Question 1 [23 Marks]

a) A cat show is being judged from pictures of the cats. The judges would like to see pictures of the following pairs of cats next to each other for their final decision: Fifi and Putih, Fifi and Suri, Fifi and Bob, Bob and Cheta, Bob and Didi, Bob and Suri, Cheta and Didi, Didi and Suri, Didi and Putih, Suri and Putih, Putih and Jeep, Jeep and Didi.

Draw a graph modeling this situation. (3 marks)

b) Given a graph as shown in Figure 1.

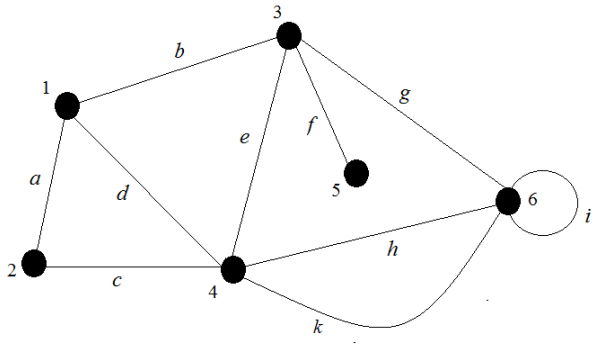

Figure 1

i. Find the incidence matrix of the graph. (4 marks)

ii. Find the adjacency matrix of the graph. (3 marks)

c) Determine whether the graphs in Figure 2 (*A* and *B*) are isomorphic. If the graphs are isomorphic, find their adjacency matrices; otherwise, give an invariant that the graphs do not share. (6 marks)

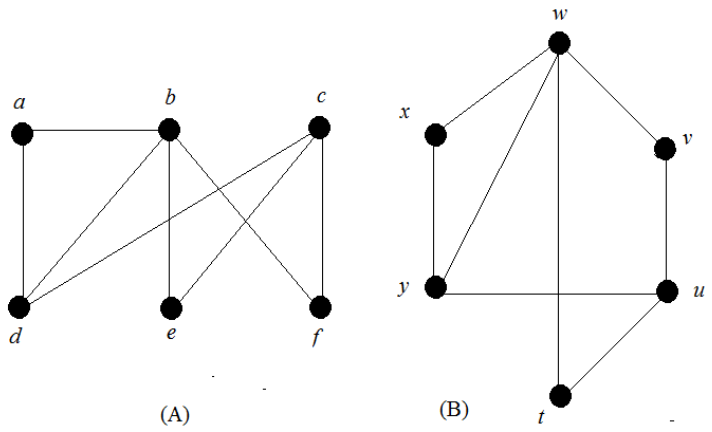

Figure 2

d) Determine whether the graph in Figure 3 has an Euler cycle or Euler path. If the graph has an Euler cycle or Euler path, exhibit one; otherwise, give an argument that shows there is no Euler path. (4 marks)

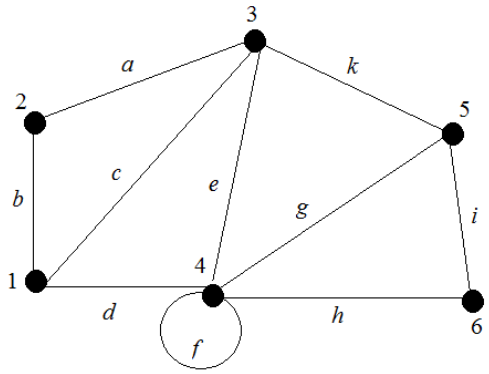

Figure 3

e) Determine whether the graph in Figure 4 has an Hamiltonian cycle. If yes, exhibit one. (3 marks)

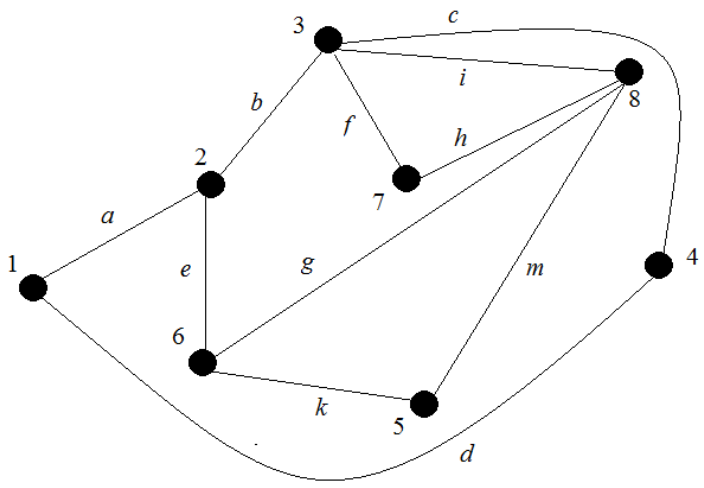

Figure 4

Question 2 [10 Marks]

The network in Figure 5 gives the distances in miles between pairs of cities A, B, …, and H.

