Test 1 Semester 1, 2014/2015

SCSI 1013 (Discrete Structure)

Please answer all questions in the answer booklet.

Question 1 **25 marks**

a) Let the universe be the set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $A=\{2, 5, 8, 9\}$, $B=\{5, 6, 7, 8, 9\}$, and $C=\{1, 3, 5, 7\}$. List the elements of the following sets.

i. $A \cap (B \cup C)$
ii. $B' \cap (C - A)$
iii. $(A \cup B) - (C - B)$
iv. $(A \cap B)' \cup C$

(8 marks)

b) Let $X=\{7, 8\}$, $Y=\{\alpha\}$, and $Z=\{c, d\}$. Find

i. $X \times Y \times Z$
ii. $Y \times X \times Y \times Z$
iii. $|X \times X \times Z|$
iv. $|Y \times X \times Z \times Z|$

(8 marks)

c) Let A and B be sets. Show that $A \cup (A' \cap B) = A \cup B$ using the properties on set.

(4 marks)

d) Students at EZ Elementary School receive ribbons at the end of each year at a school-wide awards ceremony. This year 120 students receive gold stars for perfect attendance, 180 receive certificates for participating in the science fair, and 80 students receive blue ribbons for outstanding grades. Of these, 40 students who receive the attendance star receive no other award, 50 students who receive the science fair certificate receive no other award, and 10 students who receive the blue ribbon receive no other awards. In addition, 10 students receive all three awards and 65 students receive no awards. Draw a Venn diagram for this situation, and determine how many students attend the school this year.

(5 marks)

Question 2

20 marks

a) Given $R = \neg(p \leftrightarrow q)$ and $S = (p \rightarrow q) \wedge (q \rightarrow p)$

Determine whether statement R is logically equivalent to statement S. (5 marks)

b) Let $P(x)$ be the statement " x is multiples of 3" and $Q(x)$ be the statement " x is even" where the domain of discourse is the set of integers. Express each of these sentences in terms of appropriate predicates, quantifiers and logical connections. (6 marks)

i. There exists an integer that is odd.

ii. Some even integers are multiples of 3.

iii. It is not the case that every integer is multiples of 3.

c) Determine the true value for $\exists x(x^2 - 8x + 15 = 0)$ where the domain of discourse is all positive integers.

(4 marks)

d) Prove that if $5n+6$ is odd then n is odd using an indirect proof. (5 marks)

Question 3

25 marks

a) Let $X = \{1,3,5\}$ R_1 is a relation on X, defined by : aRb *if and only if* $|a-b|$ is an even integer

i. List all possible members of set R_1 (3marks)

ii. Find the matrix of R_1 (3marks)

iii. Determine set of domain and range of the relation. (2 marks)

b) Let $A = \{x,y,z\}$ and R_2 is a relation on set defined by the following matrix

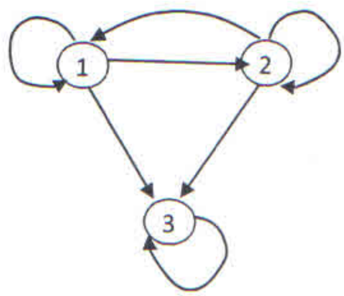
$$\begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

i. Draw the digraph for the relation R_2 (2 marks)

ii. Describes all possible properties of R_2. It is reflexive, symmetric or transitive? Justify your answer using the properties of the matrix relation (6 marks)

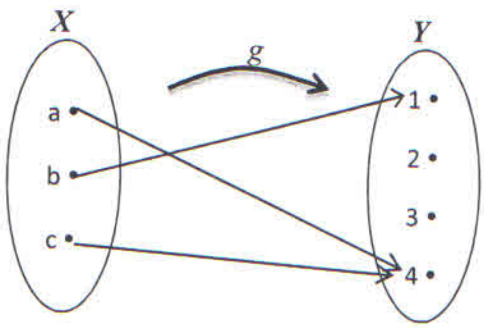
iii. Does R_2 is an equivalence relation? Justify your answer. (2 marks)

c) What are the properties of partial order relation? Does the following digraph have such relation? Justify your answer. (7 marks)



Question 4 **24 marks**

a) Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3, 4\}$. Define $g : X \to Y$ by the following arrow diagram.



i. Write the domain of g and the co-domain of g. (2 marks)

ii. Find $g(a)$, $g(b)$ and $g(c)$. (3 marks)

iii. What is the range of g? (1 marks)

iv. Represent g as a set of ordered pairs. (2 marks)

b) Let $V = \{1, 2, 3, 4\}$ and $W = \{a, b, c\}$

Function $f: V \to W$ is defined as follows: $f(1) = c, f(2) = a, f(3) = c, f(4) = b$.

Function $h: V \to W$ is defined as follows : $h(1) = c$, $h(2) = b$, $h(3) = b$, and $h(4) = c$.

Which of these functions is onto? Justify your answer. (5 marks)

c) The following function f is defined on a set of real numbers.

$$f(x) = \frac{x+1}{x}, \text{ for all real numbers } x \neq 0$$

Determine whether f is one-to-one function and justify your answer. (5 marks)

d) Define $g: \boldsymbol{R} \rightarrow \boldsymbol{R}$ by the rule $g(x) = 4x - 5$ for all real numbers x. Find $g^{-1}(y)$.

(3 marks)

e) Given the function $f(x) = 4x^3 - 5$, $g(x) = 2x$ and $h(x) = x^2$. Find the composition of $h \circ (g \circ f)$

(3 marks)

Question 5 **6 marks**

a) Write a recursive algorithm to find the n term of the sequence defined by

$w_0 = 2, w_1 = 3$ and $w_n = w_{n-1} \cdot w_{n-2} + 3$ for $n \geq 2$. (3 marks)

b) Trace the algorithm for $n = 3$. (3marks)

Properties On Sets

Commutative laws
$A \cup B = B \cup A$
$A \cap B = B \cap A$

Associative laws
$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

Distributive laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Absorption laws
$A \cap (A \cup B) = A$
$A \cup (A \cap B) = A$

Idempotent laws
$A \cup A = A$
$A \cap A = A$

Complement laws
$(A')' = A$
$A \cup A' = U$
$A \cap A' = \varnothing$
$\varnothing' = U$
$U' = \varnothing$

De Morgan's laws
$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

Properties of universal set
$A \cup U = U$
$A \cap U = A$

Properties of empty set
$A \cup \varnothing = A$
$A \cap \varnothing = \varnothing$