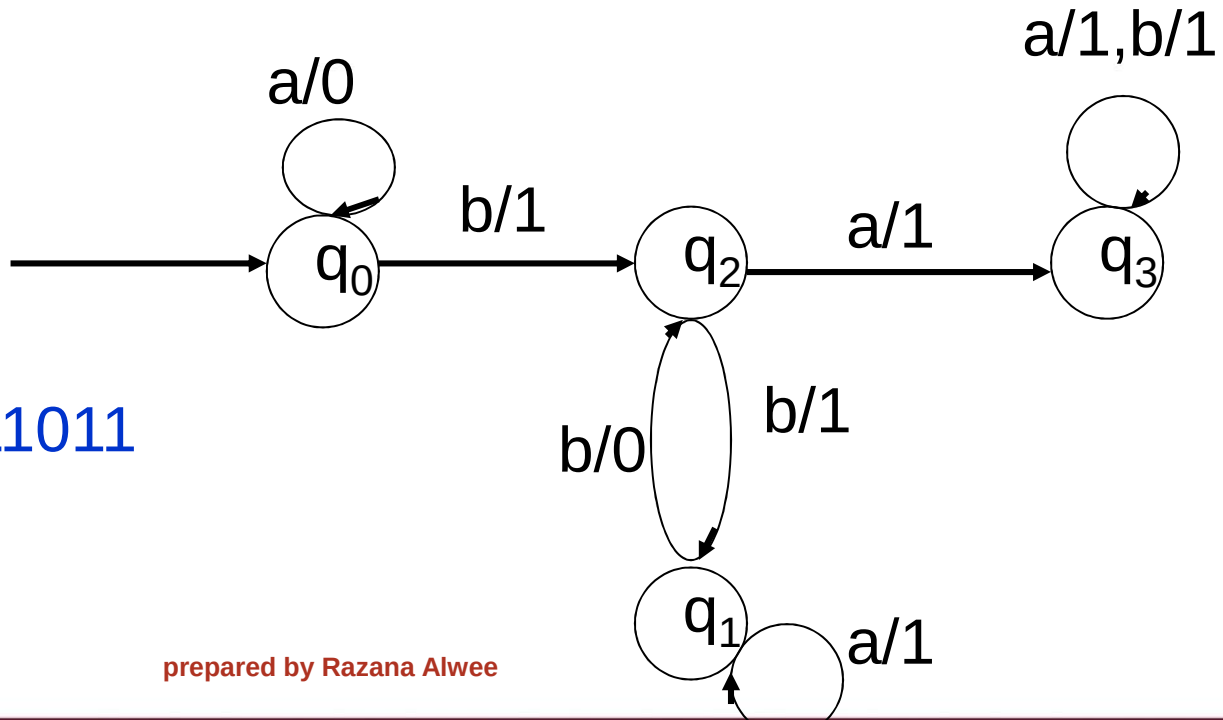
example

- What is the output string if the input string is *abbabab*?

abbabab

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_2 \xrightarrow{b} q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_3 \xrightarrow{b} q_3$

0 1 1 1 0 1 1



Output string: 0111011

example

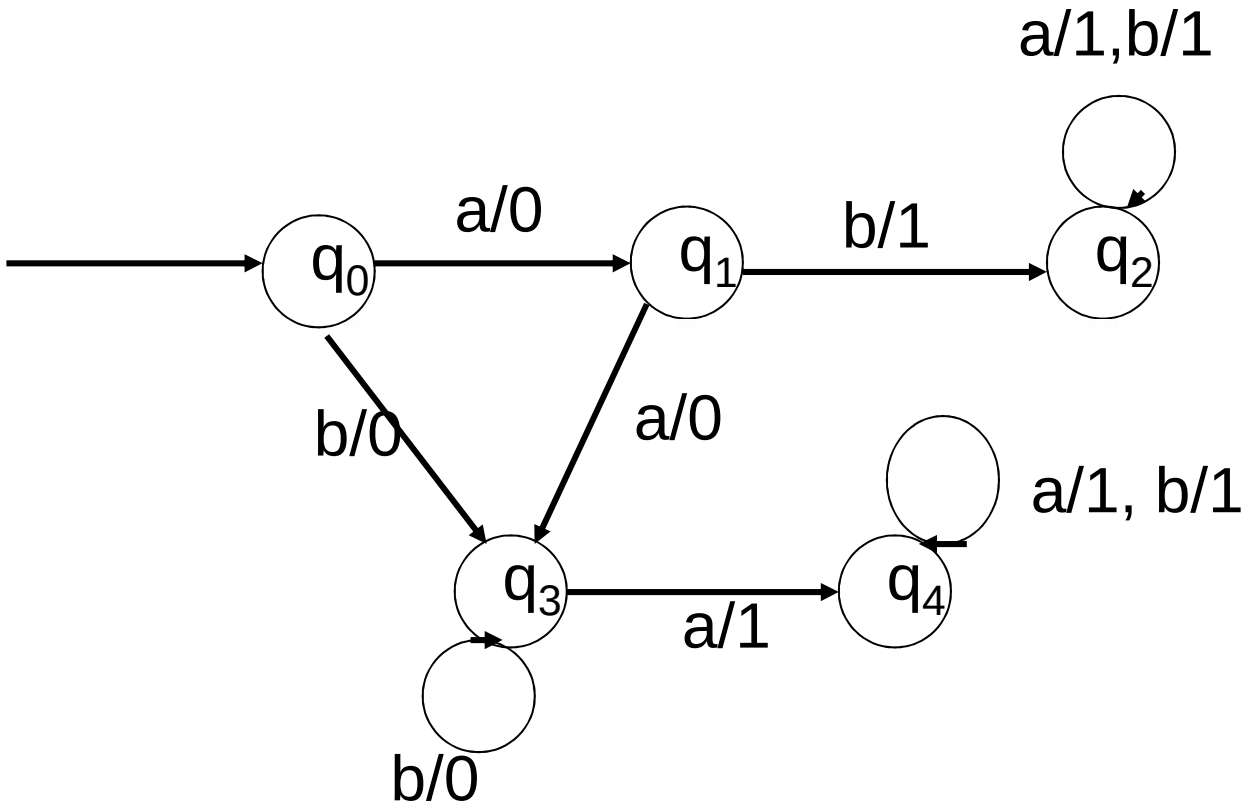
- What is the output of *abbabab*?

