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Chapter 5.2 - HYPOTHESIS TESTING

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Introduction

- In statistics, a hypothesis is a claim or statement about a property of a population.
- A hypothesis test (or test of significance) is a standard procedure for testing a claim about a property of a population.

Introduction

- To conduct a Hypothesis Testing, we must establish the following :
  - Hypothesis statement
  - Test Statistic
  - Level of confidence
  - Conclusion/Decision rule

Examples of Hypotheses that can be Tested

- **Genetics:** The Genetics & IVF Institute claims that its XSORT method allows couples to increase the probability of having a baby girl.
- **Business:** A newspaper headline makes the claim that most workers get their jobs through networking.
- **Medicine:** Medical researchers claim that when people with colds are treated with echinacea, the treatment has no effect.

Examples of Hypotheses that can be Tested

- **Aircraft Safety:** The Federal Aviation Administration claims that the mean weight of an airline passenger (including carry-on baggage) is greater than 185 lb, which it was 20 years ago.

- **Quality Control:** When new equipment is used to manufacture aircraft altimeters, the new altimeters are better because the variation in the errors is reduced so that the readings are more consistent. (In many industries, the quality of goods and services can often be improved by reducing variation.)

Components of a Formal Hypothesis Test

- Null Hypothesis: H_0
- Alternative Hypothesis: H_1

Null Hypothesis: H_0

- The null hypothesis (denoted by H_0) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is equal to some claimed value.
- We test the null hypothesis directly either reject H_0 or fail to reject H_0.

Alternative Hypothesis: H_1

- The alternative hypothesis (denoted by H_1 or H_a or H_A) is the statement that the parameter has a value that somehow differs from the null hypothesis.
- The symbolic form of the alternative hypothesis must use one of these symbols: $\neq$, $<$, $>$.

Note about Identifying H_0 and H_1

Start

↓

Identify the specific claim or hypothesis to be tested, and express it in symbolic form.

↓

Give the symbolic form that must be true when the original claim is false.

↓

Of the two symbolic expressions obtained so far, let the alternative hypothesis H_1 be the one not containing equality, so that H_1 uses the symbol $<$ or $>$ or $\neq$. Let the null hypothesis H_0 be the symbolic expression that the parameter equals the fixed value being considered.

Stating Hypothesis - Example 1

- Sharing prescription drugs with others can be dangerous. A survey of a representative sample of 592 U.S. teens age 12 to 17 reported that 118 of those surveyed admitted to having shared a prescription drug with a friend. Is this sufficient evidence that **more than 10%** of teens have shared prescription medication with friends?
- State the hypotheses :

$$H_0: p = .1$$

$$H_1: p > .1$$

Stating Hypothesis - Example 2

- Compact florescent (cfl) light bulbs are much more energy efficient than regular incandescent light bulbs. Eco bulb brand 60-watt cfl light bulbs state on the package “Average life 8000 hours”. People who purchase this brand would be unhappy if the bulbs lasted **less than 8000 hours**. A sample of these bulbs will be selected and tested.
- State the hypotheses :

H_0: $\mu = 8000$

H_1: $\mu < 8000$

The true mean (μ) life of the cfl light bulbs

One sided alternative

Stating Hypothesis - Example 3

- Because in variation of the manufacturing process, tennis balls produced by a particular machine do not have the same diameters. Suppose the machine was initially calibrated to achieve the specification of *m* = 3 inches. However, the manager is now concerned that the diameters no longer conform to this specification. If the **mean diameter is not 3** inches, production will have to be halted.
- State the hypothesis

$$H_0: \mu = 3$$

$$H_1: \mu \neq 3$$

Two sided alternative

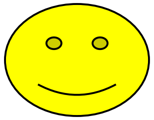
Example

- For each pair of hypotheses, indicate which are not legitimate and explain why

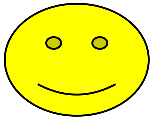
$$\text{a) } \mathrm{H}_0 : \mu = 15; \ \mathrm{H}_1 : \mu \geq 15$$

$$\text{b) } \mathrm{H}_0 : \bar{x} = 4; \ \mathrm{H}_1 : \bar{x} < 15$$

$$\text{c) } \mathrm{H}_0 : p = .1; \ \mathrm{H}_1 : p \neq .1$$



$$\text{d) } \mathrm{H}_0 : \mu = 2.3; \ \mathrm{H}_1 : \mu > 3.2$$



$$\text{e) } \mathrm{H}_0 : p \neq .5; \ \mathrm{H}_1 : p = .5$$

Exercise 1 – Identify H_0 and H_1

- The mean annual income of employees who took a statistics course is greater than $60,000.
- The proportion of people aged 18 to 25 who currently use illicit drugs is equal to 0.20 (or 20%).
- The standard deviation of human body temperatures is equal to 0.62°F.
- The standard deviation of duration times (in seconds) of the Old Faithful geyser is less than 40 sec.
- The standard deviation of daily rainfall amounts in San Francisco is 0.66 cm.
- The proportion of homes with fire extinguishers is 0.80.
- The mean weight of plastic discarded by households in one week is less than 1 kg.

Exercise 1 – Identify H_0 and H_1

- Researchers have postulated that, due to differences in diet, Japanese children have a lower mean blood cholesterol level than U.S. children. Suppose that the mean level of U.S. children is known to be 170.
- A water quality control board reports that water is unsafe for drinking if the mean nitrate concentration exceeds 30ppm. Water specimens are taken from a well.
- Last year, your company 's service technicians took an average of 2.6 hours to respond to trouble calls from business customers who had purchased service contracts. Do this year's data show a different average response time?

Test of Statistical Hypothesis - Formula

Test statistic for proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Test statistic for mean

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad \text{or} \quad t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Test statistic for standard deviation

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Example

- Let's again consider the claim that the XSORT method of gender selection increases the likelihood of having a baby girl. Preliminary results from a test of the XSORT method of gender selection involved 14 couples who gave birth to 13 girls and 1 boy. Null and alternative hypotheses H_0: p = 0.5 and H_1: p > 0.5
- Calculate the value of the test statistic.

Solution

- H_0: $p = 0.5$
- H_1: $p > 0.5$.
- The sample proportion of 13 girls in 14 births results in $\hat{p} = 13/14 = 0.929$
- Using $p = 0.5$, $\hat{p} = 0.929$ and $n = 14$, we find the value of the test statistic as follows:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.929 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{14}}} = 3.21$$

Critical Region

- The critical region (or rejection region) is the set of all values of the test statistic that cause us to reject the null hypothesis

Significance Level

- The significance level (denoted by α) is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true.
- Common choices for α are 0.05, 0.01, and 0.10.

