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SCSI1013: Discrete Structures

CHAPTER 1

**SET THEORY**  
**[Part 2: Operation on Set]**

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**Union**

- The **union** of two sets  $A$  and  $B$ , denoted by  $A \cup B$ , is defined to be the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

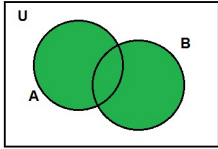
- The union consists of all elements belonging to either  $A$  or  $B$  (or both)

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**Union**

- Venn diagram of  $A \cup B$



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**Example**

$A = \{1, 2, 3, 4, 5\}$ ,  $B = \{2, 4, 6\}$  and  $C = \{8, 9\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$$

$$B \cup C = \{2, 4, 6, 8, 9\}$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$$

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**Union**

- If  $A$  and  $B$  are finite sets, the **cardinality** of  $A \cup B$ ,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

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**Intersection**

- The **intersection** of two sets  $A$  and  $B$ , denoted by  $A \cap B$ , is defined to be the set

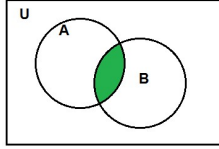
$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

- The **intersection** consists of all elements belonging to both  $A$  and  $B$ .

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**Intersection**

- Venn diagram of  $A \cap B$



The diagram shows a rectangular universal set U containing two overlapping circles, A and B. The region where the two circles overlap is shaded in green, representing the intersection  $A \cap B$ .

**Example**

$A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{2, 4, 6, 8, 10\}$  and  $C = \{1, 2, 8, 10\}$

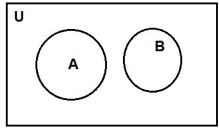
$A \cap B = \{2, 4, 6\}$   
 $A \cap C = \{1, 2\}$   
 $C \cap B = \{2, 8, 10\}$   
 $A \cap B \cap C = \{2\}$

**Disjoint**

- Two sets  $A$  and  $B$  are said to be **disjoint** if,  
 $A \cap B = \emptyset$

**Disjoint**

- Venn diagram,  $A \cap B = \emptyset$



The diagram shows a rectangular universal set U containing two separate, non-overlapping circles, A and B, illustrating that their intersection is an empty set.

**Example**

$A = \{1, 3, 5, 7, 9, 11\}$   
 $B = \{2, 4, 6, 8, 10\}$

$A \cap B = \emptyset$

**Difference**

- The set  
 $A - B = \{x \mid x \in A \text{ and } x \notin B\}$   
 is called the **difference**.
- The difference  $A - B$  consists of all elements in  $A$  that are not in  $B$ .

**Difference**

- Venn diagram of  $A-B$

**Example**

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$   
 $B = \{2, 4, 6, 8\}$

$A-B = \{1, 3, 5, 7\}$

**Symmetric Difference**

The symmetric difference of set  $A$  and set  $B$ , denoted by  $A \oplus B$  is the set  $(A - B) \cup (B - A)$

Venn Diagram

**Example**

$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$   
 $A = \{1, 2, 3, 4, 5\}; B = \{4, 5, 6, 7, 8\}$

$A \oplus B = (A - B) \cup (B - A) = \{1, 2, 3, 6, 7, 8\}$

$A - B = \{1, 2, 3\}$        $B - A = \{6, 7, 8\}$

**Complement**

- The complement of a set  $A$  with respect to a universal set  $U$ , denoted by  $A'$  is defined to be

$A' = \{x \in U \mid x \notin A\}$   
 $A' = U - A$

**Complement**

Venn diagram of  $A'$

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## Example

Let  $U$  be a universal set,

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A = \{2, 4, 6\}$$

$$A' = U - A = \{1, 3, 5, 7\}$$

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## Exercise

- Let,
 
$$U = \{a, b, c, d, e, f, g, h, i, j, k, l, m\}$$

$$A = \{a, c, f, m\}$$

$$B = \{b, c, g, h, m\}$$
- Find:
 
$$A \cup B, A \cap B, |A \cup B|, A - B \text{ dan } A'$$

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## Exercise

Let the universe be the set  $U = \{1, 2, 3, 4, \dots, 10\}$ .  
 Let  $A = \{1, 4, 7, 10\}$ ,  $B = \{1, 2, 3, 4, 5\}$  and  
 $C = \{2, 4, 6, 8\}$ .  
 List the elements of each set:

- $U'$
- $B' \cap (C - A)$
- $B - A$
- $(A \cup B) \cap (C - B)$

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## Set Identities (or Properties of Set)

- Commutative laws

$$A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

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## Set Identities (cont'd)

- Associative laws

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

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## Set Identities (cont'd)

- Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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 **Set Identities** (cont'd)

- Absorption laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

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 **Set Identities** (cont'd)

- Idempotent laws

$$A \cap A = A$$

$$A \cup A = A$$

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 **Set Identities** (cont'd)

- Complement laws

$$A \cap A' = \emptyset$$


$$A \cup A' = U$$

- Double complement laws:  $(A')' = A$
- Complement of  $U$  and  $\emptyset$ :

$$\emptyset' = U$$

$$U' = \emptyset$$

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 **Set Identities** (cont'd)

- De Morgan's laws

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

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 **Set Identities** (cont'd)

- Properties of universal set


$$A \cup U = U$$

$$A \cap U = A$$

- Set difference laws:

$$A - B = A \cap B'$$

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 **Set Identities** (cont'd)

