Chapter 5.3
Hypothesis Testing – 2 Sample Test

Example Questions on 2 samples test

- Does taking a small dose of aspirin every day reduce the risk of heart attack?
  – *Compare a group that takes aspirin with one that doesn't.*
- Do men and women in the same occupation have different salaries.
  – *Compare a sample of men with a sample of women.*
- Does fuel A lead to better gas mileage than fuel B?
  – *Compare the gas mileage for a fleet of cars when they use fuel A, to when those same cars use fuel B*

Illustrative problem for Two Samples test

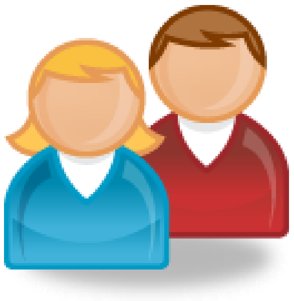
Illustrative problem

- Suppose we have a population of adult men with a mean height of 71 inches and standard deviation of 2.5 inches. We also have a population of adult women with a mean height of 65 inches and standard deviation of 2.3 inches. In this case the researcher is interested in comparing the mean height between men and women. His research question is: Does the mean height of men differ from the mean height of women?. Here, the hypotheses are

$$H_0 : \mu_1 = \mu_2 \quad \text{and} \quad H_0 : \mu_1 \neq \mu_2$$

where μ_1= mean height of men

μ_2= mean height of women



Inferences from two samples

- Two Proportions
- Two Means: Independent Samples
- Two Dependent Samples (Matched Pairs)
- Two Variances or Standard Deviations

Inferences About Two Proportion:

Note: the same methods for dealing with probabilities or the decimal equivalents of percentages.

Two Proportions – Notation symbols

For population 1:

p_1 = population proportion

n_1 = size of the sample

x_1 = number of successes in the sample

$$\hat{p}_1 = \frac{x_1}{n_1}$$ (the sample proportion)

$$\hat{q}_1 = 1 - \hat{p}_1$$

The corresponding notations apply to

$p_2, n_2, x_{2,}\ \hat{p}_2\ and\ \hat{q}_2,$ which come from population 2.

Pooled Sample Proportion

The pooled sample proportion is given by:

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$\overline{q} = 1 - \overline{p}$$

Requirement

1. We have proportions from two independent simple random samples.
2. For each of the two samples, the number of successes is at least 5 and the number of failures is at least 5.

Test Statistic for Two Proportions

For $H_0 : p_1 = p_2$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p}\overline{q}}{n_1} + \frac{\overline{p}\overline{q}}{n_2}}}$$

where (assumed in the null hypothesis) $p_1 - p_2 = 0$

Testing Two Proportions

- *P*-value: P-values are automatically provided by technology (e.g SPSS). If technology is not available, use Table.

- Critical values: Use Table A-2.

Example

Do people having different spending habits depending on the type of money they have?

89 undergraduates were randomly assigned to two groups and were given a choice of keeping the money or buying gum or mints.

The claim is that “money in large denominations is less likely to be spent relative to an equivalent amount in many smaller denominations”.

Let’s test the claim at the 0.05 significance level.

Example

- Below are the sample data and summary statistics:

	Group 1	**Group 2**
	Subjects Given $1 Bill	Subjects Given 4 Quarters
Spent the money	x_1=12	x_2=27
Subjects in group	n_1=46	n_2=43

$$\hat{p}_1 = \frac{12}{46} \qquad \hat{p}_2 = \frac{27}{43}$$

$$\bar{p} = \frac{12+27}{46+43} = 0.438202$$

Example

Requirement Check:

1. The 89 subjects were randomly assigned to two groups, so we consider these independent random samples.

2. The subjects given the $1 bill include 12 who spent it and 34 who did not. The subjects given the quarters include 27 who spent it and 16 who did not. All counts are above 5, so the requirements are all met.

Example

Step 1: The claim that "money in large denominations is less likely to be spent" can be expressed as $p_1 < p_2$.

Step 2: If $p_1 < p_2$ is false, then $p_1 \geq p_2$.

Step 3: The hypotheses can be written as:

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 < p_2$$

Example

Step 4: The significance level is $\alpha = 0.05$.