Critical Value

- A critical value is any value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to rejection of the null hypothesis.
- The critical values depend on the nature of the null hypothesis, the sampling distribution that applies, and the significance level α.

P-Value

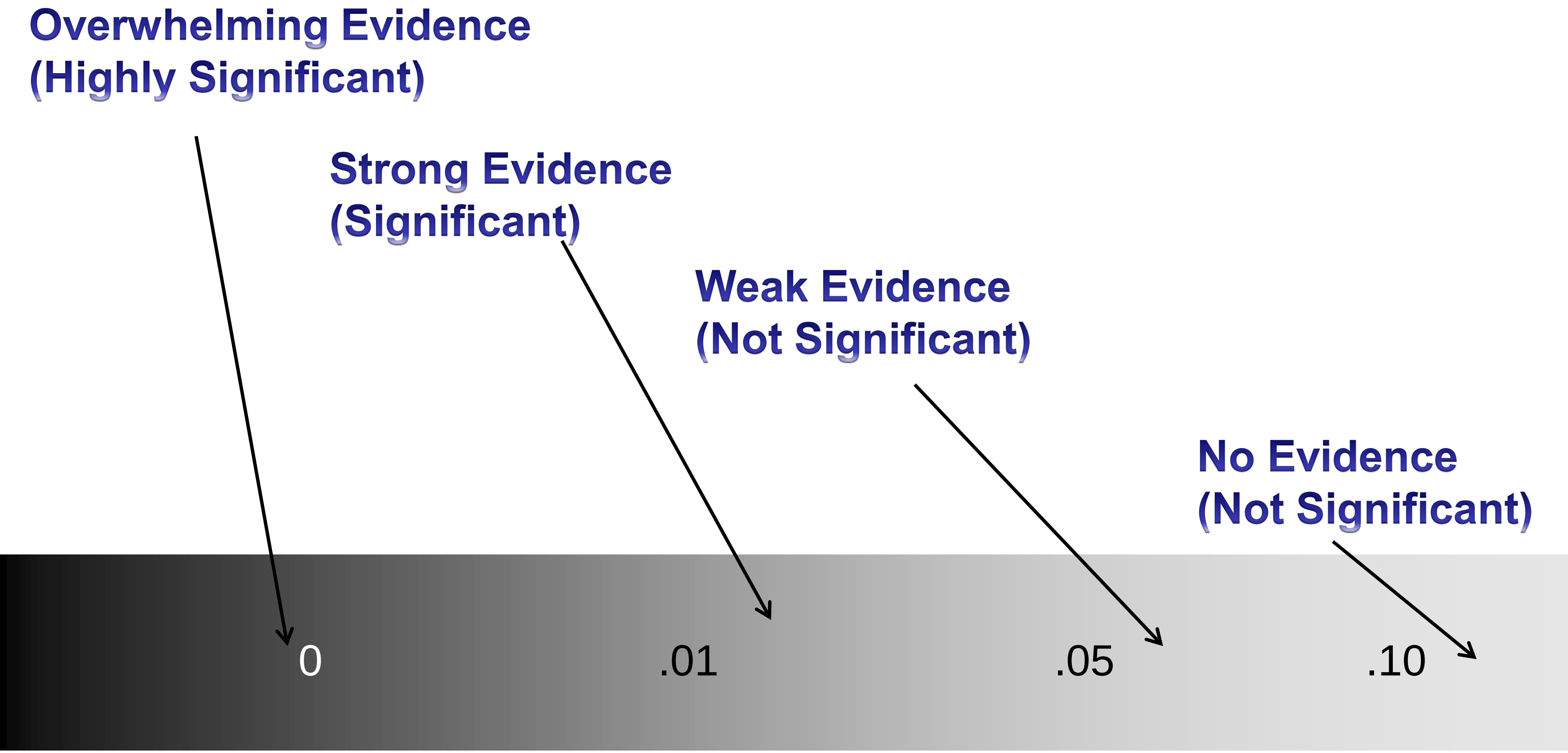
- The *P*-value (or *p*-value or probability value) is the probability of getting a value of the test statistic that is at least as extreme as the one representing the sample data, assuming that the null hypothesis is true.

Critical region in the left tail: **P-value = area to the left of the test statistic**

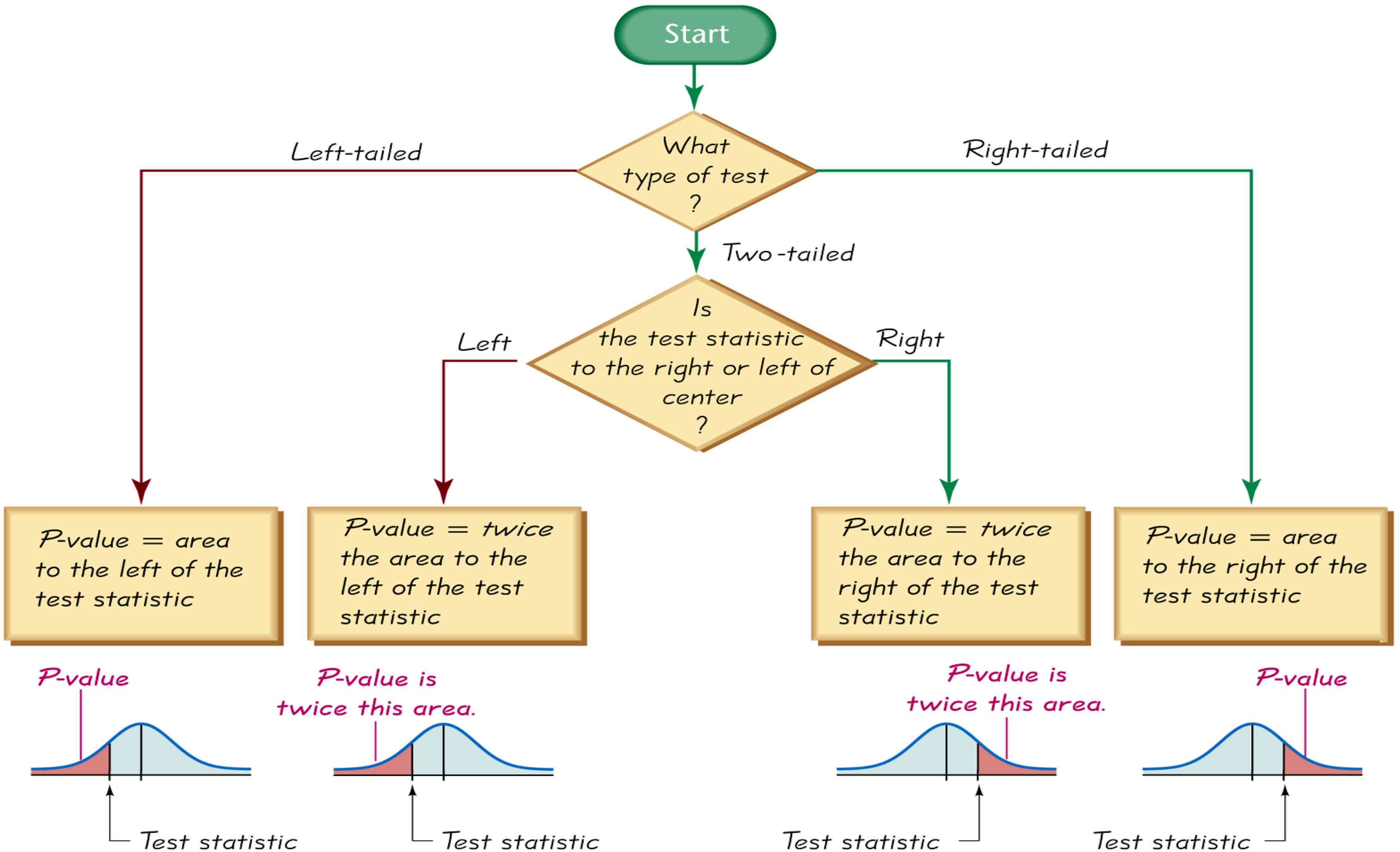
Critical region in the right tail: **P-value = area to the right of the test statistic**

