

SCSI1013: Discrete Structures


CHAPTER 3

COUNTING METHODS [Part 1]

Innovative • entrepreneurial • global

2014/2015 – Sem. 1

www.utm.my




Basic Counting Principles

- Counting principle is all about choices we might make given many possibilities.
- It is used to find the number of possible outcomes.
- It provides a basis of computing probabilities of discrete events.

Innovative • entrepreneurial • global

www.utm.my




Basic Counting Principles

Some sample of counting problems:

- Problem 1** - How many ways are there to seat n couples at a round table, such that each couple sits together?
- Problem 2** - How many ways are there to express a positive integer n as a sum of positive integers?

Innovative • entrepreneurial • global

www.utm.my




Basic Counting Principles

- Problem 3** - There are three boxes containing books. The first box contains 15 mathematics books by different authors, the second box contains 12 chemistry books by different authors, and the third box contains 10 computer science books by different authors. A student wants to take a book from one of the three boxes. In how many ways can the student do this?

Innovative • entrepreneurial • global

www.utm.my




Basic Counting Principles

Problem 4 - The department will award a free computer to either a CS student or a CS professor. How many different choices are there, if there are 530 students and 15 professors?

Innovative • entrepreneurial • global

www.utm.my




Basic Counting Principles

Problem 5 - A scholarship is available, and the student to receive this scholarship must be chosen from the Mathematics, Computer Science, or the Engineering Department. How many different choices are there for this student if there are 38 qualified students from the Mathematics Department, 45 qualified students from the Computer Science Department and 27 qualified students from the Engineering Department?

Innovative • entrepreneurial • global

www.utm.my




Basic Counting Principles

- There are a number of basic principles that we can use to solve such problems:
 - Addition Principle**
 - Multiplication Principle**

11

Innovative • entrepreneurial • global www.utm.my




Addition Principle

- Suppose that tasks T_1, T_2, \dots, T_k can be done in n_1, n_2, \dots, n_k ways, respectively.
- If all these tasks are **independent** of each other, then the number of ways to do one of these tasks is $n_1 + n_2 + \dots + n_k$

12

Innovative • entrepreneurial • global www.utm.my




Basic Counting Principles

- If a task can be done in n_1 ways and a second task in n_2 ways, and if these two tasks cannot be done at the same time, then there are $n_1 + n_2$ ways to do either task.

13

Innovative • entrepreneurial • global www.utm.my



Example 1


We want to find the number of integers between 5 and 50 that end with 1 or 7.

Solution:

- Let T denote this task. We divide T into the following tasks.
 - T_1 : find all integers between 5 and 50 that end with 1.
 - T_2 : find all integers between 5 and 50 that end with 7.

14

Innovative • entrepreneurial • global www.utm.my




- T_1 : find all integers between 5 and 50 that end with 1.
 $\rightarrow 11, 21, 31, 41 \Rightarrow 4$ ways
- T_2 : find all integers between 5 and 50 that end with 7.
 $\rightarrow 7, 17, 27, 37, 47 \Rightarrow 5$ ways

$\therefore n_1 + n_2 = 4 + 5 = 9$ ways.

15

Innovative • entrepreneurial • global www.utm.my



Example 2

We want to find the number of integers between 4 and 100 that end with 3 or 5.

Solution:

Let T denote this task. We divide T into the following tasks.

- T_1 : find all integers between 4 and 100 that end with 3.
- T_2 : find all integers between 4 and 100 that end with 5.

16

Innovative • entrepreneurial • global www.utm.my

UTM

- T_1 : find all integers between 4 and 100 that end with 3.
 $\Rightarrow 13, 23, 33, 43, 53, 63, 73, 83, 93$
 $\Rightarrow 9$ integers
- T_2 : find all integers between 4 and 100 that end with 5.
 $\Rightarrow 5, 15, 25, 35, 45, 55, 65, 75, 85, 95$
 $\Rightarrow 10$ integers

innovative • entrepreneurial • global

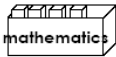
UTM

- Task T_1 can be done in 9 ways.
- Task T_2 can be done in 10 ways.
- The number of ways to do one of these tasks is,
 $9+10=19$
- Task T can be completed in 19 ways.

