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SCSI 1013: Discrete Structure

CHAPTER 6

FINITE AUTOMATA

2019/2020 – SEM. 1

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Deterministic Finite Automata (DFA)

- In computer science, we study different types of computer languages, such as Basic, Pascal, and C++.
- We will discuss a type of a language that can be recognized by special types of machines.
- A deterministic finite automaton (pl. automata) is a mathematical model of a machine that accepts languages of some alphabet.

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Deterministic Finite Automata (DFA)

- Deterministic Finite Automaton is a quintuple M= { S, I, q_0, f_s, F}
  where,
  S is a finite nonempty set of states
  I is the input alphabet (a finite nonempty set of symbols)
  q_0 is the initial state
  f_s is the state transition function
  F is the set of final states, subset of S.

Note:
Tuple: is an ordered list of elements.
Quintuple: five times as much in size; e.g., M = {S, I, O, q_0, f_s} ;
sextuple: six time as much in size, e.g., M = {S, I, O, q_0, f_s, f_0 }.

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Example 1

- Let M={ {q_0,q_1,q_2}, {0,1}, q_0, f_s, {q_2}}
  where f_s is defined as follows:
  $f_s(q_0,0)= q_1$, $f_s(q_1,1)= q_2$
  $f_s(q_0,1)= q_0$, $f_s(q_2,0)= q_0$
  $f_s(q_1,0)= q_2$, $f_s(q_2,1)= q_1$
- Note that for M:
  S={q_0,q_1,q_2 } , I= {0,1}, F={q_2}
  q_0 is the initial state

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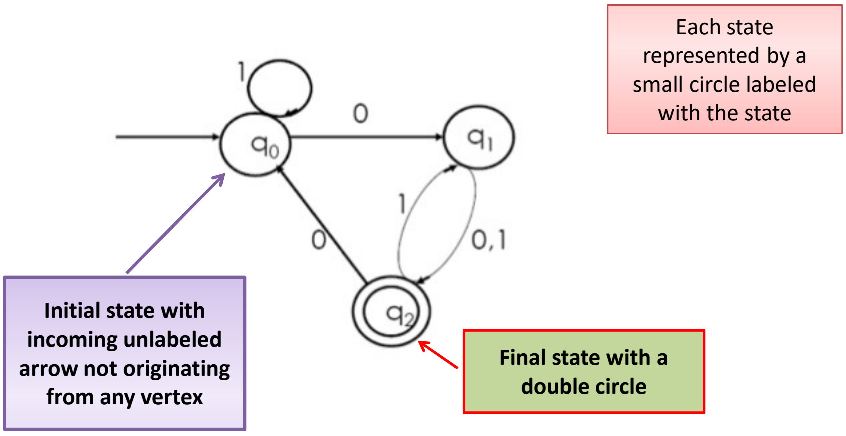
Example 1 (cont.)

- The state transition function of a DFA is often described by means of a table, called a **transition table**.

f_s	0	1
q_0	q_1	q_0
q_1	q_2	q_2
q_2	q_0	q_1

Example 1 (cont.)

- The **transition diagram** of this DFA is,



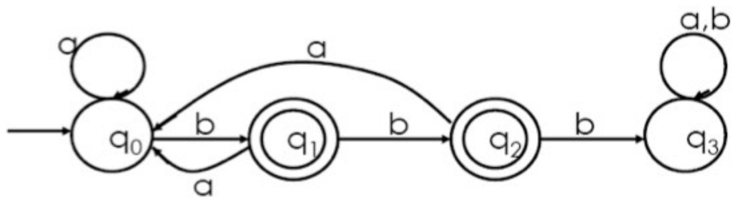
Example 2

Let $M=(\{q_0,q_1,q_2,q_3\}, \{a,b\}, q_0, f_s, \{q_1,q_2\})$
where f_s is given by the table

f_s	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

Example 2 (cont.)

