Faculty of Computing

UNIVERSITI TEKNOLOGI MALAYSIA

TEST 1

SEMESTER I - 2015/2016

SUBJECT CODE : SCSI1013

SUBJECT NAME : DISCRETE STRUCTURE

YEAR/COURSE :

TIME : 2 HOURS

DATE :

VENUE :

INSTRUCTIONS TO THE STUDENTS:
Answer all questions in the given answer booklet.

Name	
I/C No.	
Year/Course	
Section	
Lecturer Name	

This questions paper consists of **6** printed pages including this page.

Question 1 [5 Marks]

a) Calculate the size (cardinality) of these sets, given A={1, 3, 5} and B={2, 4, 6, 8}. This can be done using only the facts that $|A|$ =3 and $|B|$ =4. You do not need to list all elements of the new sets.

   i. $|A \times B|$

   ii. $|P(A) \times P(B)|$

(2 marks)

b) Let A and B be sets. Use properties of sets to show that $(A \cap B) \cup (A' \cup B)' = A$.

(3 marks)

Question 2 [15 marks]

a) Let D={1, 3, 5, 7, 8, 9}. Decide whether each of the following statements is true for all elements of D. For each that is not, give a counterexample. That is, provide a number in D for which the statement is not true.

   i. x is even and $x > 7$

   ii. x is odd or $x > 7$

   iii. x is not odd and $x \leq 7$

   iv. x is odd

(4 marks)

b) Use truth table to check if the symbolic logic statements are equivalent.

$$p \wedge (p \rightarrow q) \quad \text{and} \quad p \wedge q$$

(3 marks)

c) Let $P(x)$ be the statement "x can speak Arabic" and let $Q(x)$ be the statement "x knows computer language C++". Express each of these sentences in term of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your faculty.

   i. There is a student at your faculty who can speak Arabic and who knows C++.

   ii. Every student at your faculty either can speak Arabic or knows C++.

(4 marks)

d) For each of the compound propositions, state either TRUE or FALSE if ***p*** and ***r*** are TRUE and ***q*** is FALSE.

i. $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

ii. $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$

(4 marks)

Question 3 [20 Marks]

a) Let $A = \{3,4,5\}$ and $B = \{2,4,6,8,10\}$ and R be the relation from A *to* B defined by aRb if and only if $2a \leq b+1$.

i. List the ordered pair of the relation R.

ii. Find the domain and range.

(4 marks)

b) Let $A = \{1,2,3,4,5\}$ and the relation R on A represented by the digraph in Figure 1.

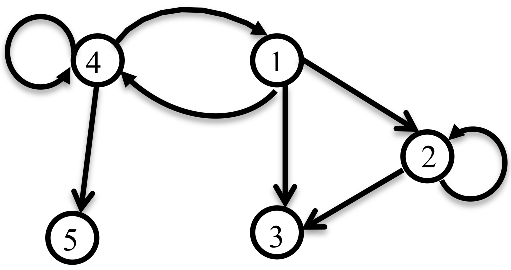

Figure 1

Based on the digraph in Figure 1, determine:

i. The in-degree and the out-degree of each vertex.

(3 marks)

ii. Matrix relation of R, M_R

(2 marks)

c) Determine whether the relation $R = \{(1,2),(1,4),(4,5),(5,3),(1,3),(2,2)\}$ on set $D = \{1,2,3,4,5\}$ is asymmetric? Explain.

(4 marks)

d) Determine whether the relation R on $A = \{a, b, c, d\}$ whose matrix M_R is given below is an equivalence relation? Explain.

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(7 marks)

Question 4 [20 marks]

a) Let $J = \{0, 1, 2, 3, 4\}$ and define a function f: $J \rightarrow J$ as follows: For each $x \in J, f(x) = (x^3 + 2x + 4) \bmod 5$. Find the following:

i. $f(0)$

ii. $f(1)$

(2 marks)

b) Let $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$ and $W = \{1,2\}$.

i. Draw the arrow diagram to define function f: $X \rightarrow Y$ that is one-to-one but not onto.

(2 marks)

ii. List three ordered pairs to define function g: $X \rightarrow W$ that is onto but not one-to-one.

(2 marks)

c) Let $A = \{p, q, r, s\}$ and $B = \{1, 2, 3, 4\}$. Which of the following can represent as bijection function from A to B? Justify your answer.

i. g = {(p,2), (q,1), (r,4), (s,3)}

ii. f = {(p,1), (q,1), (r,2), (s,2)}

iii. h = {(p,1), (p,2), (p,3), (p,4)}

(2 marks)

d) Function f and g are defined by formulas:

$f(x) = x^3$ and $g(x) = x - 1$, for all real number x.

Find $g \circ f$ and $f \circ g$. Determine whether $g \circ f$ equals $f \circ g$.

(6 marks)

e) Write a recursive algorithm to find the n term of the sequence defined by,

$$k_0 = -1, \quad k_1 = 2, \text{ and } k_n = k_{n-1} + 2k_{n-2}, \quad \text{for } n \geq 2.$$

Then, trace the algorithm for $n = 4$.

(6 marks)

** End of Questions **

Properties On Sets

Commutative laws

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Associative laws

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Distributive laws

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Absorption laws

$A \cap (A \cup B) = A$

$A \cup (A \cap B) = A$

Idempotent laws

$A \cup A = A$

$A \cap A = A$

Complement laws

$(A')' = A$

$A \cup A' = U$

$A \cap A' = \varnothing$

$\varnothing' = U$

$U' = \varnothing$

De Morgan's laws

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

Properties of universal set

$A \cup U = U$

$A \cap U = A$

Properties of empty set

$A \cup \varnothing = A$

$A \cap \varnothing = \varnothing$