Step 5: We will use the normal distribution to run the test with:

$$\hat{p}_1 = \frac{12}{46} \qquad \hat{p}_2 = \frac{27}{43}$$

$$\overline{p} = \frac{12+27}{46+43} = 0.438202$$

$$\overline{q} = 1 - \overline{p} = 1 - 0.438202 = 0.561798$$

Example

Step 6: Calculate the value of the test statistic:

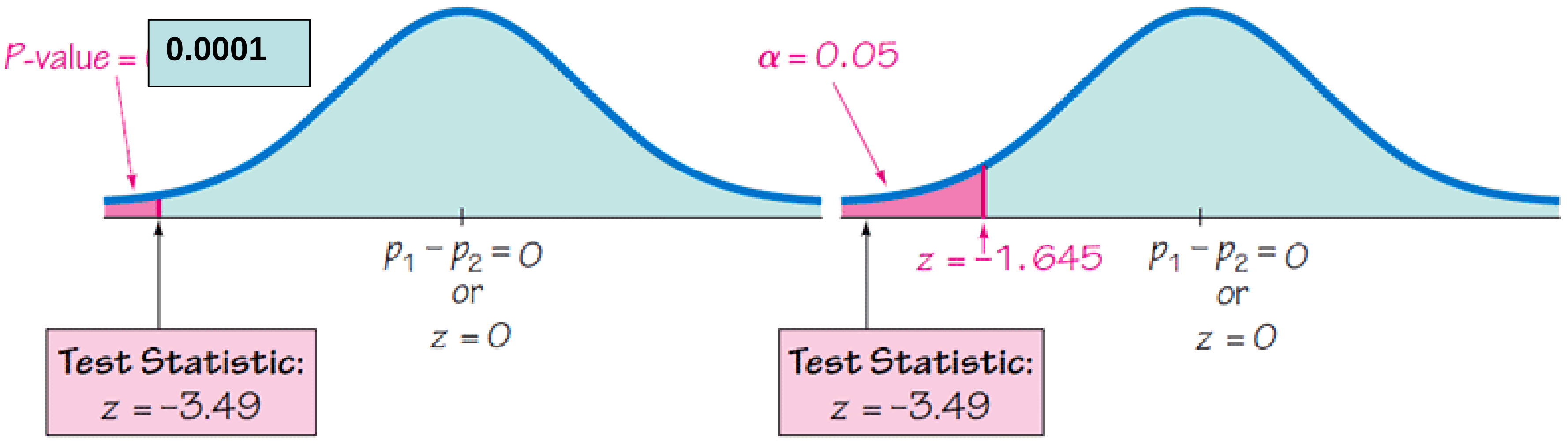
$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p}\overline{q}}{n_1} + \frac{\overline{p}\overline{q}}{n_2}}}$$

$$= \frac{\left(\frac{12}{46} - \frac{27}{43}\right) - 0}{\sqrt{\frac{(0.438202)(0.561798)}{46} + \frac{(0.438202)(0.561798)}{43}}}$$

$$= -3.49$$

Example

Step 6: This is a left-tailed test, so the *P*-value is the area to the left of the test statistic z = –3.49, or 0.0001. The critical value is also shown below.



(a) *P*-Value Method (b) Critical Value Method

Example

Step 7: Because the *P*-value of 0.0002 is less than the significance level of $\alpha = 0.05$, reject the null hypothesis.

There is sufficient evidence to support the claim that people with money in large denominations are less likely to spend relative to people with money in smaller denominations.

It should be noted that the subjects were all undergraduates and care should be taken before generalizing the results to the general population.

Exercise

- Time magazine reported the result of a telephone poll of 800 adult Americans. The question posed of the Americans who were surveyed was: "Should the federal tax on cigarettes be raised to pay for health care reform?" The results of the survey were:

Non-smoker	Smoker
N_1=605	**N_2=195**
Y1=351 said yes	**Y2=41 said yes**

- Is there sufficient evidence at the $\alpha = 0.05$ level, say, to conclude that the two populations — smokers and non-smokers — differ significantly with respect to their opinions?

Inferences About Two Means: Independent Samples

Definition

Two samples are independent if the sample values selected from one population are not related to or somehow paired or matched with the sample values selected from the other population.

Two samples are dependent (or consist of matched pairs) if the members of one sample can be used to determine the members of the other sample.