- The transition diagram of this DFA is,



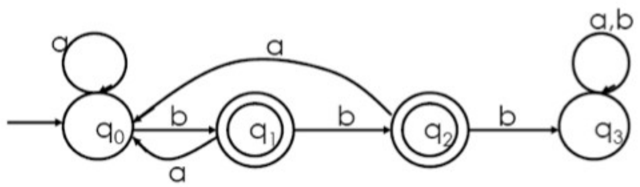
DFA – with extended transition function for M

- Let M= { S, I, q_0, f_s, F} be a DFA and *w* is an input string,
- *w* is said to be accepted by M if

$$f_s^*(q_0, w) \in F$$

- f_s^* - extended transition function for M

Example 3



w= abb

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2$$ accepted by M

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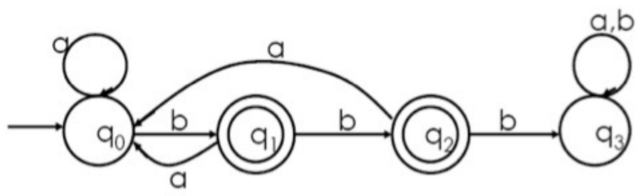
Example 4



w= abba

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0$$

not accepted by M

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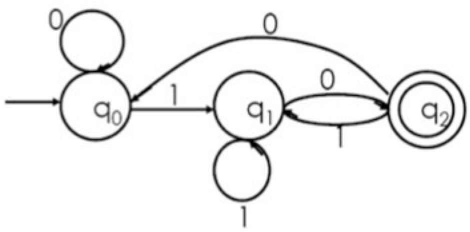
Example 5



- What are the states of M?
- Write the set of input symbols.
- Which is the initial state?

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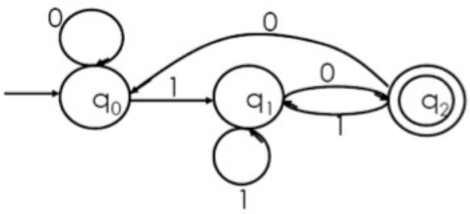
Example 5 (cont.)



- Write the set of final states.
- Write the transition table for this DFA

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Example 5 (cont.)

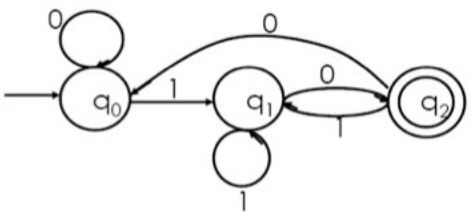
Which of the strings are accepted by M?

0111010, 00111, 111010,

0100, 1110

Example 5 - Solution

0111010

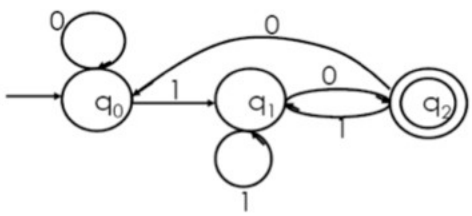


$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

Example 5 – Solution (cont.)

00111



$$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1$$

not accepted by M

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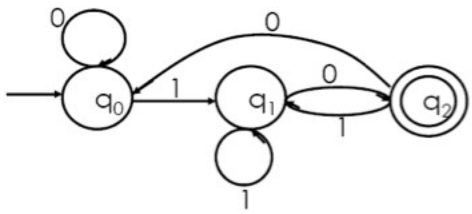
Example 5 – Solution (cont.)

111010



$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$

accepted by M

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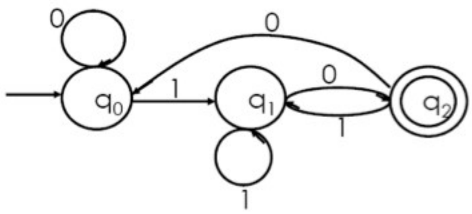
Example 5 – Solution (cont.)

0100

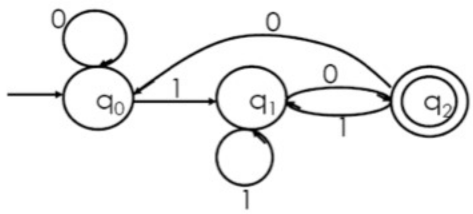


$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} q_0$

not accepted by M

Example 5 – Solution (cont.)

1110



$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2$

accepted by M

Example 6

Construct a state transition diagram of a DFA that accepts on {a,b} that contain an even number of a's and an odd number of b's.

Example of accepted strings:
aab, baa, baaabba

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Example 6 - Solution

4 states,

q_0	even num. of a's & even num. of b's.
q_1	even num. of a's & odd num. of b's.
q_2	odd num. of a's & odd num. of b's.
q_3	odd num. of a's & even num. of b's.