Output: 1

Finite State Machines (FSM)

- Let M be a FSM.
- Let x be a nonempty string in M.
- We say that x is accepted by M if and only if the output of x is 1.

example



example

- Write the transition table of M.
- What is the output string if the input string is *aaabbbb*?
- What is the output if the input string is *bbbaaaa*?

example

- Is the string *aaa* accepted by M?

- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

example

- The transition table of M.

	f_s		f_o	
	a	b	a	b
q_0	q_1	q_3	0	0
q_1	q_3	q_2	0	1
q_2	q_2	q_2	1	1
q_3	q_4	q_3	1	0
q_4	q_4	q_4	1	1

example

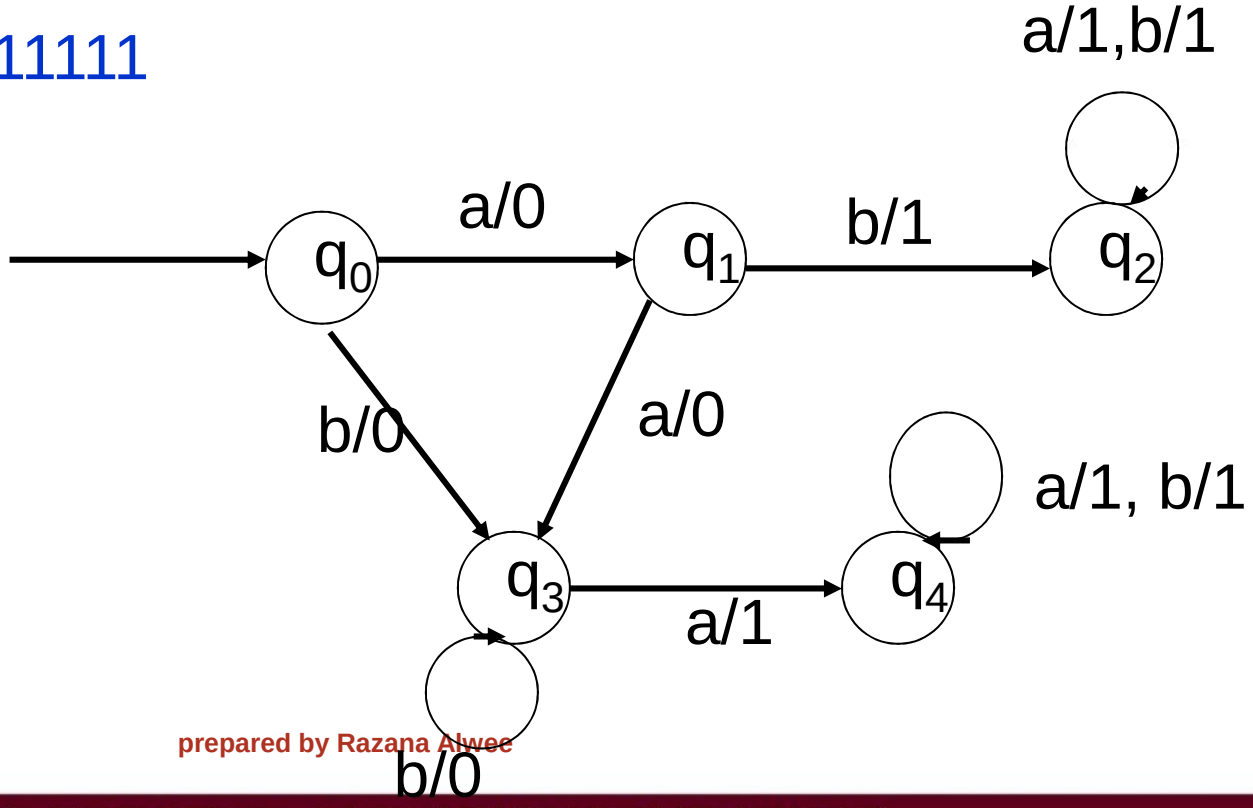
- What is the output string if the input string is *aaabbbb*?