Critical region in two tails: **P-value = twice the area in the tail beyond the test statistic**

Interpreting the *P-Value*



Procedure for Finding *P*-Value



Caution

- Don't confuse a *P*-value with a proportion *p*.
- Know this distinction:

***P*-value =** probability of getting a test statistic at least as extreme as the one representing sample data

***p* =** population proportion

Example

- Finding *P*-values. *Use a 0.05 significance level and state the conclusion about the null hypothesis (reject the null hypothesis or fail to reject the null hypothesis).*
  - The test statistic in a left-tailed test is z = - 1.25.
  - The test statistic in a right-tailed test is z = 2.50.
  - The test statistic in a two-tailed test is z = 1.75.

Solution

- The test statistic in a left-tailed test is $z = -1.25$.

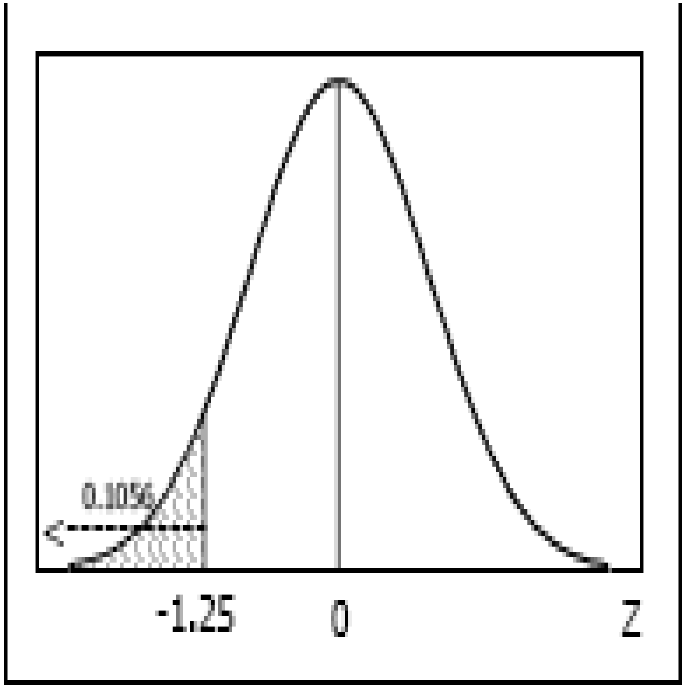
Answer:

P-value

$= P(z < -1.25)$

$= 0.1056$

Since $0.1056 > 0.05$, fail to reject H_0.



Solution

- The test statistic in a right-tailed test is $z = 2.50$.

Answer:

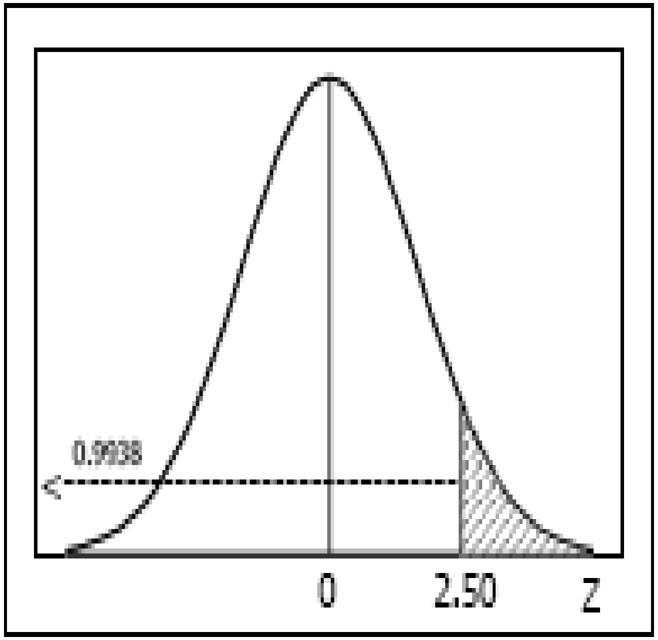
P-value

$= P(z > 2.50)$

$= 1 - 0.9938$

$= 0.0062$

Since $0.0062 < 0.05$, reject H_0.



Solution

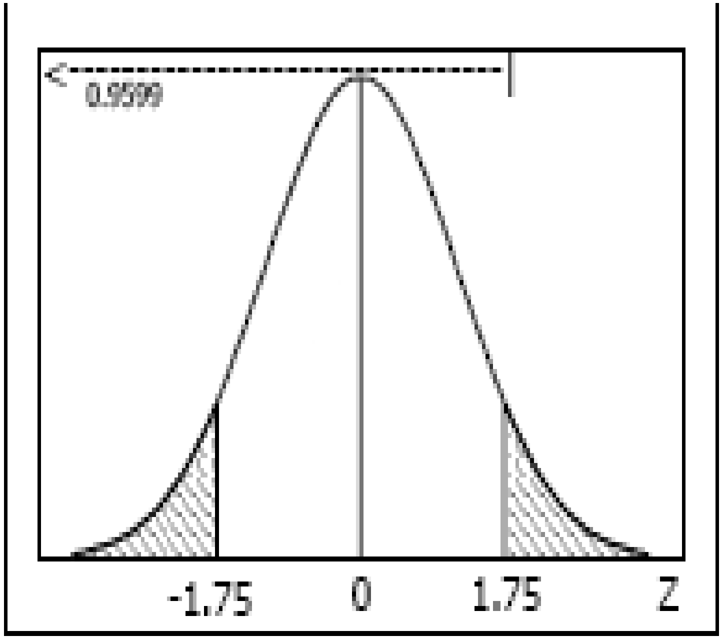
- The test statistic in a two-tailed test is $z = 1.75$.