innovative • entrepreneurial • global

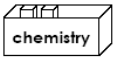
UTM

Example 3



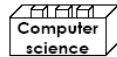
mathematics

4 books



chemistry

2 books



Computer science

3 books

- A student wants to take a book from one of the three boxes.
- In how many ways can the students do this?

innovative • entrepreneurial • global

UTM


- T_1 : choose a mathematics book
 $\Rightarrow 4$ ways
- T_2 : choose a chemistry book
 $\Rightarrow 2$ ways
- T_3 : choose a computer science book.
 $\Rightarrow 3$ ways
- The number of ways to do one of these tasks is,
 $\Rightarrow 4+2+3=9$

innovative • entrepreneurial • global

UTM

Exercise

- There are 8 male students and 21 female students in Discrete Structure class. Among all of them, 7 students are Chinese and the rest are Malay.
- \Rightarrow In how many ways can we select 1 student - a boy or a girl?
 - \Rightarrow In how many ways can we select 1 student - a Chinese or a Malay?



innovative • entrepreneurial • global

UTM

Exercise - Solution

■ In how many ways can we select 1 student - a boy or a girl?

- \Rightarrow To select a boy
 - 8 ways
- \Rightarrow To select a girl
 - 21 ways
- \Rightarrow The number of ways
 - $8+21=29$ ways

■ In how many ways can we select 1 student - a Chinese or a Malay?

- \Rightarrow To select a Chinese
 - 7 ways
- \Rightarrow To select a Malay
 - 22 ways
- \Rightarrow The number of ways
 - $7+22=29$ ways

innovative • entrepreneurial • global



Multiplication Principle

- A task T can be completed in k successive steps.
Step 1 can be completed in n_1 different ways.
Step 2 can be completed in n_2 different ways.
Step k can be completed in n_k different ways.
- Then the task T can be completed in $n_1 \cdot n_2 \cdot \dots \cdot n_k$ different ways.

Innovative • entrepreneurial • global

19

www.utm.my



Suppose that a procedure can be broken down into two successive tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are $n_1 n_2$ ways to do the procedure.

Innovative • entrepreneurial • global

20

www.utm.my



Example 4

- Morgan is a lead actor in a new movie. She needs to shoot a scene in the morning in studio A and an afternoon scene in studio C. She looks at the map and finds that there is no direct route from studio A to studio C. Studio B is located between studios A and C. Morgan's friends Brad and Jennifer are shooting a movie in studio B. There are three roads, say A_1 , A_2 , and A_3 , from studio A to studio B and four roads, say B_1 , B_2 , B_3 , and B_4 , from studio B to studio C. In how many ways can Morgan go from studio A to studio C and have lunch with Brad and Jennifer at Studio B?

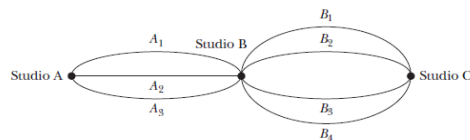


FIGURE 7.1 Routes from studio A to studio C

Innovative • entrepreneurial • global

21

www.utm.my



- There are 3 ways to go from studio A to studio B and 4 ways to go from studio B to studio C.
 $\rightarrow T_1 = 3$
 $\rightarrow T_2 = 4$
- The number of ways to go from studio A to studio C via studio B is $3 \times 4 = 12$.

Innovative • entrepreneurial • global

22

www.utm.my



Example 5

- The letters A , B , C , D , and E are to be used to form strings of length 4.
- How many strings can be formed if we do not allow repetitions?
- For example:
 – $BADE$, $ACBD$, $AEBC$..

Innovative • entrepreneurial • global

23

www.utm.my



- T_1 : choose the first letter $\rightarrow 5$ ways
- T_2 : choose the second letter $\rightarrow 4$ ways
- T_3 : choose the third letter $\rightarrow 3$ ways
- T_4 : choose the fourth letter $\rightarrow 2$ ways
- There are $5 \cdot 4 \cdot 3 \cdot 2 = 120$ strings.

Innovative • entrepreneurial • global

24

www.utm.my



Example 6

- The letters A, B, C, D , and E are to be used to form strings of length 4.
- How many strings can be formed if we allow repetitions?
- For example:
 - $BABB, AABB, ACEE \dots$

Innovative • entrepreneurial • global

25

www.utm.my



- T_1 : choose the first letter $\rightarrow 5$ ways
- T_2 : choose the second letter $\rightarrow 5$ ways
- T_3 : choose the third letter $\rightarrow 5$ ways
- T_4 : choose the fourth letter $\rightarrow 5$ ways
- There are $5.5.5.5 = 625$ strings.

