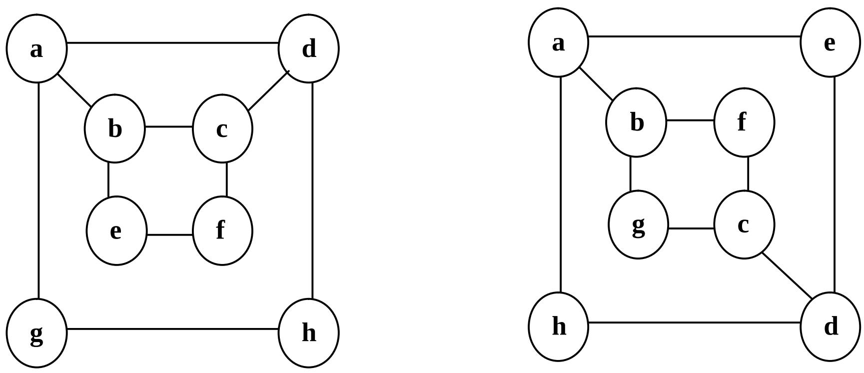
Question 1 **[25 Marks]**

i) Draw a graph with the following features: vertex set V = $\{v_1, v_2, v_3, v_4, v_5\}$ and edge set E = $\{\{v_1, v_3\}, \{v_1, v_4\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_4, v_5\}\}$ [3 marks]

ii) Based on the answer in (i), list the vertex that is not adjacent and the isolated vertex. [2 marks]

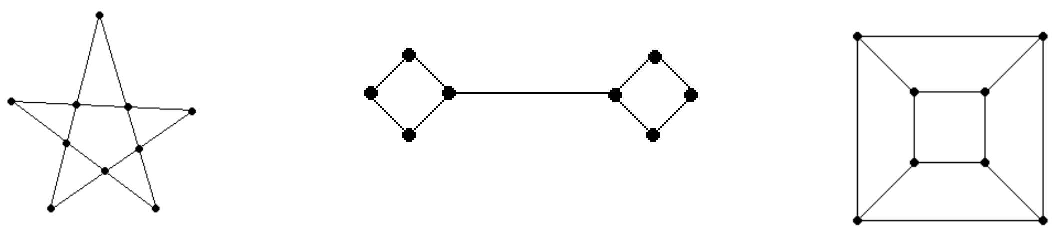
iii) Are the following graphs in Figure 1(a) and 1(b) are isomorphic? Justify your answer. [4 marks]



(a) (b)

Figure 1

iv) Given graphs in Figure 3(a), (b) and (c), determine whether they contain an Euler Circuit or an Euler path? Give your answer by completing Table 1. [6 marks]



(a) (b) (c)

Figure 2

Table 1

Graph	Number of odd vertices (vertices connected to an odd number of edges)	Number of even vertices (vertices connected to an even number of edges)	What does the path contain? (Euler path = P); Euler circuit = C; Neither = N)
2(a)			
2(b)			
2(c)			

v) Use Dijkstra's Algorithm on the graph in Figure 3 and find the shortest paths from *O* to *T*. [10 marks]