Answer:

P-value

$$= 2 \cdot P(z > 1.75)$$
$$= 2 \cdot [1 - 0.9599]$$
$$= 2 \cdot [0.0401]$$
$$= 0.0802$$

Since $0.0802 > 0.05$, fail to reject H_o.



Types of Hypothesis Test: Two-tailed, Left-tailed, Right-tailed

- Determinations of *P*-values and critical values are affected by whether a critical region is in two tails, the left tail, or the right tail. It therefore becomes important to correctly characterize a hypothesis test as two-tailed, left-tailed, or right-tailed.
- The tails in a distribution are the extreme regions bounded by critical values.

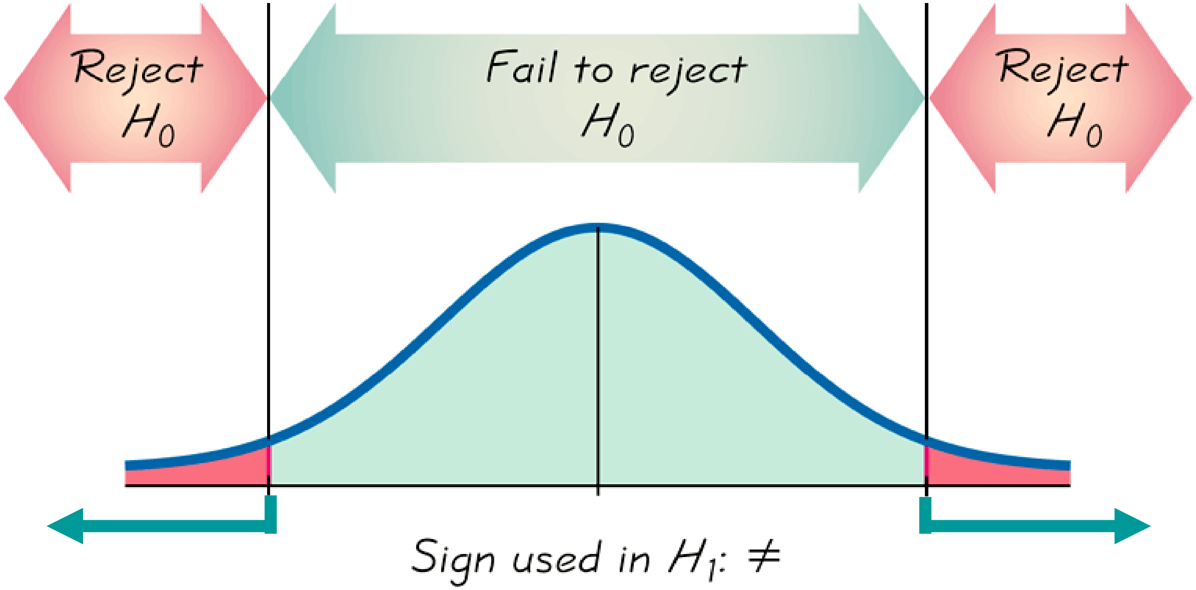
Two-tailed Test

H_0: =

H_1: ≠

α is divided equally between the two tails of the critical region

Means less than or greater than



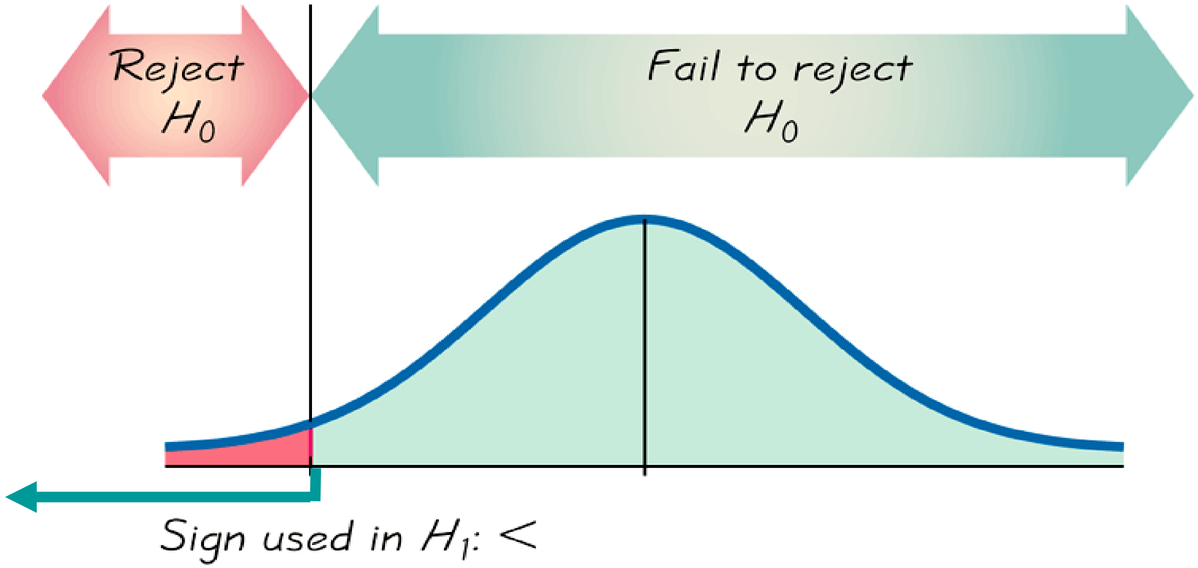
Left-tailed Test

α the left tail

H_0: =

H_1: <

Points Left

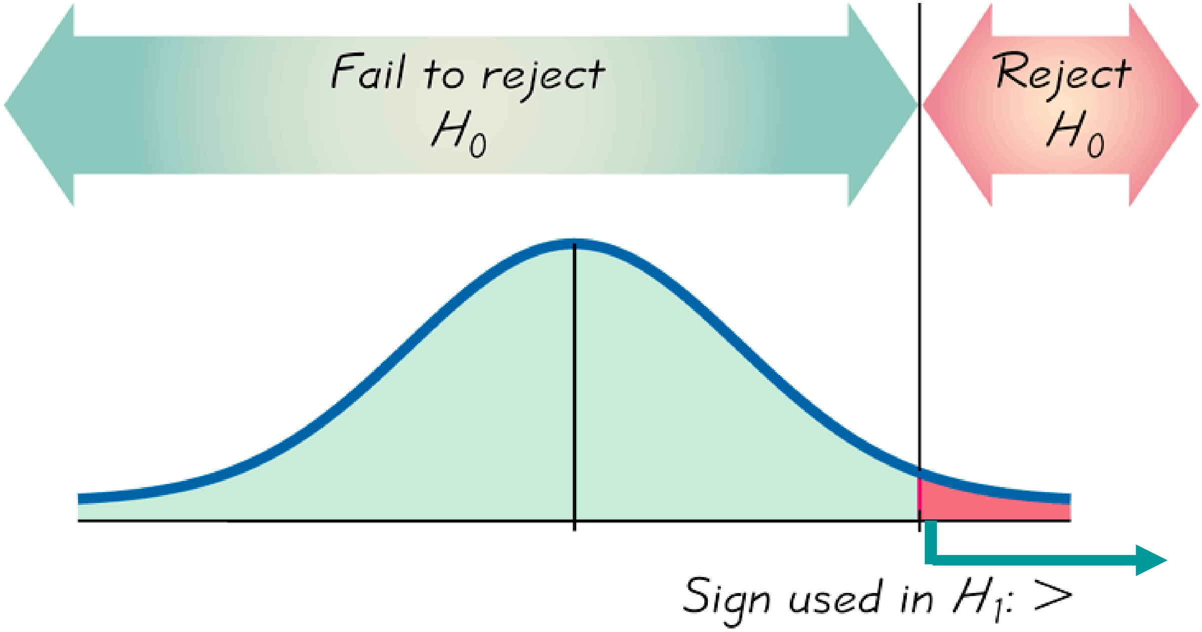


Right-tailed Test

H_0: =

H_1: >

Points Right



Conclusions in Hypothesis Testing

- We always test the null hypothesis.
- The initial conclusion will always be one of the following:

1. Reject the null hypothesis

2. Fail to reject the null hypothesis

Decision Criterion

- *P*-value method:

  Using the significance level α:

  - If *P*-value $\leq \alpha$, reject H_0.
  - If *P*-value > α , fail to reject H_0.

Decision Criterion

- Traditional method:
  - If the test statistic falls within the critical region, reject H_0.
  - If the test statistic does not fall within the critical region, fail to reject H_0.

Example

- Consider the claim that with the XSORT method of gender selection, the likelihood of having a baby girl is different from $p = 0.5$, and use the test statistic $z = 3.21$ found from 13 girls in 14 births.
- First determine whether the given conditions result in a critical region in the right tail, left tail, or two tails, then to find the *P*-value.

Solution

- The claim that the likelihood of having a baby girl is different from $p = 0.5$ can be expressed as $p \neq 0.5$ so the critical region is in two tails.
- To find the *P*-value for a two-tailed test, we see that the *P*-value is *twice* the area to the right of the test statistic $z = 3.21$.
- In this case, the *P*-value is twice the area to the right of the test statistic, so we have:

$$P\text{-value} = 2 \times 0.0007 = 0.0014$$

Error of Decision

- A Type I error is the mistake of rejecting the null hypothesis when it is actually true.