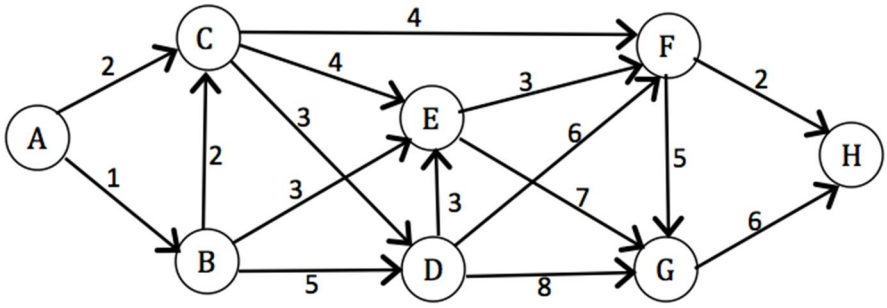

Figure 5

a) Based on Dijkstra's algorithm, complete Table 1 to find the shortest path from city A to city H. (Note: Copy Table 1 into your answer booklet).

(8 marks)

Table 1

Iteration	**S**	**N**	***L*(A)**	***L*(B)**	***L*(C)**	***L*(D)**	***L*(E)**	***L*(F)**	***L*(G)**	***L*(H)**
0										
1										
2										
3										
4										
5										
6										
7										

b) State the minimum distance and the shortest path from city 1 to city 8.

(2 marks)

Question 3 [12 Marks]

a) Consider the following Boolean function of three arguments. Derive an expression for *f(x,y,z)* as a Disjunctive Normal Form (DNF). (2 marks)

Table 2

x	*y*	*z*	*f(x,y,z)*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

b) Derive the boolean expressions of the following circuit in Figure 6. (3 marks)

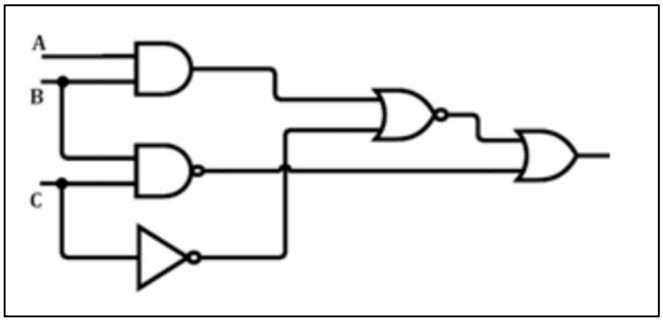

Figure 6

c) The truth table for a 4-input: **A**, **B**, **C**, **D** digital circuit give the output: **X** as shown in Table 3. If the binary number formed at the input is between 0010 and 1001, the output **X** is 1.

Table 3

	Inputs				Output
	A	B	C	D	**X**
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

i. Simplify the output expression **X** using K-Map. (3 marks)

ii. Design the logic circuit for simplified expression obtained in 3(c(i)). (4 marks)

Question 4 [25 Marks]

(a) Let $B = \{S, I, q_o, f_s, F\}$ be the Deterministic Finite Automata (DFA) machine as depicted in Figure 7.

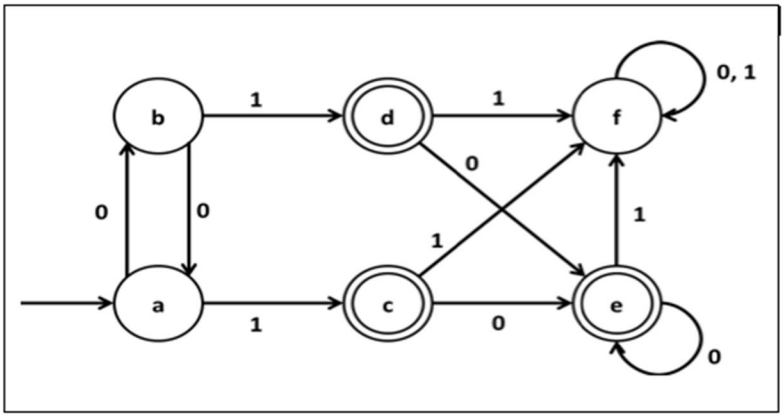

Figure 7

i. List all the components of S, I, q_o, F. (4 marks)

ii. Construct a transition table for the above machine. (4 marks)

iii. Is the string 111010 accepted by the machine? (3 marks)

iv. Find the sequence of configurations and state if the string 01011101 is accepted by the machine. (3 Marks)

(b) Let $H = \{S, I, O, q_0, f_s, f_0\}$ be the finite state machine of equipment Y. Table 4 shows the transition table for the machine.