Example

Independent Samples:

Researchers investigated the reliability of test assessment. On group consisted of 30 students who took proctored tests. A second group consisted of 32 students who took online tests without a proctor.

The two samples are independent, because the subjects were not matched or paired in any way.

Example

Dependent Samples:

Students of the author collected data consisting of the heights (cm) of husbands and the heights (cm) of their wives. Five of those pairs are listed below. The data are dependent, because each height of each husband is matched with the height of his wife.

Height of Husband	175	180	173	176	178
Height of Wife	160	165	163	162	166

Test on Difference Between Two Mean

Assumptions:-

1. The two population standard deviations are both known.

2. The two samples are independent.

   - X_{11}, X_{12}, , X_{1n1} is a random sample from population 1.

   - X_{21}, X_{22}, , X_{2n2} is a random sample from population 2

3. Both samples are simple random samples.

4. Either or both of these conditions are satisfied: The two sample sizes are both large (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions.

Test on Difference Between Two Mean

- Two population standard deviation σ_1 and σ_2 are known
- The test statistic will be a *z* and use the standard normal model.

Test on Mean with σ_1 and σ_2 (variances) are known

- consider hypothesis testing on the difference in the means $\mu_1 - \mu_2$ of two normal populations.
- The procedure of this testing can be summarized as follow:

Null hypothesis: $H_0\colon \mu_1 - \mu_2 = \Delta_0$

Test statistic: $$Z_0 = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Alternative Hypotheses	Rejection Criterion
$H_1\colon \mu_1 - \mu_2 \neq \Delta_0$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1\colon \mu_1 - \mu_2 > \Delta_0$	$z_0 > z_\alpha$
$H_1\colon \mu_1 - \mu_2 < \Delta_0$	$z_0 < -z_\alpha$

Test on Mean with σ_1 and σ_2 (variances) are known

- In the formula, $\bar{X}_1 - \bar{X}_2$ is the observed difference, and expected difference $\mu_1 - \mu_2$ is 0 when the null hypothesis is $\mu_1 = \mu_2$
- In the comparison of two sample means, the difference maybe due to chance, in which case the null hypothesis will not be rejected, and the researcher can assume that the means of the population are basically the same

Example

- A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested; formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another ten specimens are painted with formulation 2; the 20 specimens are painted in random order and normally distributed. The two sample average drying times are $\bar{x}_1$ = 121 minutes and $\bar{x}_2$ = 112 minutes, respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient, using α = 0.05?

Example

1. The quantity of interest is the difference in mean drying times, $\mu_1 - \mu_2$ and $\Delta_0 = 0$.
2. H_0: $\mu_1 - \mu_2 = 0$.
3. H_1: $\mu_1 > \mu_2$ .We want to reject H_0 if the new ingredient reduces mean drying time.
4. α =0.05.
5. The test statistic is

$$z_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{\sigma_1^2}{n1} + \frac{\sigma_2^2}{n2}}}$$

where $\sigma_1^2 = \sigma_2^2 = 8^2 = 64$ and $n1 = n2 = 10$.

Example

6. Reject H_0 if $z_0 > z_{0.05} = 1.645$.
7. Computations: Since $\overline{x_1}$ = 121 minutes and $\overline{x_2}$ = 112 minutes, we have

$$z_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{\sigma_1^2}{n1} + \frac{\sigma_2^2}{n2}}} = \frac{121 - 112}{\sqrt{\frac{64}{10} + \frac{64}{10}}} = 2.52.$$

8. Conclusion: Since $z_0 = 2.52 > 1.645$, we reject H_0 at the 0.05 level and conclude that adding the new ingredient to the paint significantly reduces the drying time.

Test on Mean, Variance unknown

- We now consider tests of hypotheses on the difference in means $\mu_1 - \mu_2$ of two normal distributions where the variances σ_1^2 and σ_2^2 are unknown.
- The variances of the two samples may be assumed to be equal or unequal.
  - **$\sigma_1^2 = \sigma_2^2 = \sigma^2$ (CASE 1)**
  - **$\sigma_1^2 \neq \sigma_2^2$ (CASE 2)**
- A *t*-statistic will be used to test these hypotheses.