$S = \{q_0, q_1, q_2, q_3\}$

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Example 6 – Solution (cont.)

set of states, $S = \{q_0, q_1, q_2, q_3\}$

set of input symbols, $I = \{a, b\}$

initial state, q_0

final state, q_1

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Example 6 – Solution (cont.)

State transition function

f_s	a	b
q_0	q_3	q_1
q_1	q_2	q_0
q_2	q_1	q_3
q_3	q_0	q_2

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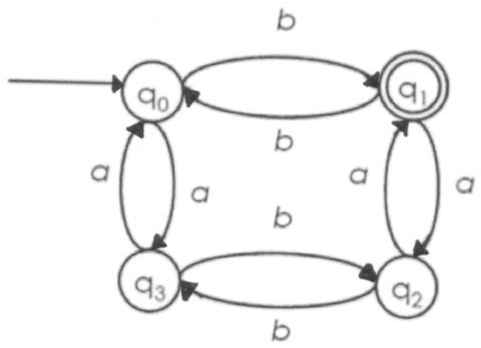
Example 6 – Solution (cont.)

State transition diagram



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Exercise 1

Let $M=(S, I, q_0, f_s, F)$ be the DFA such that $S=\{q_0,q_1,q_2\}$, $I=\{a,b\}$, $F=\{q_2\}$, q_0 =initial state, and f_s is given by,

f_s	a	b
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

Draw the state diagram of M.

Which of the strings
abaa, bbbabb, bbbaa dan bababa
are accepted by M?

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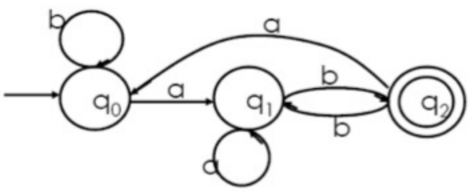
Exercise 2

The transition diagram of M is,



Construct the transition table of M.
Which of the strings
baba, baab, abab dan abaab
are accepted by M?

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Exercise 3

Construct a state transition diagram of a DFA M with the input set {0,1} such that M accepts only the string 101.

Solution:

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Exercise 4

1. Construct DFA machine over input I={0,1} that accepts, Odd number of 0's
2. Construct DFA that recognise the set of all bit string beginning with 01
3. Construct DFA that recognizes the set of all bit strings that contain exactly three 0's

Finite State Machines (FSM)

- Automata with input as well as output.
- Every state has an input and corresponding to the input the state also has an output.
- These types of automata are commonly called **finite state machines**.

Finite State Machines (FSM)

- A finite state machine is a sextuple,
  $M = \{ S, I, O, q_0, f_s, f_o \}$
  where,
  S is a finite nonempty set of states
  I is the input alphabet
  O is the output alphabet
  q_0 is the initial state
  f_s is the state transition function
  f_0 is the output function.

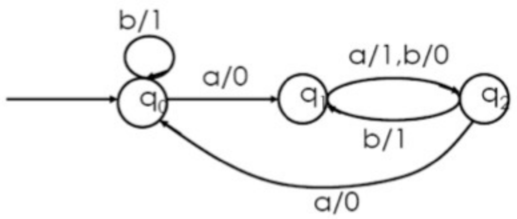
Example 1

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM
- where,
  S = {q_0,q_1,q_2},
  I = {a,b},
  O={0,1},
  q_0= initial state,
- The f_s and f_0 are =>

	f_s a	b	f_o a	b
q_0	q_1	q_0	0	1
q_1	q_2	q_2	1	0
q_2	q_0	q_1	0	1

Example 1 (cont.)

- The transition diagram:

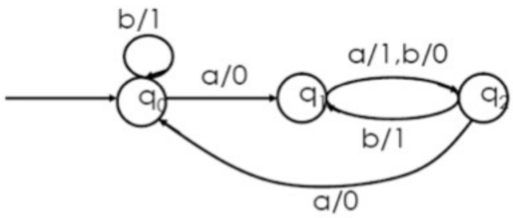


Input string: bbab

$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2$

Output string: 1100 Output: 0

Example 1 (cont.)