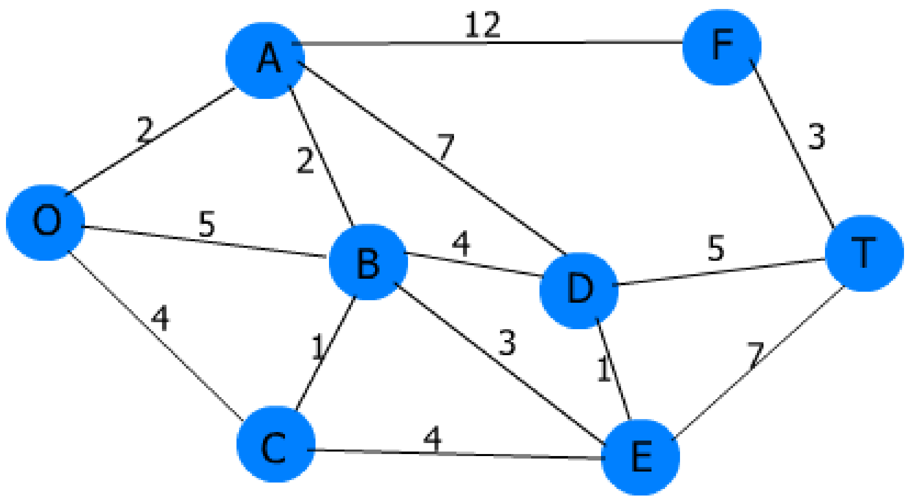


Figure 3

Question 2 **[20 Marks]**

i) Derive the Boolean expression for the logic circuit in Figure 4. [3 marks]

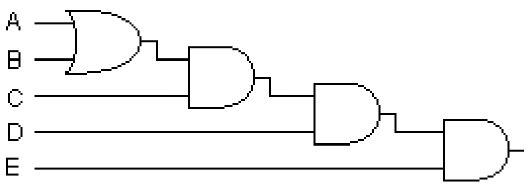


Figure 4

ii) From the truth table in Table 2, determine the DNF expression. [4 marks]

Table 2

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

iii) Generate a Karnaugh map of the following expression. [3 marks]

$$ABC\overline{D} + \overline{A}\overline{B}CD + A\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D}$$

iv) Find the minimal sum of product/ DNF for the given K-maps. [5 marks]

Y

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

v) Using appropriate properties, prove that $(a+b)(b+c)(c+a) = ab + bc + ca$

[5 marks]

Question 3 **[20 Marks]**

i) A diagram for automaton is given as follows:

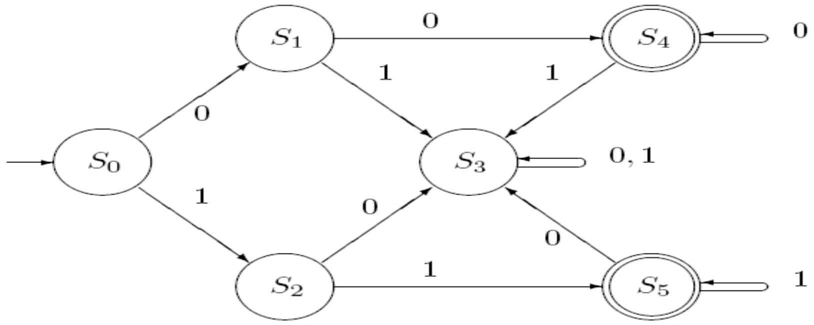


Figure 5

(a) Based on the diagram in Figure 5, determine:

1) Finite states [1 mark]

2) Initial states [1 mark]

3) Final states. [1 mark]

(b) Construct the state transition table. [3 marks]

ii) Given state transition diagram in Figure 6, find the sequence of configurations and state if the word is accepted by the language of the automaton, or not.

(a) abbbaaaba [2 marks]

(b) bbbbb [2 marks]