Case 1 : $\sigma_1^2 = \sigma_2^2 = \sigma^2$

- We wish to test

$$H_0: \mu_1 - \mu_2 = \Delta_0$$
$$H_1: \mu_1 - \mu_2 \neq \Delta_0$$

  - Let $X_{11}, X_{12}, \ldots, X_{1n1}$ be a random sample of n_1 observations from the first population
  - *Let* $X_{21}, X_{22}, \ldots, X_{2n2}$ be a random sample of n_2 observations from the second population.
  - Let S_1^2, and S_2^2 be sample variances, respectively.

Case 1 : $\sigma_1^2 = \sigma_2^2 = \sigma^2$

- The procedure of testing:

Null hypothesis: $H_0\colon \mu_1 - \mu_2 = \Delta_0$

Test statistic: $$T_0 = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Alternative Hypothesis	Rejection Criterion
$H_1\colon \mu_1 - \mu_2 \neq \Delta_0$	$t_0 > t_{\alpha/2, n_1+n_2-2}$ or $t_0 < -t_{\alpha/2, n_1+n_2-2}$
$H_1\colon \mu_1 - \mu_2 > \Delta_0$	$t_0 > t_{\alpha, n_1+n_2-2}$
$H_1\colon \mu_1 - \mu_2 < \Delta_0$	$t_0 < -t_{\alpha, n_1+n_2-2}$

Case 1 : $\sigma_1^2 = \sigma_2^2 = \sigma^2$

- A *pooled estimates of the variance* is a weighted average of the variance using the two sample variances and the degree of freedom of each variance as the weights
- The *pooled estimator* of σ^2, denoted by S_p^2 is defined by

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Note on t-test

- To used the *t*-test,
  - first used the *F* test (test difference on variances) to determine whether the variance are equal (if you don't know whether the variances are equal or not).
  - then use the appropriate *t*-test formula.

Case 1 : $\sigma_1^2 = \sigma_2^2 = \sigma^2$

- **Example**

Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. Specifically, catalyst 1 is currently in use, but catalyst 2 is acceptable. Since catalyst 2 is cheaper, it should be adopted, providing it does not change the process yield. A test is run in the pilot plant and results in the data shown in Table 5.1. Is there any difference between the mean yields? Use α = 0.05, and assume equal variances.

An Example (Cont'd)

Observation Number	Catalyst 1	Catalyst 2
1	91.50	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21
5	91.79	97.19
6	89.07	97.04
7	94.72	91.07
8	89.21	92.75
	$\bar{x}_1 = 92.255$	$\bar{x}_2 = 92.733$
	$s_1 = 2.39$	$s_2 = 2.98$

An Example (Cont'd)

1. The parameters of interest are μ_1 and μ_2, the mean process yield using catalysts1 and 2, respectively, and we want to know if
2. H_0: $\mu_1 - \mu_2 = 0$.
3. H_1: $\mu_1 \neq \mu_2$.
4. α =0.05.
5. The test statistic is $$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{s_p \sqrt{\frac{1}{n1} + \frac{1}{n2}}}$$

An Example (Cont'd)

6. Reject H_0 if $t_0 > t_{0.025,\,14} = 2.145$ or if $t_0 < -\,t_{0.025,\,14} = 2.145$.
7. Computations: Since $\bar{x}_1$ = 92.255 , s_1= 2.39, *n1* = 8, = 92.733, s_2= 2.98, *n2* = 8, we have

$$s_p^2 = \frac{(n1-1)s_1^2 + (n2-1)s_2^2}{n1+n2-2} = \frac{(7)(2.39)^2 + (7)(2.398)^2}{8+8-2} = 7.30,$$

$$s_p = \sqrt{7.30} = 2.70,$$

and

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{2.70\sqrt{\frac{1}{n1} + \frac{1}{n2}}} = \frac{92.255 - 92.733}{2.70\sqrt{\frac{1}{8} + \frac{1}{8}}} = -0.35.$$

An Example (Cont'd)

Conclusion: Since $-2.145 < t_0 = -0.35 < 2.145$, H_0 cannot be rejected. That is, at the 0.05 level of significance, we do not have strong evidence to conclude that catalyst 2 results in a mean yield that differs from the mean yield when catalyst 1 is used.

