

UTM
UNIVERSITI TEKNIK MALAYSIA

SCSI1013: Discrete Structures

CHAPTER 1

SET THEORY
[Part 2: Operation on Set]

innovative • entrepreneurial • global 2017/2018 – Sem. 1 www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Union

- The **union** of two sets A and B , denoted by $A \cup B$, is defined to be the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

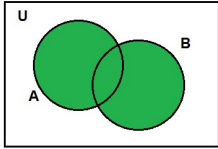
- The union consists of all elements belonging to either A or B (or both)

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Union

- Venn diagram of $A \cup B$



innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Example

$A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$ and $C = \{8, 9\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$$

$$B \cup C = \{2, 4, 6, 8, 9\}$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Union

- If A and B are finite sets, the **cardinality** of $A \cup B$,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Intersection

- The **intersection** of two sets A and B , denoted by $A \cap B$, is defined to be the set

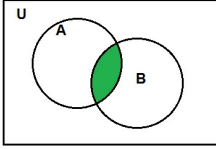
$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

- The **intersection** consists of all elements belonging to both A and B .

innovative • entrepreneurial • global www.utm.my

Intersection

- Venn diagram of $A \cap B$



The diagram shows a rectangular universal set U containing two overlapping circles, A and B. The region where the two circles overlap is shaded in green, representing the intersection $A \cap B$.

Example

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$ and $C = \{1, 2, 8, 10\}$

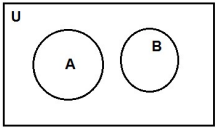
$A \cap B = \{2, 4, 6\}$
 $A \cap C = \{1, 2\}$
 $C \cap B = \{2, 8, 10\}$
 $A \cap B \cap C = \{2\}$

Disjoint

- Two sets A and B are said to be **disjoint** if,
 $A \cap B = \emptyset$

Disjoint

- Venn diagram, $A \cap B = \emptyset$



The diagram shows a rectangular universal set U containing two separate, non-overlapping circles, A and B, illustrating that their intersection is an empty set.

Example

$A = \{1, 3, 5, 7, 9, 11\}$
 $B = \{2, 4, 6, 8, 10\}$

$A \cap B = \emptyset$

Difference

- The set $A - B = \{x \mid x \in A \text{ and } x \notin B\}$ is called the **difference**.
- The difference $A - B$ consists of all elements in A that are not in B .

Difference

- Venn diagram of $A-B$

Example

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 $B = \{2, 4, 6, 8\}$

$A-B = \{1, 3, 5, 7\}$

Symmetric Difference

The symmetric difference of set A and set B , denoted by $A \oplus B$ is the set $(A-B) \cup (B-A)$

Venn Diagram

Example

$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 $A = \{1, 2, 3, 4, 5\}; B = \{4, 5, 6, 7, 8\}$

$A \oplus B = (A-B) \cup (B-A) = \{1, 2, 3, 6, 7, 8\}$

$A-B = \{1, 2, 3\}$ $B-A = \{6, 7, 8\}$

Complement

- The complement of a set A with respect to a universal set U , denoted by A' is defined to be

$A' = \{x \in U \mid x \notin A\}$
 $A' = U - A$

Complement

Venn diagram of A'

UTM
UNIVERSITI TEKNIK MALAYSIA

Example

Let U be a universal set,

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A = \{2, 4, 6\}$$

$$A' = U - A = \{1, 3, 5, 7\}$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Exercise

- Let,

$$U = \{a, b, c, d, e, f, g, h, i, j, k, l, m\}$$

$$A = \{a, c, f, m\}$$

$$B = \{b, c, g, h, m\}$$
- Find:

$$A \cup B, A \cap B, |A \cup B|, A - B \text{ dan } A'$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Exercise

Let the universe be the set $U = \{1, 2, 3, 4, \dots, 10\}$.
 Let $A = \{1, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and
 $C = \{2, 4, 6, 8\}$.
 List the elements of each set:

- U'
- $B' \cap (C - A)$
- $B - A$
- $(A \cup B) \cap (C - B)$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (or Properties of Set)

- Commutative laws

$$A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (cont'd)

- Associative laws

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (cont'd)

- Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (cont'd)

- Absorption laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (cont'd)

- Idempotent laws

$$A \cap A = A$$

$$A \cup A = A$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (cont'd)

- Complement laws

$$A \cap A' = \emptyset$$

$$A \cup A' = U$$

- Double complement laws: $(A')' = A$
- Complement of U and \emptyset :

$$\emptyset' = U$$

$$U' = \emptyset$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (cont'd)

- De Morgan's laws

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (cont'd)

- Properties of universal set

$$A \cup U = U$$

$$A \cap U = A$$

- Set difference laws:

$$A - B = A \cap B'$$

innovative • entrepreneurial • global www.utm.my

UTM
UNIVERSITI TEKNIK MALAYSIA

Set Identities (cont'd)

- Identity laws:

$$A \cup \emptyset = A$$

$$A \cap U = A$$

- Properties of empty set:

$$A \cup \emptyset = A$$

$$A \cap \emptyset = \emptyset$$

innovative • entrepreneurial • global www.utm.my

Example

- Let A, B and C denote the subsets of a set S and let C' denote a complement of C in S .
- If $A \cap C = B \cap C$ and $A \cap C' = B \cap C'$, then prove that $A=B$

Example

$$\begin{aligned}
 A &= A \cap S \\
 &= A \cap (C \cup C') \\
 &= (A \cap C) \cup (A \cap C') && \text{by distributivity} \\
 &= (B \cap C) \cup (B \cap C') && \text{by the given conditions} \\
 &= B \cap (C \cup C') && \text{by distributivity} \\
 &= B \cap S \\
 &= B
 \end{aligned}$$