aaabbbb

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4$

0 0 1 1 1 1 1

Output string: 0011111



example

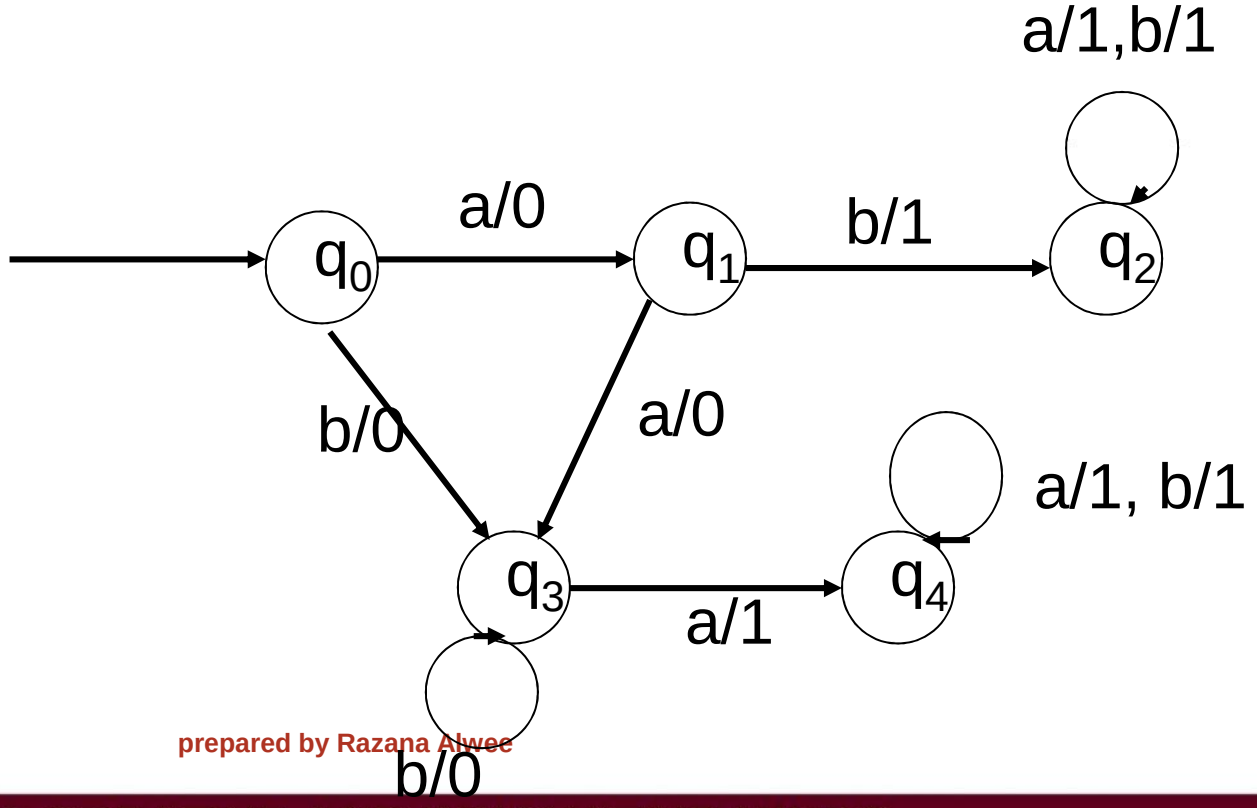
- What is the output if the input string is *bbbaaaa*?

bbbaaaa

$$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{a} q_4 \xrightarrow{a} q_4 \xrightarrow{a} q_4$$