Independent Samples with σ_1 and σ_2 Unknown and Not Assumed Equal

Case 2 : $\sigma_1^2 \neq \sigma_2^2$

- In some situations, we cannot reasonably assume that the unknown σ_1^2 and σ_2^2 are equal.
- There is not an exact *t*-statistic available for testing H_0: $\mu_1 - \mu_2 = \Delta_0$ in this case.
- However, H_0: $\mu_1 - \mu_2 = \Delta_0$ true, the statistic

$$T_0^* = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{S_1^2}{n1} + \frac{S_2^2}{n2}}}$$

is distributed approximately as *t.*

Case 2 : $\sigma_1^2 \neq \sigma_2^2$

- The degrees of freedom given by

$$v = \frac{\left(\dfrac{S_1^2}{n1} + \dfrac{S_2^2}{n2}\right)^2}{\dfrac{\left(S_1^2 / n1\right)^2}{n1 - 1} + \dfrac{\left(S_2^2 / n2\right)^2}{n2 - 1}}.$$

Example

- Arsenic concentration in public drinking water supplies is a potential health risk. An article in the *Arizona Republic* (Sunday, May 27, 2001) reported drinking water arsenic concentrations in parts per billion (ppb) for 10 metropolitan Phoenix communities and 10 communities in rural Arizona. The data is given as follows:

An example (Cont'd)

Metro Phoenix ($\bar{x}_1 = 12.5, s_1 = 7.63$)	Rural Arizona ($\bar{x}_2 = 27.5, s_2 = 15.3$)
Phoenix, 3	Rimrock, 48
Chandler, 7	Goodyear, 44
Gilbert, 25	New River, 40
Glendale, 10	Apachie Junction, 38
Mesa, 15	Buckeye, 33
Paradise Valley, 6	Nogales, 21
Peoria, 12	Black Canyon City, 20
Scottsdale, 25	Sedona, 12
Tempe, 15	Payson, 1
Sun City, 7	Casa Grande, 18

We wish to determine it there is any difference in mean arsenic concentrations between metropolitan Phoenix communities and communities in rural Arizona.

An Example (Cont'd)

1. The parameters of interest are the mean arsenic concentrations for the two geographic regions, say, μ_1 and μ_2, and we are interested in determining whether
2. *H_0*: $\mu_1 - \mu_2 = 0$.
3. *H_1*: $\mu_1 \neq \mu_2$.
4. $\alpha = 0.05$.
5. The test statistic is $$t_0^* = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{s_1^2}{n1} + \frac{s_2^2}{n2}}}$$

An Example (Cont'd)

6. The degrees of freedom on t_0^* are found using

$$v = \frac{\left(\frac{S_1^2}{n1} + \frac{S_2^2}{n2}\right)^2}{\frac{\left(S_1^2/n1\right)^2}{n1-1} + \frac{\left(S_2^2/n2\right)^2}{n2-1}} = 13.2 \simeq 13.$$