Input string: bababaa

$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2 \xrightarrow[0]{a} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[1]{a} q_2$

Output string: 1000101 Output: 1

Example 2

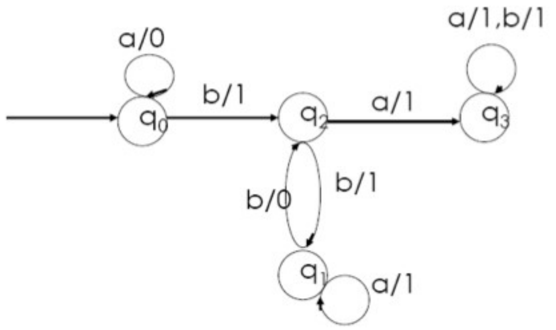
- Let M= { S, I, O, q_0, f_s, f_o} be the FSM
- where,
  S ={q_0,q_1,q_2 ,q_3},
  I ={a,b},
  O ={0,1},
  q_0 = initial state,

- f_s and f_0

	f_s a	b	f_o a	b
q_0	q_0	q_2	0	1
q_1	q_1	q_2	1	0
q_2	q_3	q_1	1	1
q_3	q_3	q_3	1	1

Example 2 (cont.)

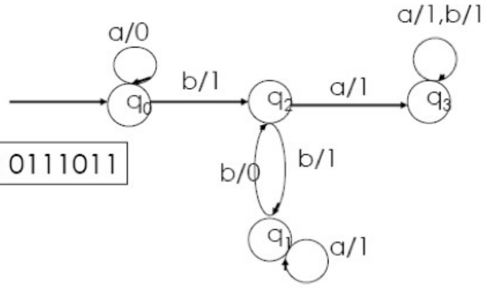
- Draw the transition diagram of M.



Example 2 (cont.)

- What is the output string if the input string is *abbabab*?

$$q_0 \xrightarrow[0]{a} q_0 \xrightarrow[1]{b} q_2 \xrightarrow[1]{b} q_1 \xrightarrow[1]{a} q_1 \xrightarrow[0]{b} q_2 \xrightarrow[1]{a} q_3 \xrightarrow[1]{b} q_3$$



Output string: 0111011

Output : 1

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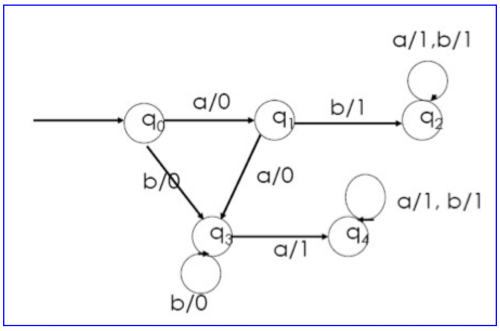
Example 3

- Let M be a FSM.
- Let x be a nonempty string in M.
- We say that x is accepted by M if and only if the output of x is 1.

State transition diagram



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Example 3 (cont.)

Based on the given information, answer the following:

- Write the transition table of M.
- What is the output string if the input string is *aaabbbb*?
- What is the output if the input string is *bbbaaaa*?
- Is the string *aaa* accepted by M?
- Which of the strings *ba, aabbba, bbbb, aaabbbb* are accepted by M?

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Example 3 - Solution

- The transition table of M.

	f_s a	b	f_o a	b
q_0	q_1	q_3	0	0
q_1	q_3	q_2	0	1
q_2	q_2	q_2	1	1
q_3	q_4	q_3	1	0
q_4	q_4	q_4	1	1

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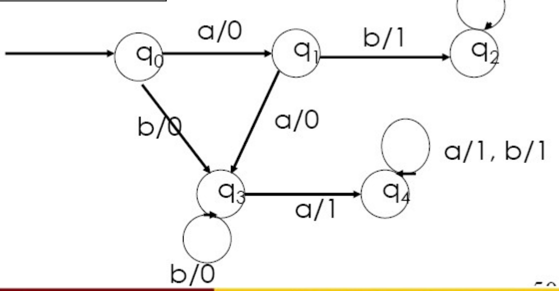
Solution (cont.)

- What is the output string if the input string is **aaabbbb**?

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4$$

0 0 1 1 1 1 1

Output string: 0011111



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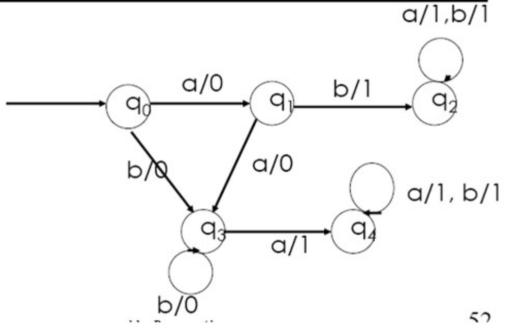
Solution (cont.)