  - The symbol α (alpha) is used to represent the probability of a type I error.
- A Type II error is the mistake of failing to reject the null hypothesis when it is actually false.
  - The symbol β (beta) is used to represent the probability of a type II error.

Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	**Type I error** (rejecting a true null hypothesis) $P(\text{type I error}) = \alpha$	Correct decision
	We fail to reject the null hypothesis	Correct decision	**Type II error** (failing to reject a false null hypothesis) $P(\text{type II error}) = \beta$

Example

- Assume that we are conducting a hypothesis test of the claim that a method of gender selection increases the likelihood of a baby girl, so that the probability of a baby girls is $p > 0.5$. Here are the null and alternative hypotheses: H_0: $p = 0.5$, and H_1: $p > 0.5$.

  a) Identify a type I error.

  b) Identify a type II error.

Solution

a) A type I error is the mistake of rejecting a true null hypothesis, so this is a type I error: Conclude that there is sufficient evidence to support $p > 0.5$, when in reality $p = 0.5$.

b) A type II error is the mistake of failing to reject the null hypothesis when it is false, so this is a type II error: Fail to reject $p = 0.5$ (and therefore fail to support $p > 0.5$) when in reality $p > 0.5$.

Hypothesis Testing Using One Sample Test

One Sample Test

- Test on mean, variance known
- Test on mean, variance unknown
- Test on variance/standard deviation

Steps in Hypothesis Testing

1. From the problem context, identify the parameter of interest.
2. State the null hypothesis, H_0.
3. Specify an appropriate alternative hypothesis, H_1 .
4. Choose a significance level α.
5. Determine an appropriate test statistic.
6. State the rejection region for the statistic.
7. Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
8. Decide whether or not H_0 should be rejected and report that in the problem context.

Steps 1 – 4 should be completed prior to examination of the sample data.

Test on Mean, Variance Known

$$z = \frac{x - \mu_x}{\frac{\sigma}{n}}$$

Test on Mean, Variance Known

- Assumption: Random Sample is normally distributed. The sample size usually small. We want to test:

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0 \qquad (5.1a)$$

or

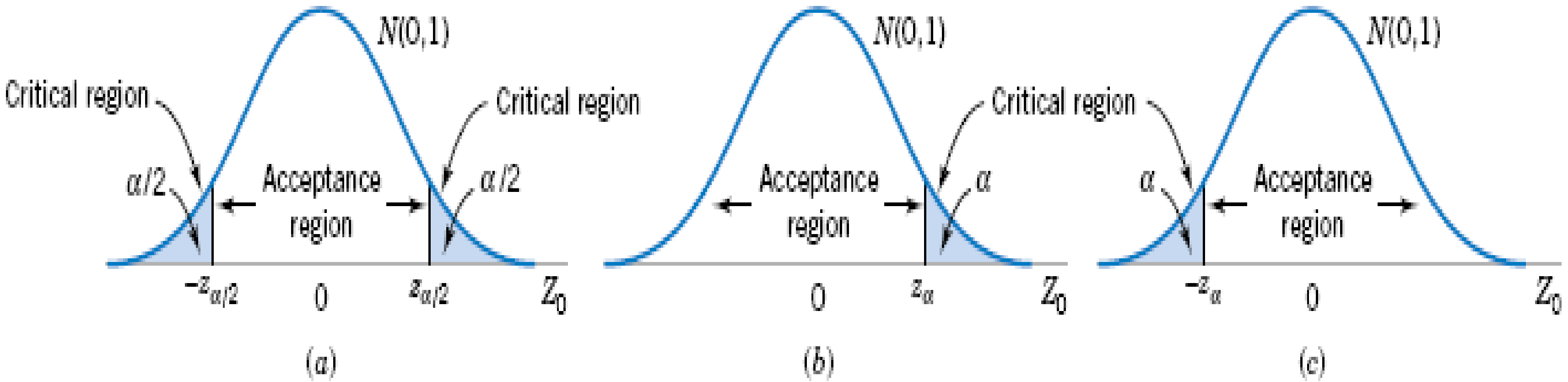
$$H_0: \mu = \mu_0$$
$$H_1: \mu > \mu_0 \qquad (5.1b)$$

or

$$H_0: \mu = \mu_0$$
$$H_1: \mu < \mu_0 \qquad (5.1c)$$

with variance σ is known.