0 0 0 1 1 1 1

Output: 1



example

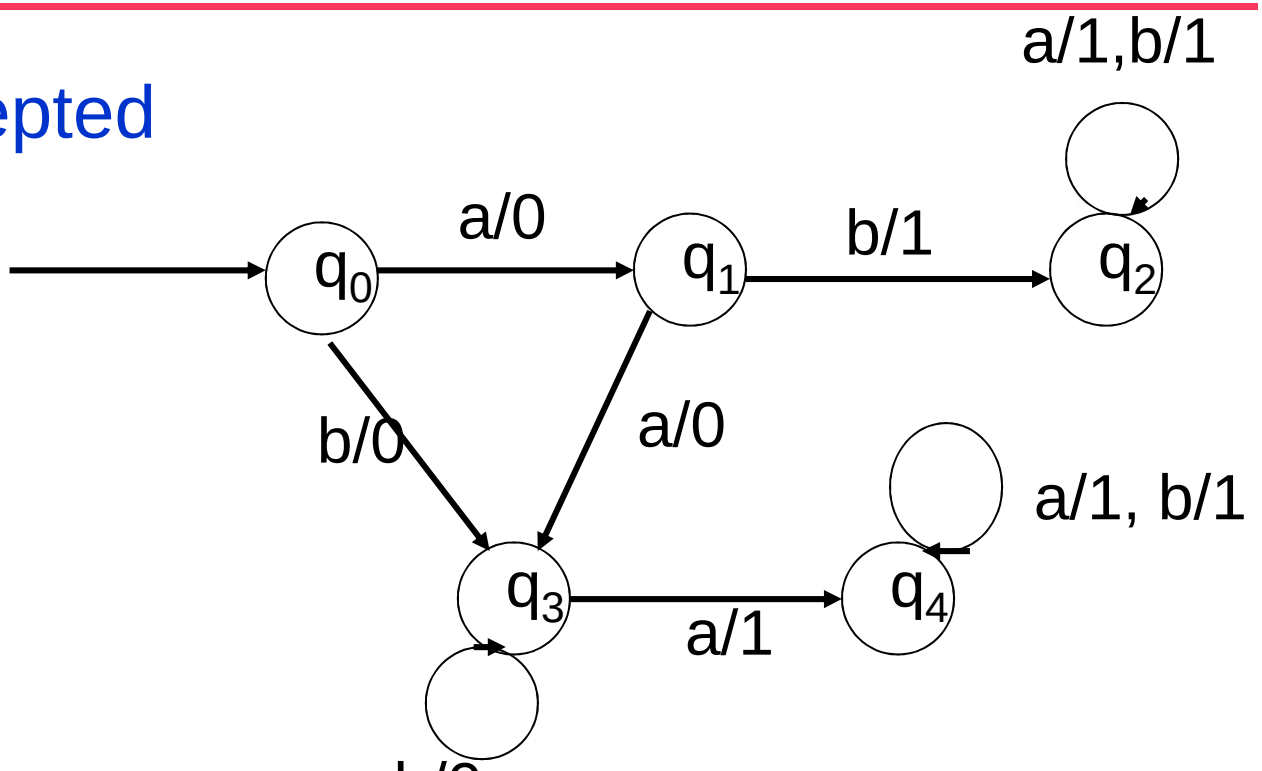
- Is the string *aaa* accepted by M?