- What is the output string if the input string is **bbbaaaa**?

$$q_0 \xrightarrow[0]{b} q_3 \xrightarrow[0]{b} q_3 \xrightarrow[0]{b} q_3 \xrightarrow[1]{a} q_4 \xrightarrow[1]{a} q_4 \xrightarrow[1]{a} q_4 \xrightarrow[1]{a} q_4$$

Output: 1



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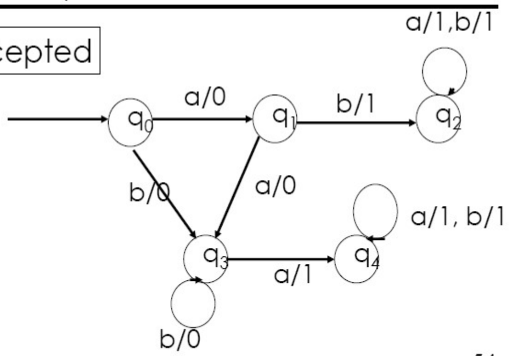
Solution (cont.)

- Is the string **aaa** accepted by M?

$$q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{a} q_3 \xrightarrow[1]{a} q_4$$

Output: 1,accepted



Solution (cont.)

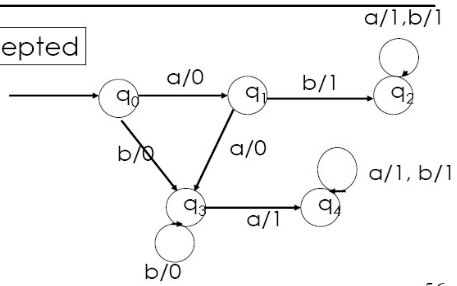
- Which of the strings **ba, aabbba, bbbb, aaabbbb** are accepted by M?

ba

$$q_0 \xrightarrow{b} q_3 \xrightarrow{a} q_4$$

0 1

Output: 1,accepted

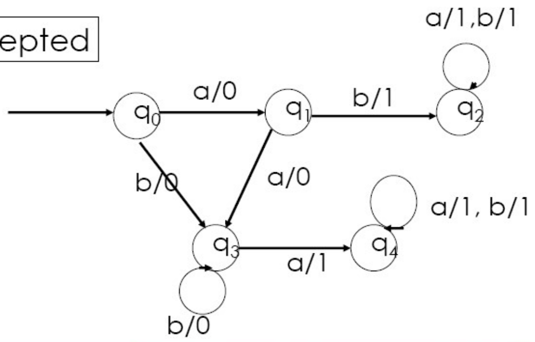


aabbba

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4$$

0 0 0 0 0 1

Output: 1,accepted



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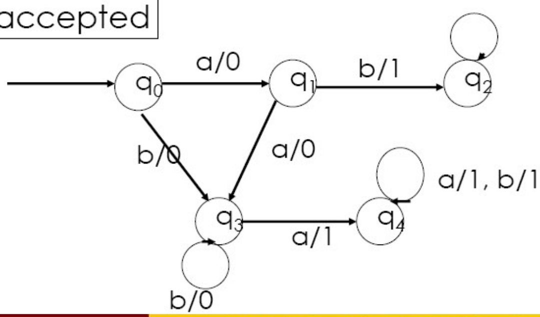
bbbb

$$q_0 \xrightarrow[0]{b} q_3 \xrightarrow[0]{b} q_3 \xrightarrow[0]{b} q_3 \xrightarrow[0]{b} q_3$$

Output: 0,not accepted



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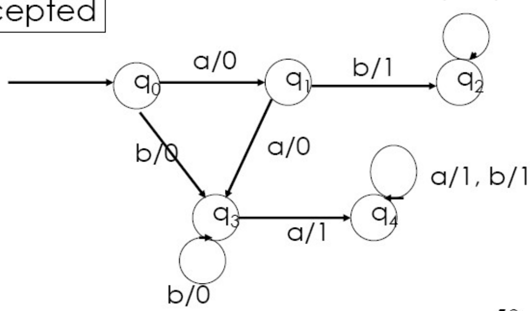
aaabbbb

$$q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{a} q_3 \xrightarrow[1]{a} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4$$

Output: 1,accepted



Example 4

- Consider a vending machine that sells candy and the cost of a candy is 50 cents.
- The machine accepts any sequence of 10-, 20-, or 50 cent coins.
- After inserting at least 50 cents, the customer can press the button to release the candy.