Therefore, using α =0.05, we reject H_0

if $t_0^* > t_{0.025,13} = 2.160$

or

$$t_0^* < -t_{0.025,13} = -2.160$$

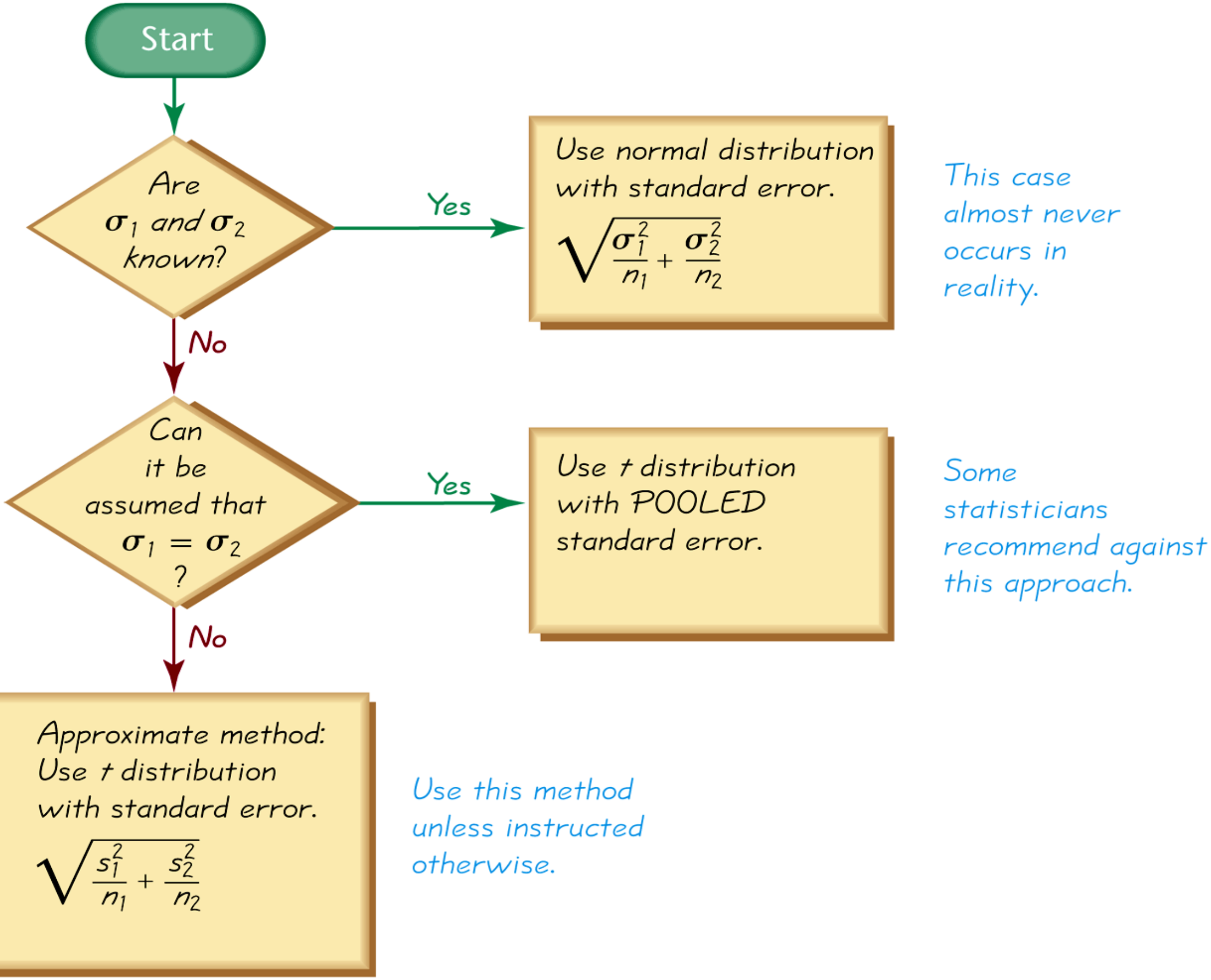
An example (Cont'd)

7. Computations: Using the sample data we find

$$t_0^* = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{s_1^2}{n1} + \frac{s_2^2}{n2}}} = -2.77. \quad t_0^*$$

8. Conclusion: Since t_0^* =-2.77<- $t_{0.025,\ 13}$ = -2.160, we reject the null hypothesis. Therefore, there is evidence to conclude that mean arsenic concentration in the drinking water in rural Arizona is different from the mean arsenic concentration in metropolitan Phoenix drinking water.

Methods for Inferences About Two Independent Means



Statistical Inferences for Two Sample Test on Variance/Standard Deviation

Test on Difference Between Two Variances

- In addition to comparing two means, statisticians are interested in comparing two variances or standard deviation. For example, is the variation in the temperature for a certain month for two cities different?
- For the comparison of two variances or standard deviations, an *F* test is used

Test on difference Between Two Variance/Standard Deviation

- When can this test be used?
- There are two samples from two populations
  - These samples can be different size
- The two samples are independent
- Both population are normally distributed
- Both population variances σ_1^2 and σ_2^2 are unknown
- We wish to test the hypotheses

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Test on difference Between Two Variance/Standard Deviation

- Let S_1^2 and S_2^2 be the sample variances.
- The statistic used is the ratio of the two sample variances

$$F = \frac{S_1^2 / \sigma_1^2}{S_2^2 / \sigma_2^2}$$

- Has an F distribution with degree of freedom

$v_1 = n_1 - 1$ (numerator) and $v_2 = n_2 - 1$ (denominator)