Test on Mean, Variance Known



The distribution of Z_0 when H_0: $\mu = \mu_0$ is true, with critical region (a) the two-sided alternative H_1: $\mu \neq \mu_0$ (b) the one-sided alternative H_1: $\mu > \mu_0$ and (c) the one-sided alternative H_1: $\mu < \mu_0$.

Test on Mean, Variance Known

- Null hypothesis : *H: m=m$_o$*
- Test statistic

Alternative hypothesis	Rejection Region
$H_1 : \mu \neq \mu_0$	$\lvert z \rvert \geq z_{\alpha/2}$
$H_1 : \mu > \mu_0$	$z \geq z_{\alpha}$
$H_1 : \mu < \mu_0$	$z \leq -z_{\alpha}$

Example

- **Writing a Hit Song** In the manual "How to Have a Number One the Easy Way," by KLF Publications, it is stated that a song "must be no longer than three minutes and thirty seconds" (or 210 seconds). A simple random sample of 40 current hit songs results in a mean length of 252.5 sec. (The songs are by Timberlake, Furtado, Daughtry, Stefani, Fergie, Akon, Ludacris, etc.) Assume that the standard deviation of song lengths is 54.5 sec. Use a 0.05 significance level to test the claim that the sample is from a population of songs with a mean greater than 210 sec. What do these results suggest about the advice given in the manual?

Solution

original claim: $\mu > 210$ sec

H_o: $\mu = 210$ sec

H_1: $\mu > 210$ sec

$\alpha = 0.05$

C.V. $z = z_\alpha = z_{0.05} = 1.645$

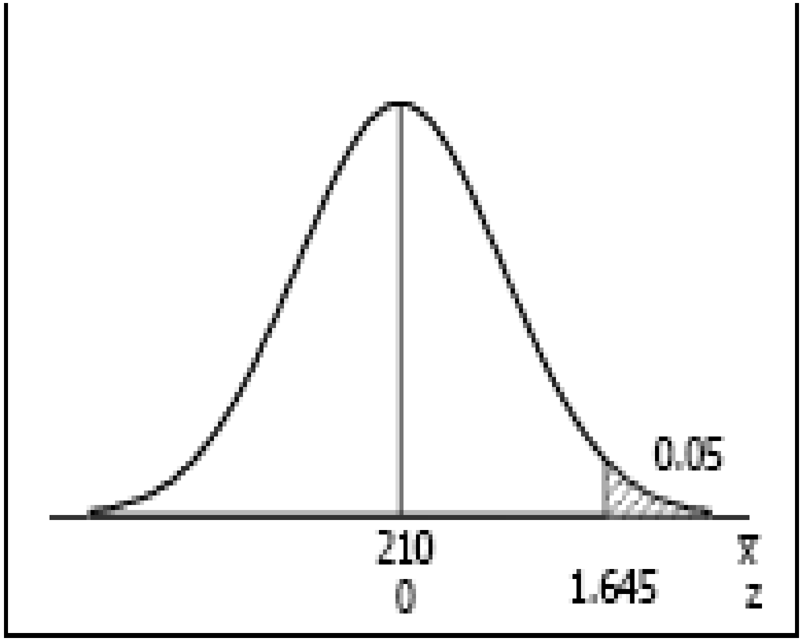
calculations:

$$z_{\bar{x}} = (\bar{x} - \mu_{\bar{x}})/\sigma_{\bar{x}}$$

$$= (252.5 - 210)/(54.5/\sqrt{40})$$

$$= 42.5/8.6172 = 4.93$$

P-value = $P(z>4.93) = 1 - 0.9999 = 0.0001$



conclusion:

Reject H_o; there is sufficient evidence to conclude that $\mu > 210$. There is sufficient evidence to support the claim that the sample is from a population of songs with a mean greater than 210 seconds. These results suggest that the advice given in the manual is not good advice.

Exercise 2

- **Red Blood Cell Count** A simple random sample of 50 adults is obtained, and each person's red blood cell count (in cells per microliter) is measured. The sample mean is 5.23. The population standard deviation for red blood cell counts is 0.54. Use a 0.01 significance level to test the claim that the sample is from a population with a mean less than 5.4, which is a value often used for the upper limit of the range of normal values. What do the results suggest about the sample group?

Exercise 3

- **M&M Weights** A simple random sample of the weights of 19 green M&Ms has a mean of 0.8635 g. Assume that is known to be 0.0565 g. Use a 0.05 significance level to test the claim that the mean weight of all green M&Ms is equal to 0.8535 g, which is the mean weight required so that M&Ms have the weight printed on the package label. Do green M&Ms appear to have weights consistent with the package label?

Test on Mean, Variance Unknown

- Assumption: Random Sample is normally distributed.
- We want to test

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0 \qquad (5.2a)$$

or

$$H_0: \mu = \mu_0$$
$$H_1: \mu > \mu_0 \qquad (5.2b)$$

or

$$H_0: \mu = \mu_0$$
$$H_1: \mu < \mu_0 \qquad (5.2c)$$

with variance σ is unknown.