- If the customer inputs more than 50 cents, the machine does not return the change.
- After selling the candy, the machine returns to initial state.
- Construct a finite state machine that models this vending machine.

Example 4 - Solution

1. List the states, S:

States,

q_0,	initial state (0)
q_1,	10 cents
q_2,	20 cents
q_3,	30 cents
q_4,	40 cents
q_5,	≥ 50 cents

2. List the elements of M:

$S= \{q_0,q_1,q_2,q_3,q_4,q_5\}$,

I = {10,20,50,B},

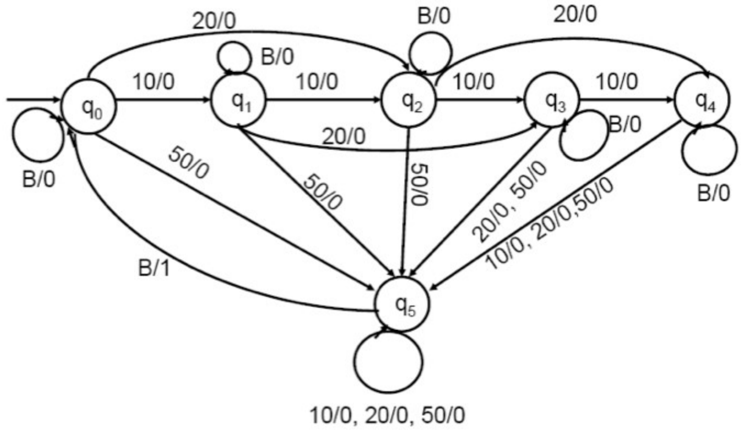
O= {0,1},

q_0 = initial state,

3. List the f_s and f_0:

	f_s 10	20	50	B	f_o 10	20	50	B
q_0	q_1	q_2	q_5	q_0	0	0	0	0
q_1	q_2	q_3	q_5	q_1	0	0	0	0
q_2	q_3	q_4	q_5	q_2	0	0	0	0
q_3	q_4	q_5	q_5	q_3	0	0	0	0
q_4	q_5	q_5	q_5	q_4	0	0	0	0
q_5	q_5	q_5	q_5	q_0	0	0	0	1

4. Draw the transition diagram of M:



Exercise #1

Let M= { S, I, O, q_0, f_s, f_o} be a FSM
where,
S ={q_0,q_1,q_2},
I ={a,b},
O ={0,1},
q_0 = initial state,

f_s and f_0

	f_s a	b	f_o a	b
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

- Draw the transition diagram of M.
- What is the output string if the input string is *aabbb*?
- What is the output string if the input string is *ababab*?
- What is the output if the input string is *abbbaba*?
- What is the output if the input string is *bbbababa*?

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Exercise #2

Let $M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b, c\}, q_0, f_s, \{q_1, q_3, q_5\})$ be the Deterministic Finite Automaton (DFA) with state transition function, f_s defined as follows:

$f(q_0, a) = q_1$ $\quad f(q_0, b) = q_0$ $\quad f(q_0, c) = q_0$

$f(q_1, a) = q_1$ $\quad f(q_1, b) = q_2$ $\quad f(q_1, c) = q_1$

$f(q_2, a) = q_2$ $\quad f(q_2, b) = q_3$ $\quad f(q_2, c) = q_4$

$f(q_3, a) = q_3$ $\quad f(q_3, b) = q_3$ $\quad f(q_3, c) = q_3$

$f(q_4, a) = q_4$ $\quad f(q_4, b) = q_5$ $\quad f(q_4, c) = q_4$

$f(q_5, a) = q_5$ $\quad f(q_5, b) = q_5$ $\quad f(q_5, c) = q_5$

(a) Draw the transition table for the above machine.

(b) Determine the final state for the input string *abcc*.

(c) Is the input string *abcb* accepted by the DFA?

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Exercise #3

Let $A = (S, I, O, Z, f, g)$ be a finite state machine (FSM) defined by the transition table shown in Table 1.

Table 1: Transition table of FSM *A*

	f			*g*		
State \ Input	*a*	*b*	*c*	*a*	*b*	*c*
X	*Z*	*X*	*Y*	1	0	1
Y	*X*	*X*	*Z*	0	1	0
Z	*Y*	*X*	*Z*	1	0	1

(a) Draw the transition diagram of the finite state machine *A*.