aaa

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4$

0 0 1

Output: 1, accepted



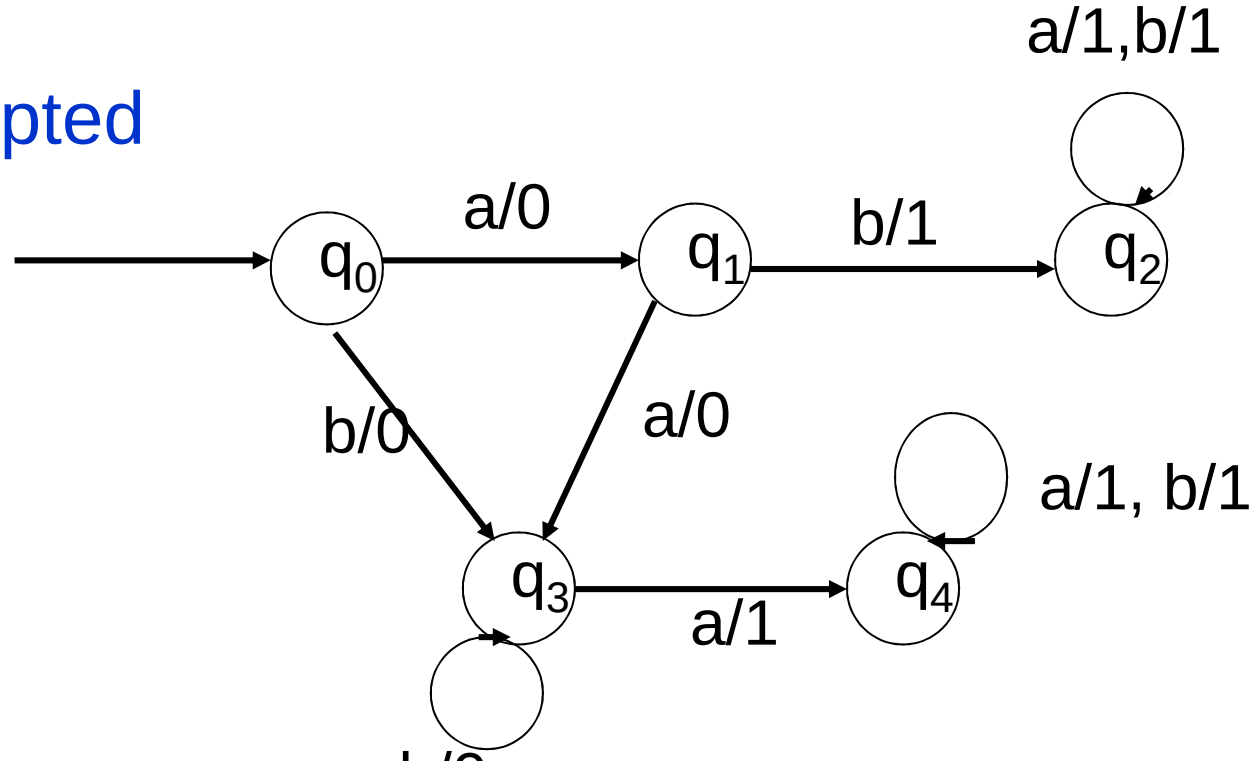
example

- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

ba

$$q_0 \xrightarrow[0]{b} q_3 \xrightarrow[1]{a} q_4$$

Output: 1, accepted

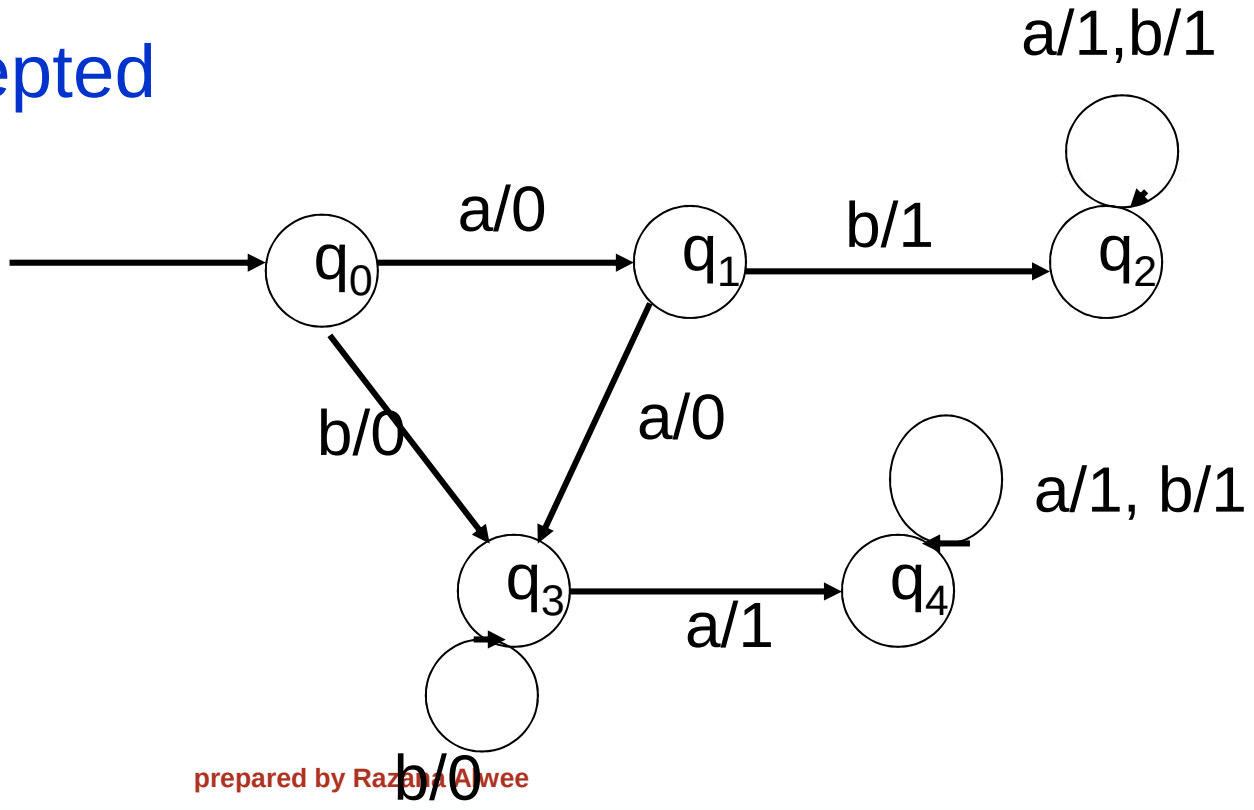


aabbba

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4$

0 0 0 0 0 1

Output: 1, accepted