Table 4

State	f_s		f_o	
	0	1	0	1
A	*B*	*C*	0	0
B	*D*	*B*	1	0
C	*E*	*C*	0	0
D	*B*	*F*	0	0
E	*D*	*B*	0	1
F	*E*	*C*	0	1

i. Draw the transition diagram with $q_0 = \{A\}$ for the above machine. (5 marks)

ii. What is the output string if the input string is 01110001. (2 marks)

iii. Is input string 11101010 accepted by the machine? Explain using the sequence of configurations and its output. (4 marks)

Question 5 [10 Marks]

Pac-Man is one of the few games to have been consistently published for over three decades, having been remade on numerous platforms and spawned many sequels. The typical version of Pac-Man is a one player game where he/she manoeuvres the Pac-Man around the maze, attempting to avoid four 'ghosts' characters while eating dots that distributed throughout the maze. Among the dots, there are four super dots that located at four corners of the maze. If the Pac-Man collides with the ghost, he loses one of his three lives and play resumes with the ghosts reassigned to their initial starting location. When Pac-Man eats a super dot, he is able to chase the ghosts for a few seconds of time before the super dot expires. The game ends when Pac-Man has lost all his three lives. Figure 8 shows a screenshot of the Pac-Man game.

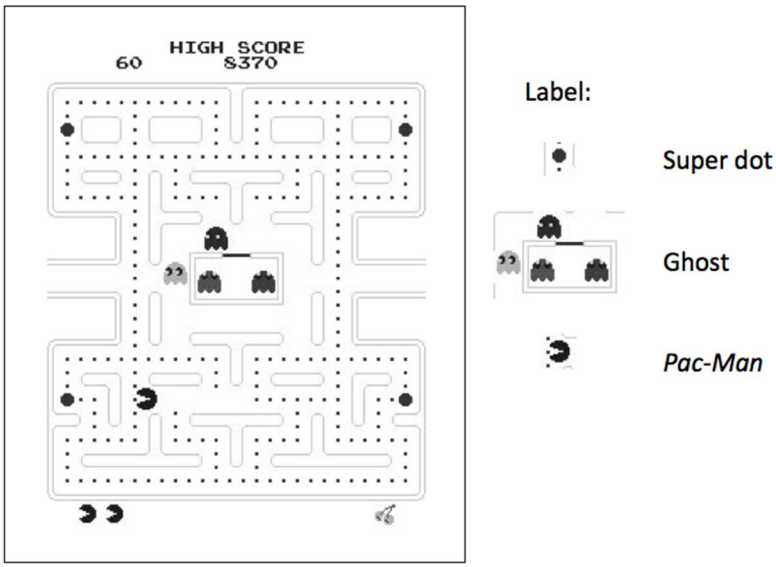

Figure 8

Noted that the **ghosts** in Pac-Man have four behaviours:

S_1 : randomly wander the maze

S_2 : chase Pac-Man when he is within the line of sight

S_3 : avoid Pac-Man when has consumed a super dot

S_4 : return to the initial position to restart the game

The inputs are:

A : spot Pac-Man (Pac-Man is within the line of sight)

B : lose Pac-Man (Pac-Man is not within the line of sight)

C : Pac-Man eats super dot

D : super dot expires

E : collides with Pac-Man

F : reach the initial position

The outputs are:

0 : nothing happened

1 : Pac-Man loses his life

2 : number of ghosts reduces by 1

Complete the transition table below. (Note: Copy Table 5 in your answer booklet and complete the unshaded cells only). (10 Marks)

Table 5

State	Input, f_s						Output, f_o					
	A	*B*	*C*	*D*	*E*	*F*	*A*	*B*	*C*	*D*	*E*	*F*
S_1												
S_2												
S_3												
S_4												

Question 6 [20 Marks]

a) Let ***C*** be the set of code words {00000000, 11111000, 01010111, 10101111}. How many errors can ***C*** detect? (6 marks)

b) Suppose the encoding function be $f_H : B^4 \rightarrow B^7$ and the parity check matrix, *H* is given by

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine the code words representing word **1110** and **1011**. (4 marks)

c) Let an encoding function, $f : B^2 \rightarrow B^5$, with *f* given by,

$$f(00) = 00000$$

$$f(01) = 01011$$

$$f(10) = 10110$$

$$f(11) = 11101$$

Decode the word **11110** and **10101** using the maximum likelihood method. (Note: Copy Table 6 in your answer booklet and complete the '?' cells). (10 marks)

Table 6

	$e_i \oplus 00000$	$e_i \oplus 01011$	$e_i \oplus 10101$	$e_i \oplus 11101$
e_1 = 00000	00000	01011	10101	11101
e_2 = 10000	10000	11011	00110	01100
e_3 = ?	?	?	?	?
e_4 = 00100	00100	01111	10010	11001
e_5 = ?	?	?	?	?
e_6 = 00001	00001	01010	10111	11100
e_7 = ?	?	?	?	?
e_8 = 10100	10100	10111	00010	01001

*** End of Questions ***