- Identity laws:

$$A \cup \emptyset = A$$

$$A \cap U = A$$

- Properties of empty set:

$$A \cup \emptyset = A$$

$$A \cap \emptyset = \emptyset$$

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**Example**

- Let  $A, B$  and  $C$  denote the subsets of a set  $S$  and let  $C'$  denote a complement of  $C$  in  $S$ .
- If  $A \cap C = B \cap C$  and  $A \cap C' = B \cap C'$ , then prove that  $A=B$

**Example**

$$\begin{aligned}
 A &= A \cap S \\
 &= A \cap (C \cup C') \\
 &= (A \cap C) \cup (A \cap C') && \text{by distributivity} \\
 &= (B \cap C) \cup (B \cap C') && \text{by the given conditions} \\
 &= B \cap (C \cup C') && \text{by distributivity} \\
 &= B \cap S \\
 &= B
 \end{aligned}$$

**Example**

By referring to the properties of set operations (Set Identities), show that:

$$A - (A \cap B) = A - B$$

**Solution**

From set identities:  $A - B = A \cap B'$

$$\begin{aligned}
 A - (A \cap B) &= A \cap (A \cap B)' && \text{[set difference laws]} \\
 &= A \cap (A' \cup B') && \text{[De Morgan's laws]} \\
 &= (A \cap A') \cup (A \cap B') && \text{[distributive laws]} \\
 &= \emptyset \cup (A \cap B') && \text{[complement laws]} \\
 &= (A \cap B') \cup \emptyset && \text{[commutative]} \\
 &= A \cap B' && \text{[Identity laws]} \\
 &= A - B
 \end{aligned}$$

Property (law) applied

**Exercise**

- Let  $A, B$  and  $C$  be sets.
- Show that

$$(A \cup (B \cap C))' = A' \cap (B' \cup C')$$

**Exercise**

- Let  $A, B$  and  $C$  be sets such that  $A \cap B = A \cap C$  and  $A \cup B = A \cup C$
- Prove that  $B = C$

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Generalized Unions and Intersections

The **union** of a collection of sets is the set that contains those elements that are members of **at least one set** in the collection.

**Notation:**

$$\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one } i = 0, 1, 2, \dots, n\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one nonnegative integer } i\}$$

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Generalized Unions and Intersections (cont'd)

The **intersection** of a collection of sets is the set that contains those elements that are members of **all the sets** in the collection.

**Notation:**

$$\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all } i = 0, 1, 2, \dots, n\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all nonnegative integer } i\}$$

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Example

For  $i = 1, 2, \dots$ , let  $A_i = \{i, i+1, i+2, \dots\}$ . Then,

$$\bigcap_{i=1}^n A_i = \bigcap_{i=1}^n \{i, i+1, i+2, \dots\} = \{1, 2, 3, \dots\},$$

and

$$\bigcap_{i=1}^n A_i = \bigcap_{i=1}^n \{i, i+1, i+2, \dots\} = \{n, n+1, n+2, \dots\} = A_n$$

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Cartesian Product

- Let  $A$  and  $B$  be sets.
- An **ordered pair** of elements  $a \in A$  dan  $b \in B$  written  $(a, b)$  is a listing of the elements  $a$  and  $b$  in a specific order.
- The ordered pair  $(a, b)$  specifies that  $a$  is the first element and  $b$  is the second element.

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Cartesian Product

- An ordered pair  $(a, b)$  is considered distinct from ordered pair  $(b, a)$ , unless  $a=b$ .
- Example  $(1, 2) \neq (2, 1)$

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Cartesian Product

- The Cartesian product of two sets  $A$  and  $B$ , written  $A \times B$  is the set,
 
$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$
- For any set  $A$ ,
 
$$A \times \emptyset = \emptyset \times A = \emptyset$$

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### Example

$A = \{a, b\}, B = \{1, 2\}.$

$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$

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### Cartesian Product

- if  $A \neq B$ , then  $A \times B \neq B \times A$ .
- if  $|A| = m$  and  $|B| = n$ , then  $|A \times B| = mn$ .

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### Example

- $A = \{1, 3\}, B = \{2, 4, 6\}.$

$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$

$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$

$A \neq B, A \times B \neq B \times A$

$|A| = 2, |B| = 3, |A \times B| = 2 \cdot 3 = 6.$

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### Cartesian Product

- The Cartesian product of sets  $A_1, A_2, \dots, A_n$  is defined to be the set of all  $n$ -tuples  $(a_1, a_2, \dots, a_n)$  where  $a_i \in A_i$  for  $i=1, \dots, n$ ;
- It is denoted  $A_1 \times A_2 \times \dots \times A_n$   
 $|A_1 \times A_2 \times \dots \times A_n| = |A_1| \cdot |A_2| \cdot \dots \cdot |A_n|$

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### Example

- $A = \{a, b\}, B = \{1, 2\}, C = \{x, y\}$

$A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$

- $|A \times B \times C| = 2 \cdot 2 \cdot 2 = 8$

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### Exercise

- Let  $A = \{w, x\}, B = \{1, 2\}$  and  $C = \{KB, SD, PS\}.$
- Find  $|A \times B|, |B \times C|, |A \times C|, |A \times B \times C|, |B \times C \times A|, |A \times B \times A \times C|$
- Determine the following set,
  - $A \times B, B \times C, A \times C$
  - $A \times B \times C$
  - $B \times C \times A$
  - $A \times B \times A \times C$

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**Exercise**

- Let  $X = \{1,2\}$ ,  $Y = \{a\}$  and  $Z = \{b,d\}$ .
- List the elements of each set.
  - a)  $X \times Y$
  - b)  $Y \times X$
  - c)  $X \times Y \times Z$
  - d)  $X \times Y \times Y$
  - e)  $X \times X \times X$
  - f)  $Y \times X \times Y \times Z$

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