Test on Mean, Variance Unknown

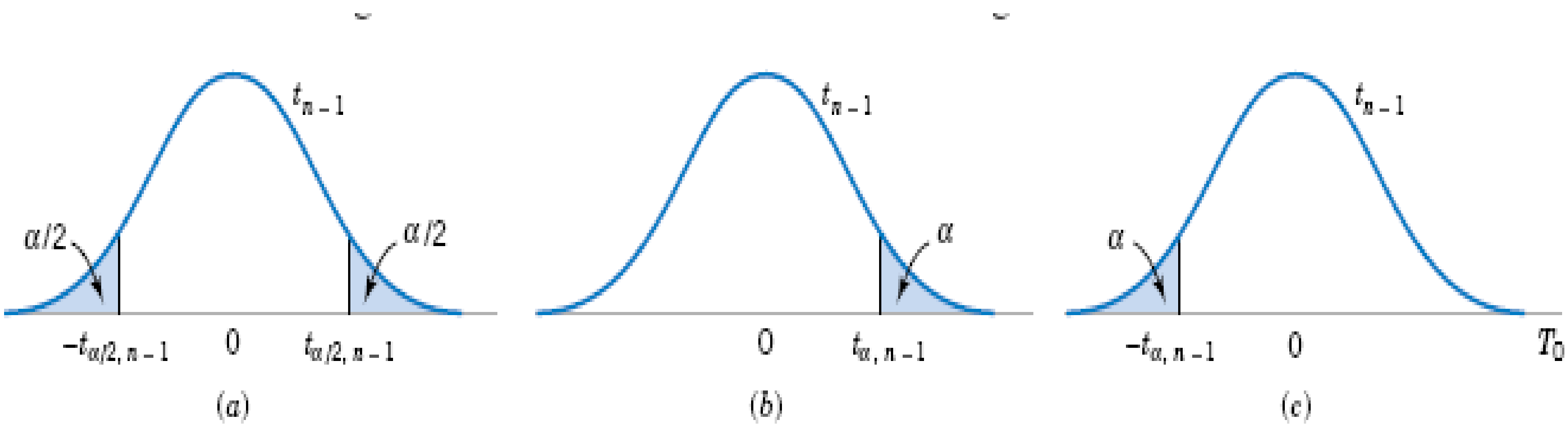
- Test statistic concerning one mean for data that can be assumed to follow a normal distribution but s is unknown and the sample is small (n < 30), the test statistic is:

$$T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

- When testing hypotheses concerning one mean for data that can be assumed to follow a normal distribution and s is unknown, but the sample is large (n >30), the test statistic is:

$$Z = \frac{\bar{x} - \mu_o}{s/\sqrt{n}}$$

Test on Mean, Variance Unknown



The reference distribution for H_0: $\mu = \mu_0$ with critical region for (a) the two-sided alternative H_1: $\mu \neq \mu_0$ (b) the one-sided alternative H_1: $\mu > \mu_0$ and (c) the one-sided alternative H_1: $\mu < \mu_0$.

Example

The increased availability of light materials with high strength has revolutionized the design and manufacture of golf clubs, particularly drivers. Clubs with hollow heads and very thin faces can result in much longer tee shots, especially for players of modest skills. This is due partly to the "spring-like effect" that the thin face imparts to the ball. Firing a golf ball at the head of the club and measuring the ratio of the outgoing velocity of the ball to the incoming velocity can quantify this spring-like effect. The ratio of velocities is called the coefficient of restitution of the club. An experiment was performed in which 15 drivers produced by a particular club maker were selected at random and their coefficients of restitution measured. In the experiment the golf balls were fired from an air cannon so that the incoming velocity and spin rate of the ball could be precisely controlled. It is of interest to determine if there is evidence (with $\alpha = 0.05$) to support a claim that the mean coefficient of restitution exceeds 0.82. The observations follow:

0.8411	**0.8191**	**0.8182**	**0.8125**	**0.8750**
0.8580	**0.8532**	**0.8483**	**0.8276**	**0.7983**
0.8042	**0.8730**	**0.8282**	**0.8359**	**0.8660**

Solution

- From these data, we have

  $\overline{x}$ = 0.83725 and s = 0.02456

- Our objective is to demonstrate that the mean coefficient of restitution exceeds 0.82
- Therefore, a one-sided alternative hypothesis is appropriate.

Solution

- The parameter of interest is the mean coefficient of restitution, μ.
- Hypothesis

  H_0: μ = 0.82

  H_1: μ >0.82.

  We want to reject H_0 if the mean coefficient of restitution exceeds 0.82.
- α=0.05.
- The test statistic is $$T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

Solution

- Reject H_0 if $t_0 > t_{0.05,14} = 1.761$.
- Computations: Since = 0.83725, s = 0.02456, $\mu_0 = 0.82$ and $n = 15$, we have

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{0.83725 - 0.82}{0.02456/\sqrt{15}} = 2.72.$$

- Conclusion: Since $t_0 = 2.72 > 1.761$, we reject H_0: $\mu = 0.82$ at the 0.05 level of significance that the mean coefficient of restitution exceeds 0.82.

Exercise 4

- **Cigarette Tar** A simple random sample of 25 filtered 100 mm cigarettes is obtained, and the tar content of each cigarette is measured. The sample has a mean of 13.2 mg and a standard deviation of 3.7 mg. Use a 0.05 significance level to test the claim that the mean tar content of filtered 100 mm cigarettes is less than 21.1 mg, which is the mean for unfiltered king size cigarettes. What do the results suggest about the effectiveness of the filters?

Exercise 5

- **Analysis of Pennies** In an analysis investigating the usefulness of pennies, the cents portions of 100 randomly selected credit card charges are recorded. The sample has a mean of 47.6 cents and a standard deviation of 33.5 cents. If the amounts from 0 cents to 99 cents are all equally likely, the mean is expected to be 49.5 cents. Use a 0.01 significance level to test the claim that the sample is from a population with a mean equal to 49.5 cents. What does the result suggest about the cents portions of credit card charges?

Test on Mean, Variance / Standard Deviation

- Suppose that we wish to test the hypothesis that the variance of a normal population σ^2 equals a specified value, say σ_0^2, or equivalently, that the standard deviation σ is equal to σ_0. Let $X_1, X_2, \ldots, X_n$ be a random sample of n observations from this population.

Test on Mean, Variance / Standard Deviation

We want to test:

$$H_0: \sigma^2 = \sigma_0^2$$
$$H_1: \sigma^2 \neq \sigma_0^2 \qquad (5.3a)$$

or

$$H_0: \sigma^2 = \sigma_0^2$$
$$H_1: \sigma^2 > \sigma_0^2 \qquad (5.3b)$$

or

$$H_0: \sigma^2 = \sigma_0^2$$
$$H_1: \sigma^2 < \sigma_0^2 \qquad (5.3c)$$

Test on Mean, Variance / Standard Deviation

- Chi-Square Distribution

Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Test on Mean, Variance / Standard Deviation

