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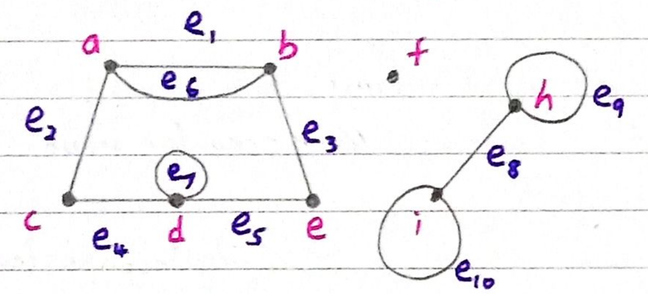
Chapter 4 : Graph Theory

4.1 Definition of Graph

Graph < Set of vertices $V(G)$
Set of edges $E(G)$

Eg 1:



Vertex set = $\{a, b, c, d, e, f, h, i\}$

Edge set = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$

Edge-endpoint function:

Edge	Endpoints
e_1	$\{a,b\}$
e_2	$\{a,c\}$
e_3	$\{b,e\}$
e_4	$\{c,d\}$
e_5	$\{d,e\}$
e_6	$\{a,b\}$
e_7	$\{d\}$
e_8	$\{h,i\}$
e_9	$\{h\}$
e_{10}	$\{i\}$

parallel endpoints : e_1, e_6

loops : e_7, e_9, e_{10}

Incident on a : e_1, e_2, e_6

Vertices adjacent to a : b, c

Edges adjacent to e_2 : e_1, e_4, e_6

Isolated vertex : f

Vertex adjacent to themselves : d, h, i
(loop)

* Directed graph : Edge connecting vertices in only one direction

Undirected graph : Edge connecting vertices in both directions

4.2 The Concept of Degree

From eg 1

Vertex	a	b	c	d	e	f	h	i
Degree	3	3	2	4	2	0	3	3

Types of Graphs

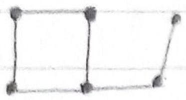
a) Simple Graphs

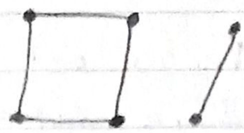
× Parallel × Loop

b) Regular Graphs // k-regular graphs, k = degree of vertices

ALL vertices has SAME degree

c) i) Connected Graphs // 整个图连得起来 ii) Unconnected Graphs





d) Complete Graphs // k_n , n = no. of vertices

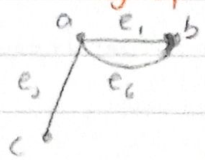
All vertices in graphs connected to each other

e) Discrete Graphs

All isolated vertices , × edges

f) Subgraph

From eg 1:



Subgraph of eg 1

Graph Representation

Adjacency matrix

From eg 1:

$$A_G = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ h \\ i \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & h & i \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Sum of row & column = degree of vertices

Incidence matrix

$$I_G = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ h \\ i \end{array} \begin{array}{c} \begin{array}{cccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

Sum of column = 2

Sum of row = degree of each vertices

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Isomorphism of Graphs

- Same number of vertices & edges
- Same degrees for corresponding vertices
- Same number of connected components
- Same number of loops & parallel edges
- Pairs of connected vertices must have the corresponding pair of vertices connected

Trails, Paths & Circuits

repeated	v	e
walk	✓	✓
trail	✓	✗
path	✗	✗

walk : finite alternating sequence of adjacent vertices and edges of G

Trail : Walk that DOES NOT contain repeated edges

Path : Walk that DOES NOT contain repeated vertices and edges

Cyclic

repeated	v	e
Closed walk	✓	✓
Circuit	✓	✗
Simple circuit	✗	✗

Closed walk : Walk that starts and ends at the SAME VERTEX WITH repeated vertex

Circuit : Walk that contains AT LEAST 1 edge, starts and ends at the SAME VERTEX but DOES NOT contain repeated edges.

Simple circuit : Circuit that DOES NOT have repeated vertex except the 1st and last

Euler circuit and Euler Trail

Euler circuit ⇒ Starts & end at same vertex
⇒ contains ALL vertex & edges ⇐ Euler trail
⇒

	v	e
Repeat	✓	✗

⇐

ALL vertex has EVEN degree

All vertex has even degree EXCEPT 2 vertex has ODD degree

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Hamiltonian Circuits

- include all VERTEX only ONCE
- start & end at same vertex

/* Doesn't need to include all edges */