Innovative • entrepreneurial • global

26

www.utm.my



Example 7

- The letters A, B, C, D , and E are to be used to form strings of length 4.
- How many strings begin with A , if repetitions are not allowed?
- For example:
 - $ADEC, ACBD, AEBC \dots$

Innovative • entrepreneurial • global

27

www.utm.my



- T_1 : choose the first letter $\rightarrow A$ (1 way)
- T_2 : choose the second letter $\rightarrow 4$ ways
- T_3 : choose the third letter $\rightarrow 3$ ways
- T_4 : choose the fourth letter $\rightarrow 2$ ways
- There are $1.4.3.2 = 24$ strings.

Innovative • entrepreneurial • global

28

www.utm.my



Exercise

- Danial, Kenny and Joseph are fighting over a turn to play a game that can only has 2 players at a time. In how many ways can we select 2 players at a time?
- Diana, Sherry, Devi and Mary are going to DSI using a motorcycle. In how many ways can we select 2 peoples to ride the motorcycle?

Innovative • entrepreneurial • global

29

www.utm.my



Exercise - Solution

- In how many ways can we select 2 players at a time?
 - \rightarrow To select 1st player
 - 3 ways
 - \rightarrow To select 2nd player
 - 2 ways
 - \rightarrow The number of ways
 - $3 \times 2 = 6$ ways

- The possible order:

Player 1	Player 2
Danial	Kenny
Danial	Joseph
Kenny	Joseph
Kenny	Danial
Joseph	Kenny
Joseph	Danial



Innovative • entrepreneurial • global

30

www.utm.my

UTM

In how many ways can we select 2 peoples to ride the motorcycle?

- To select 1st rider
 - 4 ways
- To select 2nd rider
 - 3 ways
- The number of ways
 - $4 \times 3 = 12$ ways

The possible order

Rider 1	Rider 2
Diana	Sherry
Diana	Devi
Diana	Mary
Sherry	Diana
Sherry	Devi
Sherry	Mary
Devi	Diana
Devi	Sherry
Devi	Mary
Mary	Diana
Mary	Sherry
Mary	Devi

Innovative • entrepreneurial • global

UTM

Exercise

- There are 8 male students and 21 female students in Discrete Structure class. Among all of them, 7 students are Chinese and the rest are Malay.
 - In how many ways can we select 2 students - a boy and a girl?
 - In how many ways can we select 2 students - a Chinese and a Malay?

Innovative • entrepreneurial • global

UTM

Exercise - Solution

In how many ways can we select 2 students - a boy and a girl?

- To select a boy
 - 8 ways
- To select a girl
 - 21 ways
- The number of ways
 - $8 \times 21 = 168$ ways

In how many ways can we select 2 students - a Chinese and a Malay?

- To select a Chinese
 - 7 ways
- To select a Malay
 - 22 ways
- The number of ways
 - $7 \times 22 = 154$ ways

Innovative • entrepreneurial • global

UTM

Exercise

- Suppose that, in order to declare a variable name in a computer language, the name must have five characters. The first character must be a letter, and the remaining characters can be letters or digits. How many different variable names are possible?

Innovative • entrepreneurial • global

UTM

Exercise - Solution

- How many different variable names are possible?
 - To select 1st character
 - 26 ways (26 alphabets)
 - To select 2nd character
 - 36 ways (26 alphabets + 10 numbers)
 - To select 3rd character
 - 36 ways (26 alphabets + 10 numbers)
 - To select 4th character
 - 36 ways (26 alphabets + 10 numbers)
 - To select 5th character
 - 36 ways (26 alphabets + 10 numbers)
 - The number of ways
 - $26 \times 36 \times 36 \times 36 \times 36 = 43,670,016$ ways

Innovative • entrepreneurial • global

UTM

Basic Counting Principles

- The counting problem that we have considered so far involved either the addition principle or the multiplication principle.
- Sometimes, however, we need to use both of these counting principles to solve a particular problem.