(b) Find the output string for the input string *babccaab*.

(c) Find the output generated from the input string *cabcccba*.

(d) Determine whether the input string *abcbcbcabcc* is accepted.

Exercise #4

A sliding barrier turnstile (shown in Figure 1), used to control access to subways, is an automated gate at waist height with a barrier across the entryway. Initially the barrier is locked, barring the entry, preventing passengers from passing through. Depositing a token in a slot on the turnstile unlocks the barrier, allowing a single customer to push through. After the customer passes through, the barrier is locked again until another coin is inserted.

Figure 1

Considered as a state machine, the turnstile has two states: ***Locked*** and ***Unlocked***. There are two inputs that affect its state: putting a token in the slot (***token***) and retract the barrier (***retract***). In the locked state, retracting the arm has no effect; no matter how many times the input ***retract*** is given, it stays in the locked state. Putting a ***token*** as an input, shift the state from ***Locked*** to ***Unlocked***. In the unlocked state, putting additional tokens does not change the state. However, a customer passing through the retracted barrier, giving a ***retract*** input, shifts the state back to ***Locked***.

Given
- State
  - L: Locked
  - U: Unlocked
- Input
  - T: Token
  - R: Retract
- Output
  - 0: Nothing happened
  - 1: Retract the barrier so passenger can pass through
  - 2: Lock the barrier when passenger has passed through

(a) Complete the transition table below.

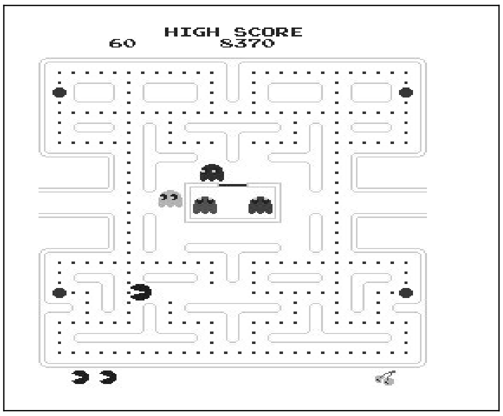
Table 1

State	Input, f_s		Output, f_o	
	T	R	T	R
L				
U				

(b) Draw the transition diagram for the turnstile system described above.

Exercise

- *Pac-Man* is an arcade game developed by a young Namco employee named Toru Iwatani and first released in Japan in 1980. It is considered one of the classics of the medium, virtually synonymous with video games, and an icon of 1980s. *Pac-Man* is one of the few games to have been consistently published for over three decades, having been remade on numerous platforms and spawned many sequels. Re-releases include ported and updated versions of the original arcade game.
- The typical version of Pac-Man is a one player game where he/she manoeuvres the Pac-Man around the maze, attempting to avoid four 'ghosts' characters while eating dots that distributed throughout the maze. Among the dots, there are four super dots that located at four corners of the maze. If the Pac-Man collides with the ghost, he loses one of his three lives and play resumes with the ghosts reassigned to their initial starting location. When Pac-Man eats a super dot, he is able to chase the ghosts for a few seconds of time before the super dot expires. The game ends when Pac-Man has lost all his three lives. Figure x shows a screenshot of the Pac-Man game.



Noted that the ghosts in Pac-Man have four behaviors:

S_1 : randomly wander the maze
S_2 : chase Pac-Man when he is within the line of sight
S_3 : avoid Pac-Man when has consumed a super dot
S_4 : return to the initial position to restart the game

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Exercise (cont)

The inputs are:

A : spot Pac-Man (Pac-Man is within the line of sight)
B : lose Pac-Man (Pac-Man is not within the line of sight)
C : Pac-Man eats super dot
D : super dot expires
E : collides with Pac-Man
F : reach the initial position

The outputs are:

0 : nothing happened
1 : Pac-Man loses his life
2 : number of ghosts reduces by 1

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Exercise (cont)

State	Input, f_s						Output, f_o					
	A	B	C	D	E	F	A	B	C	D	E	F
S_1				S_1		S_1				0		0
pS_2	S_2	S_1	S_3				0	0	0			
S_3		S_1					0	0	0	0		0
S_4	S_4	S_4	S_4	S_4	S_4	S_4			0	0	0	0