n = sample size

s = sample standard deviation

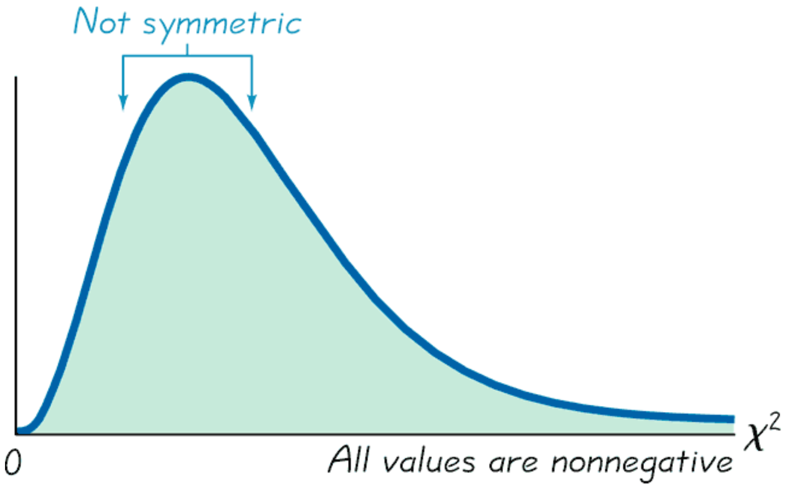
s^2 = sample variance

σ = claimed value of the population standard deviation

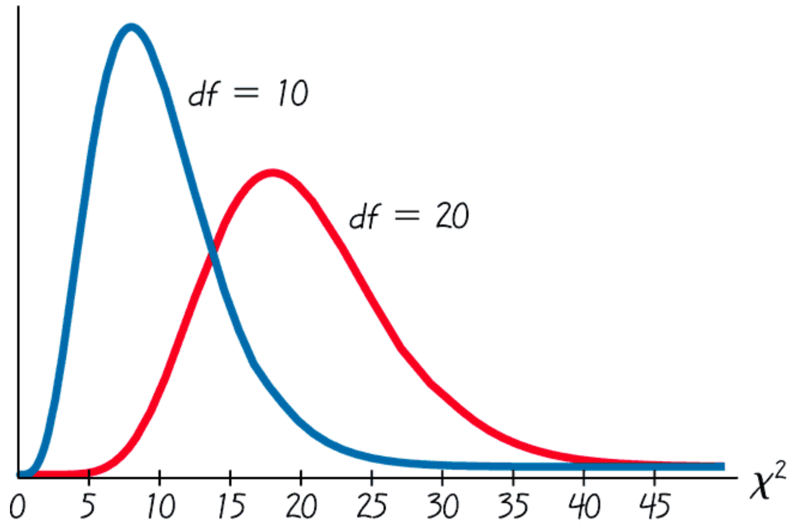
σ^2 = claimed value of the population variance

Properties of Chi-Square Distribution

Properties of the Chi-Square Distribution

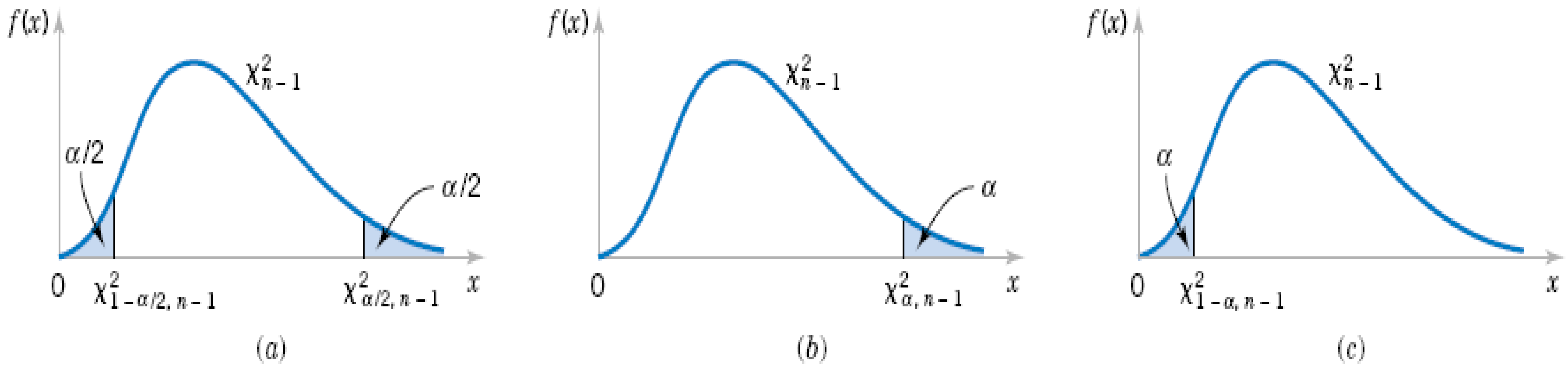


Chi-Square Distribution for 10 and 20 *df*



Different distribution for each number of *df*.

Test on Mean, Variance / Standard Deviation



The reference distribution for H_0: $\sigma^2 = \sigma_0^2$ with critical region for (a) the two-sided alternative H_1: $\mu \neq \mu_0$ (b) the one-sided alternative H_1: $\mu > \mu_0$ and (c) the one-sided alternative H_1: $\mu < \mu_0$.

Example

- A police chief claims that the standard deviation in the length of response times is less than 3.7 minutes.
- A random sample of nine response times has a standard deviation of 3.0 minutes.
- At a = 0.05, is there enough evidence to support the police chief's claim?
- Assume the population is normally distributed.

Solution

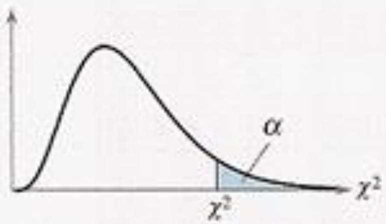
- The hypothesis ,

  $H_0: \sigma = 3.7$

  $H_1: \sigma < 3.7$

$n = 9$
$df = 8$
$s = 3.0$
$s^2 = 9$
$\sigma = 3.7$
$\sigma^2 = 13.69$
$\alpha = 0.05$

Solution



Right tail

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188

Why are we using right tail?
This is the way the chi-square table is designed.
So if its left-tail 5%, use the number for right-tail 95%.
So our critical value is 2.733.
Since it is a left-tailed test, our rejection region will be less than 2.733.

Solution

- Test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{8(3)^2}{3.7^2} = 5.2593$$

Since 5.2593 is not less than 2.733, it is not in our rejection region. Therefore, we will fail to reject the null.

Exercise 6

- A dairy processing company claims that the variance of the amount of fat in the whole milk processed by the company is no more than 0.25. You suspect this is wrong and find that a random sample of 41 milk containers has a variance of 0.27. At $\alpha = 0.05$, is there enough evidence to reject the company's claim? Assume the population is normally distributed.

Exercise 7

- A restaurant claims that the standard deviation in the length of serving times is less than 2.9 minutes. A random sample of 23 serving times has a standard deviation of 2.1 minutes. At $\alpha = 0.10$, is there enough evidence to support the restaurant's claim? Assume the population is normally distributed.