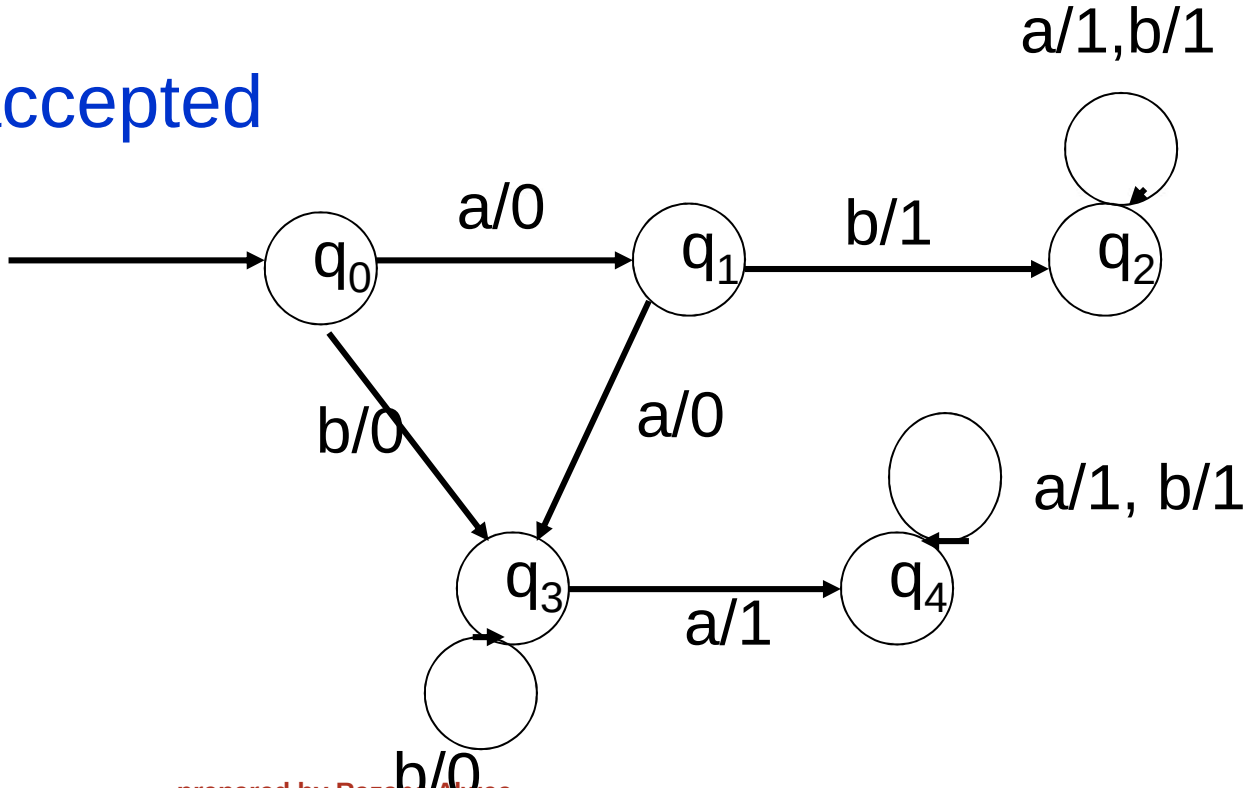


bbbb

$$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3$$

0 0 0 0

Output: 0, not accepted

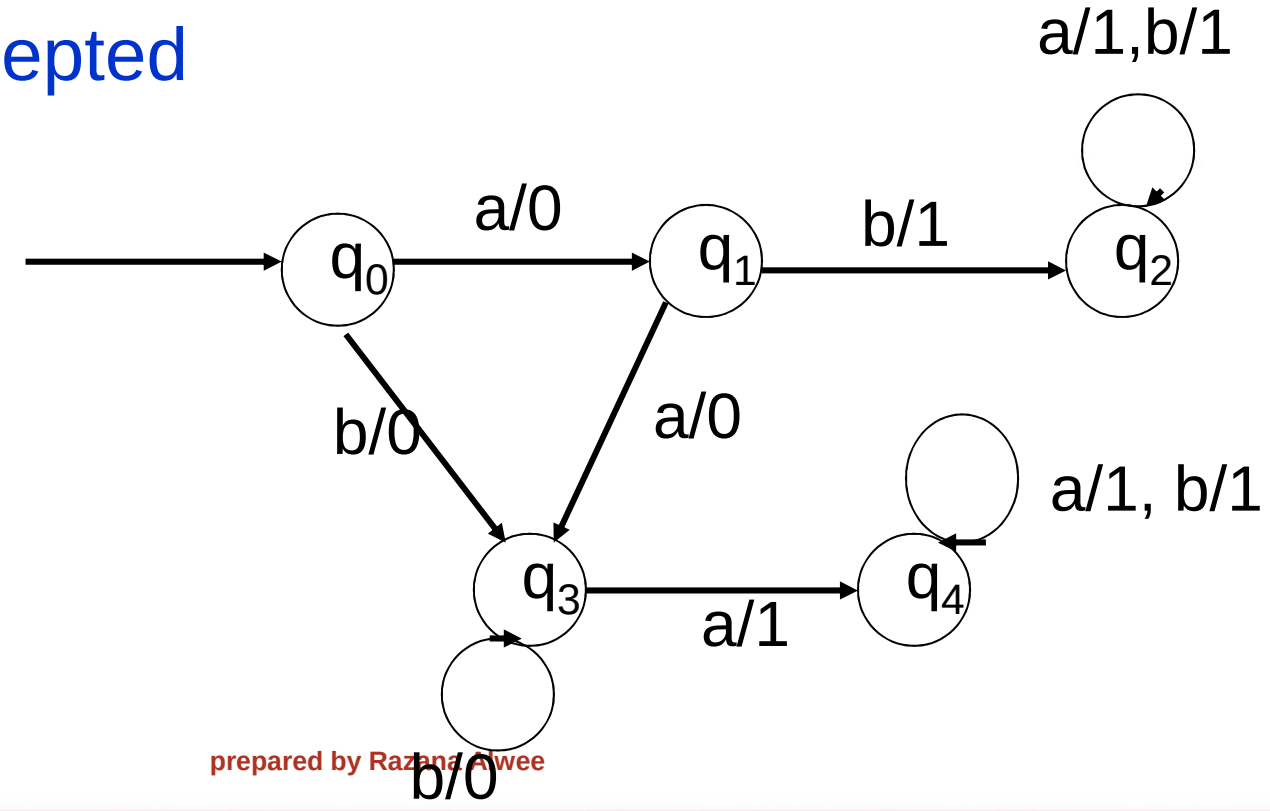


aaabbbb

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4$

0 0 1 1 1 1 1

Output: 1, accepted



example

- Consider a vending machine that sells candy and the cost of a candy is 50 cents.
- The machine accepts any sequence of 10-, 20-, or 50 cent coins.
- After inserting at least 50 cents, the customer can press the button to release the candy.

example

- If the customer inputs more than 50 cents, the machine does not return the change.
- After selling the candy, the machine returns to initial state.
- Construct a finite state machine that models this vending machine.

example

States,

- q_0, initial state (0)
- q_1, 10 cents
- q_2, 20 cents
- q_3, 30 cents
- q_4, 40 cents
- q_5, $\geq$ 50 cents