Or written $F_{(n_1 - 1, n_2 - 1)}$

Test on difference Between Two Variance/Standard Deviation

Null hypothesis: $H_0\colon \sigma_1^2 = \sigma_2^2$

Test statistic: $F_0 = \dfrac{S_1^2}{S_2^2}$

Alternative Hypotheses	**Rejection Criterion**
$H_1\colon \sigma_1^2 \neq \sigma_2^2$	$f_0 > f_{\alpha/2, n_1-1, n_2-1}$ or $f_0 < f_{1-\alpha/2, n_1-1, n_2-1}$
$H_1\colon \sigma_1^2 > \sigma_2^2$	$f_0 > f_{\alpha, n_1-1, n_2-1}$
$H_1\colon \sigma_1^2 < \sigma_2^2$	$f_0 < f_{1-\alpha, n_1-1, n_2-1}$

Test on difference Between Two Variance/Standard Deviation

Example

- A researcher wanted to see if women varied more than men in weight. Nine women and fifteen men were weighed. The variance for the women was 525 and the variance for the men was 142. What can be concluded at the 0.05 level of significance?

An Example (Cont'd)

- Since we are testing to see if the variance for the women is larger than the variance for the men, we let s_1^2 be the women's sample variance and s_2^2 the men's sample variance.

- The null and alternative hypotheses are

$$H_0 : \sigma_1 = \sigma_2$$
$$H_1 : \sigma_1 > \sigma_2$$

An Example (Cont'd)

- Since $s_1^2 = 525, \quad s_2^2 = 142$
- Test statistic

$$F = \frac{s_1^2}{s_2^2} = \frac{525}{142} = 3.70$$

- Degree of freedom
  - Numerator, n_1-1=9-1=8
  - Denominator, n_2-1=15-1=14

An Example (Cont'd)

- Refer to *F*-table, α=0.05

	p	1	2	3	4	5	6	7	8	12
		Degrees of freedom in numerator (df1)								
10	0.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.28
	0.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91
	0.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.62
	0.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.71
	0.001	21.04	14.90	12.55	11.28	10.48	9.93	9.52	9.20	8.45
12	0.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.15
	0.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69
	0.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.28
	0.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.16
	0.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.00
14	0.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.05
	0.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.53
	0.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.05
	0.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.80
	0.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.13

$$F_{0.05,8,14} = 2.70$$

- Reject H_o due to $F = 3.70 > F_{0.05,8,14} = 2.70,$

An Example (Cont'd)

- We can reject the null hypothesis and accept the alternative hypothesis.
- Hence, we have significant evidence to conclude that the variance for all female weights is larger than the variance for all male weights.

Statistical Inferences for Two Dependent Samples (Matched Pair)

Dependent Sample

- Dependent Samples- are samples that are paired or matched in some way
- Example
  - Samples in which the same subjects are used in a pre-post situation are dependent
  - Another type of dependent sample are samples matched on the basis of extraneous to the study

Is this an example of paired samples?

- An engineering association wants to see if there is a difference in the mean annual salary for electrical engineers and chemical engineers. A random sample of electrical engineers is surveyed about their annual income. Another random sample of chemical engineers is surveyed about their annual income.

No, there is no pairing of individuals, you have two **independent** samples

Is this an example of paired samples?

- A pharmaceutical company wants to test its new weight-loss drug. Before giving the drug to volunteers, company researchers weigh each person. After a month of using the drug, each person's weight is measured again.

Yes, you have two observations on each individual, resulting in paired data.

Notation for Dependent Samples

D	=	individual difference between the two values of a single matched pair
μ_D	=	mean value of the differences *d* for the population of all matched pairs of data
$\overline{D}$	=	mean value of the $\overline{D}$ differences d for the paired sample data
s_D	=	standard deviation of the differences *d* for the paired sample data
n	=	number of pairs of sample data

Requirements

1. The sample data are dependent.

2. The samples are simple random samples.

3. Either or both of these conditions is satisfied: The number of pairs of sample data is large ($n > 30$) or the pairs of values have differences that are from a population having a distribution that is approximately normal.