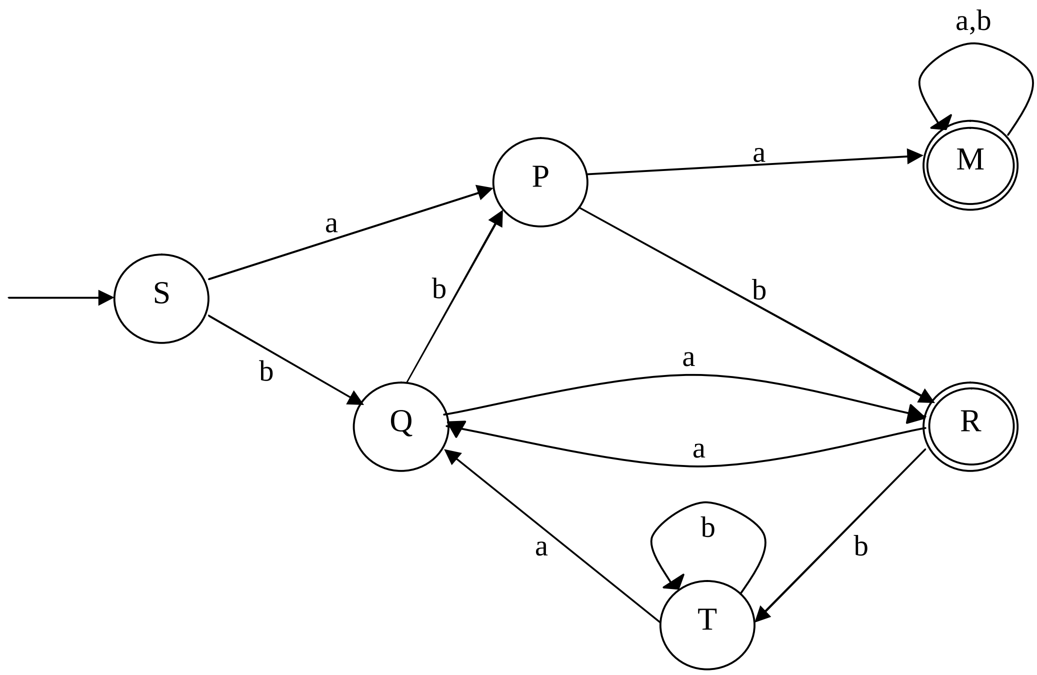


Figure 6

iii) A transition diagram of M is given in Figure 7.

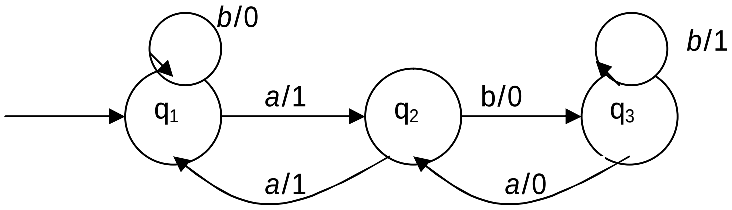


Figure 7

(a) Write the transition table of M. [3 marks]

(b) What is the output string if the input string is *aabb*? [2 marks]

(c) What is the output if the input string is *baba* [2 marks]

(d) Is the string *abba* accepted by M? [3 marks]

Question 4 **[10 Marks]**

A description of an automatic telephone answering machine is shown in Table 3. When a call arrives, the phone rings. If the phone is not picked up, then on the third ring, the machine answers. It plays a pre-recorded greeting requesting that the caller leave a message, then records the caller's message, and then automatically hangs up. If the phone is answered before the third ring, the machine does nothing.

Table 3

States		Input		Output	
q_0	idle (nothing is happening)	i_1	incoming ringing signal	0	default output when there is nothing interesting to say
q_1	one ring has arrived	i_2	a telephone is picked up	1	answer the phone and start the greeting message
q_2	two rings have arrived	i_3	greeting message is finish playing	2	start recording the incoming message
q_3	playing the greeting message	i_4	end of message detected	3	recorded an incoming message
q_4	recording the message	i_5	no input of interest		

i) Construct a state transition table by completing Table 4. [7 marks]

Table 4

	f_s					f_o				
	i_1	i_2	i_3	i_4	i_5	i_1	i_2	i_3	i_4	i_5
q_0	q_1					0				
q_1										
q_2										
q_3										
q_4										

ii) Based on answer in (i), construct a state transition diagram for the telephone answering machine. [3 marks]

Question 5 **[25 Marks]**

i) Consider the following encoding function $f: B^2 \rightarrow B^6$ defined by

$f(00) = 000000$

$f(01) = 010110$

$f(10) = 100101$

$f(11) = 110001$

(a) What is the minimum distance of the code? [7 marks]

(b) Find the error detection capability of this code. [2 marks]

ii) Consider a (3,6) code with Boolean matrix, $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Determine

(a) $f_H(010)$ [2 marks]

(b) $f_H(111)$ [2 marks]

iii) Given the (2,4) group encoding function $f: B^2 \rightarrow B^4$ defined by

$f(00) = 0000$

$f(01) = 0111$

$f(10) = 1001$

$f(11) = 1111$

Decode word 1101 based on the maximum likelihood decoding function. [12 marks]