example

$S = \{q_0, q_1, q_2, q_3, q_4, q_5\}$,

$I = \{10, 20, 50, B\}$,

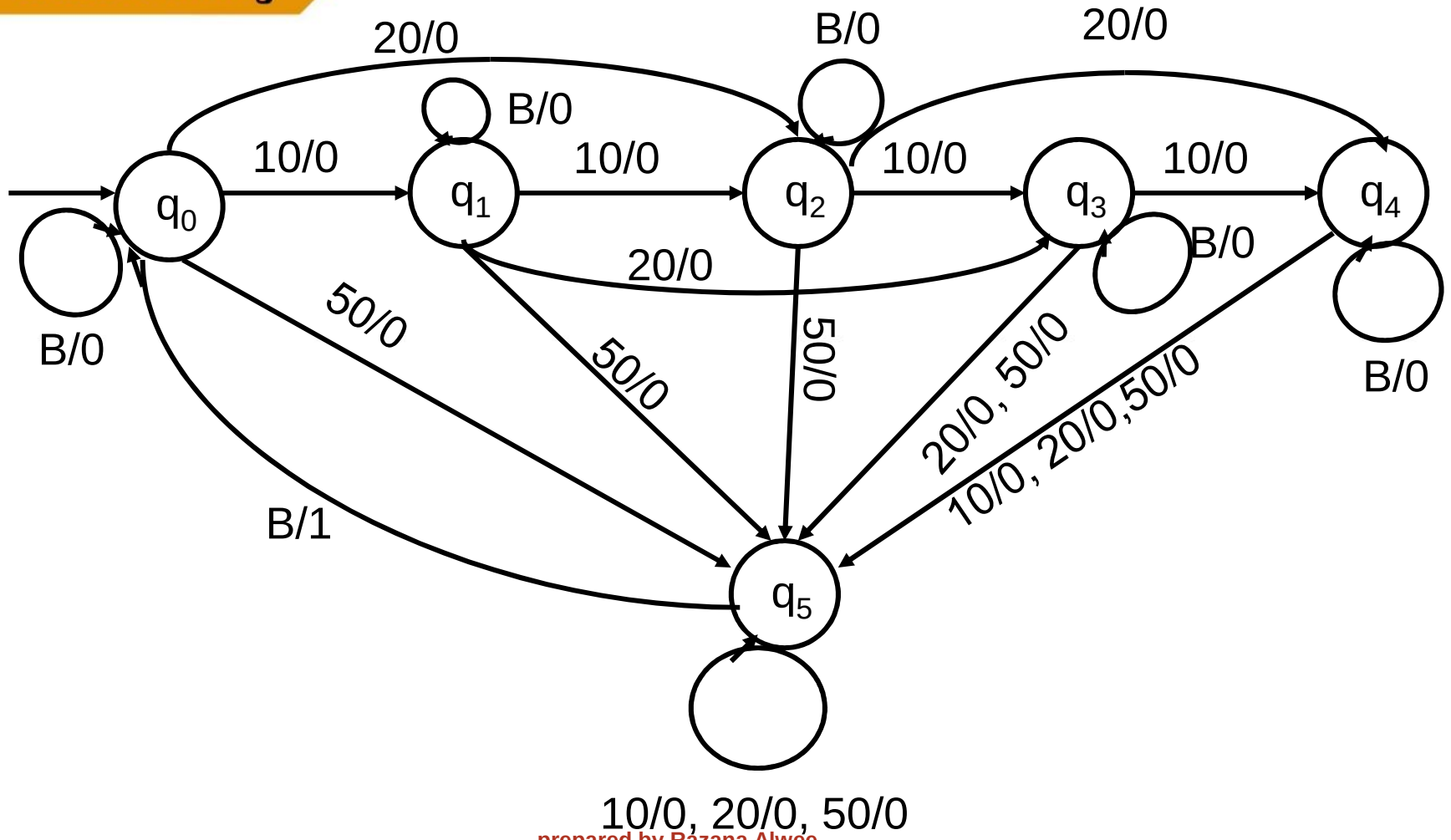
$O = \{0, 1\}$,

q_0 = initial state,

example

	f_s				f_o			
	10	20	50	B	10	20	50	B
q_0	q_1	q_2	q_5	q_0	0	0	0	0
q_1	q_2	q_3	q_5	q_1	0	0	0	0
q_2	q_3	q_4	q_5	q_2	0	0	0	0
q_3	q_4	q_5	q_5	q_3	0	0	0	0
q_4	q_5	q_5	q_5	q_4	0	0	0	0
q_5	q_5	q_5	q_5	q_0	0	0	0	1

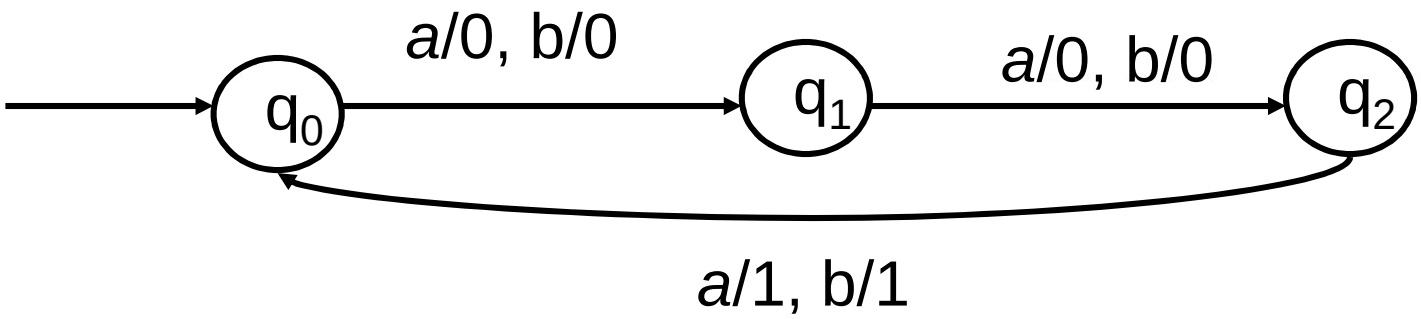
example


20/0
B/0
20/0
B/0
10/0
10/0
10/0
10/0
q_0
q_1
q_2
q_3
q_4
B/0
20/0
B/0
50/0
50/0
50/0
20/0, 50/0
10/0, 20/0,50/0
B/0
B/1
q_5
10/0, 20/0, 50/0

example

- Design a FSM, with input alphabet I={a, b}, that outputs a 1 if the number of input symbols read so far is divisible by 3.

example



exercise

Let M= { S, I, O, q_0, f_s, f_o} be a FSM

where,

S ={q_0,q_1,q_2},

I ={a,b},

O ={0,1},

q_0 = initial state,

exercise

f_s and f_0

	f_s		f_o	
	a	b	a	b
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

exercise

- Draw the transition diagram of M.
- What is the output string if the input string is *aabbb*?
- What is the output string if the input string is *ababab*?
- What is the output if the input string is *abbbaba*?
- What is the output if the input string is *bbbababa*?

exercise

- Design a FSM that accepts all string over {a,b} that begin with aa.
- For example: aaab, aabba, aababab

Exercise

- Design a FSM that accepts all string over {a,b} that end with aba.
- For example: aaba, aababa, bbbaba

Exercise

- Design a FSM that accepts all string over {a,b} that contain bbb and end in ab