Hypotheses

- Hypotheses:

Two –tailed	Left-tailed	Right-tailed
$H_{0:} : \mu_D = 0$	$H_{0:} : \mu_D = 0$	$H_{0:} : \mu_D = 0$
$H_1 : \mu_D \neq 0$	$H_1 : \mu_D < 0$	$H_1 : \mu_D > 0$

- μ_D the expected mean of the differences of the matched pair

Special *t*-test for dependent mean

$$t = \frac{\overline{D} - \mu_D}{\frac{s_D}{\sqrt{n}}}$$

where degrees of freedom = $n - 1$ and where

$$\overline{D} = \frac{\sum D}{n}$$

and

$$s_D = \sqrt{\frac{\sum D^2 - \frac{\left(\sum D\right)^2}{n}}{n-1}}$$

General Procedure – Finding the Test Value

Step 1 Find the differences of the values of the pairs of data, *D*

Step 2 Find the mean of the differences, $\overline{D}$

Step 3 Find the standard deviation of the differences , s_D

Step 4 Find the estimated standard error of the differences, $s_{\overline{D}}$

Step 5 Find the test value, *t*

Example

- Example

A physical education director claims by taking a special vitamin a weight lifter can increase his strength. Eight athletes are selected and given a test of strength, using the standard bench press. After two weeks of regular training, supplemented with the vitamin, they are tested again. Test the effectiveness of the vitamin regime at α =0.05. Each value in these data represents the maximum numbers of pounds the athlete can bench-press. Assume that the variables is approximately normally distributed

Athlete	1	2	3	4	5	6	7	8
Before (x_1)	210	230	182	205	262	253	219	216
After (x_2)	219	236	179	204	270	250	222	216

Solution

- Step 1 : State the hypotheses. In order for the vitamin to be effective, the before weight must be significantly less than the after weights; hence the mean of the differences must be less than zero

$$H_o : \mu_D = o \qquad \text{and} \qquad H_o : \mu_D > o \qquad \text{(claim)}$$

- Step 2 : Find the critical value. The degree of freedom are n-1. d.f=7. The critical value for a left-tailed test with a = 0.05 is -1.895
- Step 3 :Compute the test value

Solution

Before(x_1)	After(x_2)	$D=x_1-x_2$	$D^2=(x_1-x_2)^2$
210	219	-9	81
230	236	-6	36
182	179	3	9
205	204	1	1
262	270	-8	64
253	250	-3	9
219	222	3	9
216	216	0	0
		$\sum D = -19$	$\sum D^2 = 209$

$$\overline{D} = \frac{\sum D}{n} = \frac{-19}{8} = -2.375$$

solution

- Standard deviation of the differences

$$s_D = \sqrt{\frac{\sum D^2 - \frac{\left(\sum D\right)^2}{n}}{n-1}} = \sqrt{\frac{209 - \frac{(-19)^2}{8}}{8-1}} = 4.84$$

solution

$$t = \frac{\overline{D} - \mu_D}{\frac{s_D}{\sqrt{n}}} = \frac{-2.375 - 0}{\frac{4.84}{\sqrt{8}}} = -1.388$$

- Step 4 : make the decision. The decision is not to reject the null hypothesis at a=0.05 since -1.388> -1.895
- Step 5: summarise the results. There is not enough evidence to support the claim that vitamin increases the strength of weight lifter

Example 2

Can playing chess improve your memory? In a study, students who had not previously played chess participated in a program in which they took chess lessons and played chess daily for 9 months. Each student took a memory test before starting the chess program and again at the end of the 9-month period.

Student	1	2	3	4	5	6	7	8	9	10	11	12
Pre-test	510	610	640	675	600	550	610	625	450	720	575	675
Post-test	850	790	850	775	700	775	700	850	690	775	540	680
Difference	-340	-180	-210	-100	-100	-225	-90	-225	-240	-55	35	-5

H_0: $\mu_d = 0$

H_1: $\mu_d < 0$

Where μ_d is the mean memory score difference between students with no chess training and students who have completed chess training

An Example (Cont'd)

- H_0: $\mu_d = 0$
- H_1: $\mu_d < 0$
- Test Statistic: $t = \dfrac{-144.6 - 0}{109.74 / \sqrt{12}} = -4.56$
- T- value from table -1.796
- Since -4.56 > -1.796, we reject H_0. There is convincing evidence to suggest that the mean memory score after chess training is higher than the mean memory score before training.