Example

By referring to the properties of set operations (Set Identities), show that:

$$A - (A \cap B) = A - B$$

Solution

From set identities: $A - B = A \cap B'$

$$\begin{aligned}
 A - (A \cap B) &= A \cap (A \cap B)' && \text{[set difference laws]} \\
 &= A \cap (A' \cup B') && \text{[De Morgan's laws]} \\
 &= (A \cap A') \cup (A \cap B') && \text{[distributive laws]} \\
 &= \emptyset \cup (A \cap B') && \text{[complement laws]} \\
 &= (A \cap B') \cup \emptyset && \text{[commutative]} \\
 &= A \cap B' && \text{[Identity laws]} \\
 &= A - B
 \end{aligned}$$

Property (law) applied

Exercise

- Let A, B and C be sets.
- Show that

$$(A \cup (B \cap C))' = A' \cap (B' \cup C')$$

Exercise

- Let A, B and C be sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$
- Prove that $B = C$

UTM
Generalized Unions and Intersections

The **union** of a collection of sets is the set that contains those elements that are members of **at least one set** in the collection.

Notation:

$$\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one } i = 0, 1, 2, \dots, n\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one nonnegative integer } i\}$$

innovative • entrepreneurial • global www.utm.my

UTM
Generalized Unions and Intersections (cont'd)

The **intersection** of a collection of sets is the set that contains those elements that are members of **all the sets** in the collection.

Notation:

$$\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all } i = 0, 1, 2, \dots, n\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all nonnegative integer } i\}$$

innovative • entrepreneurial • global www.utm.my

UTM
Example

For $i = 1, 2, \dots$, let $A_i = \{i, i+1, i+2, \dots\}$. Then,

$$\bigcap_{i=1}^n A_i = \bigcap_{i=1}^n \{i, i+1, i+2, \dots\} = \{1, 2, 3, \dots\},$$

and

$$\bigcap_{i=1}^n A_i = \bigcap_{i=1}^n \{i, i+1, i+2, \dots\} = \{n, n+1, n+2, \dots\} = A_n$$

innovative • entrepreneurial • global www.utm.my

UTM
Cartesian Product

- Let A and B be sets.
- An **ordered pair** of elements $a \in A$ dan $b \in B$ written (a, b) is a listing of the elements a and b in a specific order.
- The ordered pair (a, b) specifies that a is the first element and b is the second element.

innovative • entrepreneurial • global www.utm.my

UTM
Cartesian Product

- An ordered pair (a, b) is considered distinct from ordered pair (b, a) , unless $a=b$.
- Example $(1, 2) \neq (2, 1)$

innovative • entrepreneurial • global www.utm.my

UTM
Cartesian Product

- The Cartesian product of two sets A and B , written $A \times B$ is the set,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$
- For any set A ,

$$A \times \emptyset = \emptyset \times A = \emptyset$$

innovative • entrepreneurial • global www.utm.my

UTM
Innovative • Entrepreneurial • Global

Example

$A = \{a, b\}, B = \{1, 2\}.$

$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$

Innovative • Entrepreneurial • Global www.utm.my

UTM
Innovative • Entrepreneurial • Global

Cartesian Product

- if $A \neq B$, then $A \times B \neq B \times A$.
- if $|A| = m$ and $|B| = n$, then $|A \times B| = mn$.

Innovative • Entrepreneurial • Global www.utm.my

UTM
Innovative • Entrepreneurial • Global

Example

- $A = \{1, 3\}, B = \{2, 4, 6\}.$

$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$

$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$

$A \neq B, A \times B \neq B \times A$

$|A| = 2, |B| = 3, |A \times B| = 2 \cdot 3 = 6.$

Innovative • Entrepreneurial • Global www.utm.my

UTM
Innovative • Entrepreneurial • Global

Cartesian Product

- The Cartesian product of sets A_1, A_2, \dots, A_n is defined to be the set of all n -tuples (a_1, a_2, \dots, a_n) where $a_i \in A_i$ for $i=1, \dots, n$;
- It is denoted $A_1 \times A_2 \times \dots \times A_n$
 $|A_1 \times A_2 \times \dots \times A_n| = |A_1| \cdot |A_2| \cdot \dots \cdot |A_n|$

Innovative • Entrepreneurial • Global www.utm.my

UTM
Innovative • Entrepreneurial • Global

Example

- $A = \{a, b\}, B = \{1, 2\}, C = \{x, y\}$

$A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$

- $|A \times B \times C| = 2 \cdot 2 \cdot 2 = 8$


Innovative • Entrepreneurial • Global www.utm.my

UTM
Innovative • Entrepreneurial • Global

Exercise

- Let $A = \{w, x\}, B = \{1, 2\}$ and $C = \{KB, SD, PS\}.$
- Find $|A \times B|, |B \times C|, |A \times C|, |A \times B \times C|, |B \times C \times A|, |A \times B \times A \times C|$
- Determine the following set,
 - $A \times B, B \times C, A \times C$
 - $A \times B \times C$
 - $B \times C \times A$
 - $A \times B \times A \times C$

Innovative • Entrepreneurial • Global www.utm.my



Exercise

- Let $X = \{1,2\}$, $Y = \{a\}$ and $Z = \{b,d\}$.
- List the elements of each set.
 - a) $X \times Y$
 - b) $Y \times X$
 - c) $X \times Y \times Z$
 - d) $X \times Y \times Y$
 - e) $X \times X \times X$
 - f) $Y \times X \times Y \times Z$

Innovative • entrepreneurial • global www.utm.my