Innovative • entrepreneurial • global

Example 8

- How many 8-bit strings begin either 101 or 111?
 - $\underline{1} \underline{0} \underline{1} _ _ _ _ _ _ \rightarrow 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
 - $\underline{1} \underline{1} \underline{1} _ _ _ _ _ _ \rightarrow 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
 - $32 + 32 = 64$

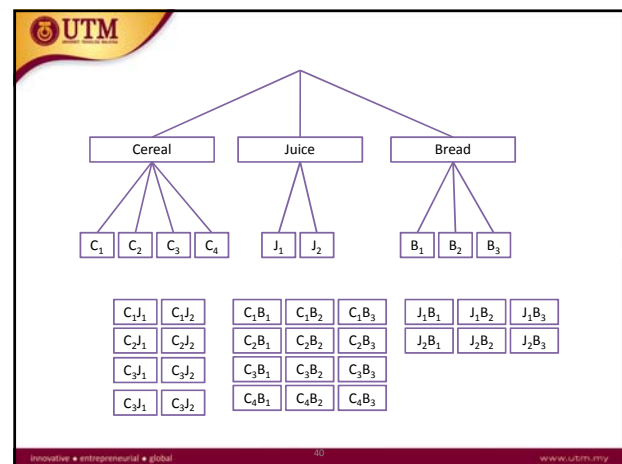
Example 9

- The following items are available for breakfast.
 - 4 types of cereal
 - 2 types of juice
 - 3 types of bread
- How many ways a breakfast can be prepared if exactly 2 items are selected from 2 different groups?

A breakfast can be prepared in either of the following 3 ways:

- A cereal & a juice
 - $4 \cdot 2 = 8$
- A cereal & a bread
 - $4 \cdot 3 = 12$
- A juice & a bread
 - $2 \cdot 3 = 6$

➢ $8 + 12 + 6 = 26$ ways




Example 10

A six-person committee composed of Aina, Wan, Chan, Tan, Syed and Helmi is to be selected to hold as a chairperson, secretary, and treasurer.

- In how many ways can this be done?
- In how many ways can this be done if either Aina or Wan must be chairperson?
- In how many ways can this be done if Syed must hold one of the position?
- In how many ways can this be done if Tan and Helmi must hold any position?


Example 10 - Solution

- In how many ways can this be done?
 - Select the chairperson (6 ways)
 - Select the secretary (5 ways)
 - Select the treasurer (4 ways)
 - $6 \cdot 5 \cdot 4 = 120$




- In how many ways can this be done if either Aina or Wan must be chairperson?
 - If Aina is chairperson $5 \cdot 4 = 20$
 - If Wan is chairperson $5 \cdot 4 = 20$
 - $20 + 20 = 40$

Innovative • entrepreneurial • global www.utm.my




- OR
 - ✓ Select the chairperson (2 ways)
 - ✓ Select the secretary (5 ways)
 - ✓ Select the treasurer (4 ways)
 - ✓ $2 \cdot 5 \cdot 4 = 40$

Innovative • entrepreneurial • global www.utm.my




- In how many ways can this be done if Syed must hold one of the position?
 - If Syed is chairperson $5 \cdot 4 = 20$
 - If Syed is secretary $5 \cdot 4 = 20$
 - If Syed is treasurer $5 \cdot 4 = 20$
 - $20 + 20 + 20 = 60$

Innovative • entrepreneurial • global www.utm.my




- OR
 - Assign Syed for any position is 3 ways
 - Fill the highest remaining position is 5 ways
 - Fill the last position is 4 ways
 - $3 \cdot 5 \cdot 4 = 60$

Innovative • entrepreneurial • global www.utm.my



- In how many ways can this be done if Tan and Helmi must hold any position?
 - Assign Tan - 3 ways
 - Assign Helmi - 2 ways
 - Fill the remaining position - 4 ways
 - $3 \cdot 2 \cdot 4 = 24$


Innovative • entrepreneurial • global www.utm.my



Exercise

- How many eight-bit strings have either the second or the fourth bit 1 (or both)?
- How many license plates of 2 letters from A to Z, followed by 3 digits from 0 to 9 can be made, if repetition of letters is not allowed?

Innovative • entrepreneurial • global www.utm.my



Exercise

- How many 5-digit telephone numbers can be constructed using digit 0-9 if each number starts with 67 and no digit appears more than once?

Innovative • Entrepreneurial • Global www.utm.my