SULIT

UTM UNIVERSITI TEKNOLOGI MALAYSIA
Fakulti Sains Komputer & Sistem Maklumat
UNIVERSITI PENYELIDIKAN

FINAL EXAMINATION SEMESTER I 2011/2012

SUBJECT CODE	**:**	**SCI 1013**
SUBJECT NAME	**:**	**DISCRETE STRUCTURE**
YEAR/COURSE	**:**	**1 SCJ/V/R/D/B/**
TIME	**:**	
DATE	**:**	
VENUE	**:**	

INSTRUCTION:

PLEASE ANSWER ALL QUESTIONS

(Please write down the lecturer's name and section in the answer booklet)

Name	
IC No / Metric Card No	
Year/Course	
Section	
Lecturer's Name	

THIS BOOKLET CONTAINS 8 PAGES ONLY EXCLUDING THIS PAGE.

Question 1 **[15 Marks]**

a) Find the number of six-letter words that can be formed from the letters of the word COMPUTER if no letter is used more than once in any word subject to the condition below.

i) The first letter of each word is either C or R (2 marks)

ii) The word starts with COM (2 marks)

iii) The word starts with P and ends with E (2 marks)

iv) The word contains COM as a substring (2 marks)

b) In a computer club, there are 10 male members and 12 female members. How many ways can we form a committee of 7 members subject to the following conditions:

i) There must be exactly 3 males and 4 females (1 marks)

ii) There must be exactly 4 males (1 marks)

iii) The committee must contain at least 2 females (2 marks)

iv) The committee must contain at least 4 males (3 marks)

Question 2 **[15 Marks]**

a) Two fair dice are rolled and the outcome of each roll is recorded.

i) Let S be the sample space. Find $|S|$. (½ marks)

ii) Let E be the event that the first die shows a 2 and the second die shows an odd number. Find the event E and $P(E)$. (2½ marks)

iii) What is the probability of getting the sum of the numbers rolled is an even number? (4 marks)

b) Table 1 refer to a company that buys computers from three vendors and tracks the number of defective machines.

Table 1: Percent of purchased and defective

	Vendor		
	Acme	DotCom	Nuclear
Percent purchased	55	10	35
Percent defective	1	3	3

Let:

A denote the event "the computer was purchased from Acme"

D denote the event "the computer was purchased from DotCom"

N denote the event "the computer was purchased from Nuclear"

B denote the event "the computer was defective"

Find:

i) $P(B)$ (2 marks)

ii) $P(A|B)$ (2 marks)

iii) $P(D|B)$ (2 marks)

iv) $P(N|B)$ (2 marks)

Question 3 **[15 Marks]**

Two graphs, G_1 and G_2, are given in Figure 1 and Figure 2 as follows.

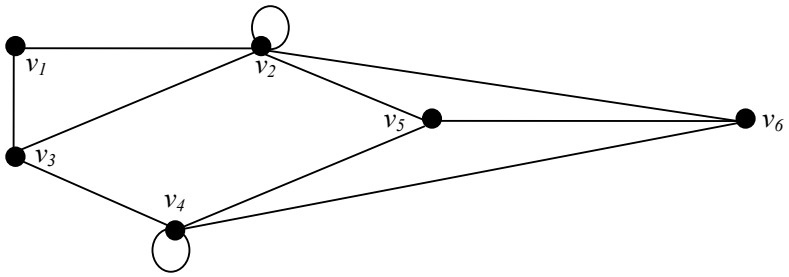


Figure 1: G_1

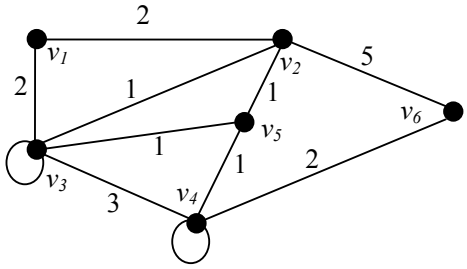


Figure 2: G_2

a) Construct the incidence matrix of G_1. (3 marks)

b) Verify whether G_1 and G_2 are isomorphic. (2 marks)

c) State whether TRUE or FALSE for the following questions.

i) G_1 is a simple graph.

ii) G_2 is a connected graph.

iii) G_1 is a K_6 regular graph.

iv) v_1 and v_5 are adjacent in G_1.

v) v_2 and v_4 are isolated in G_2.

vi) G_2 is a sub-graph of G_1.

vii) G_1 is a simple path.

viii) G_2 is a simple cycle.

ix) G_1 has Eular cycle.

x) G_2 has Hamiltonian path. (5 marks)

d) Find the shortest path from v_1 to v_6 of G_2 using Dijkstra's shortest algorithm. (5 marks)

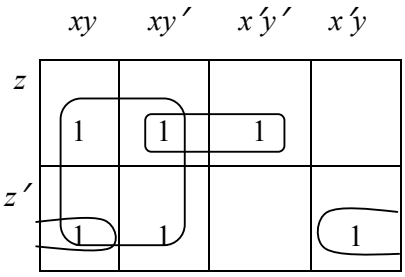
Question 4 **[20 Marks]**

a) Show the the following boolean expressions α and β are equal (6 marks)

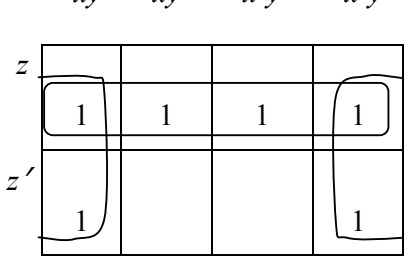
$$\alpha = x_1 + x_2 \bullet x'_2 + x_3 \qquad \beta = x_1 + x_3$$

b) Find the minimized sum-of-product Boolean expression corresponding to the *K*-maps in 3(i) and 3(ii). (4 marks)

i)



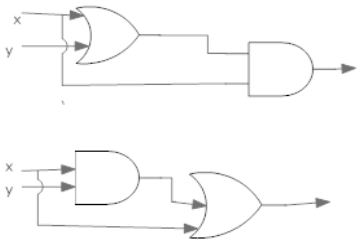
ii)



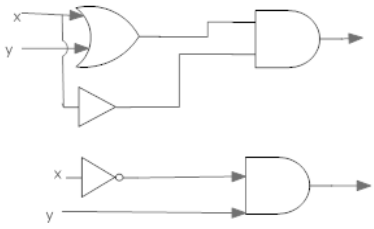
c) With the help of input-ouput table, determine if the pairs of circuits are equivalent.

(6 marks)

i)



ii)



d) Construct the circuit by using NOT, OR and AND gates corresponding to each of the given Boolean expressions

i) $xy' + (x' + y)\, y$ (2 marks)

ii) $xy + (x + y)'y$ (2 marks)

Question 5 **[15 Marks]**

a) Let $M = (Q, I, q_0, \delta, \sigma)$ be the Deterministic Finite Automaton (DFA) with state transition diagram shown in Figure 3.

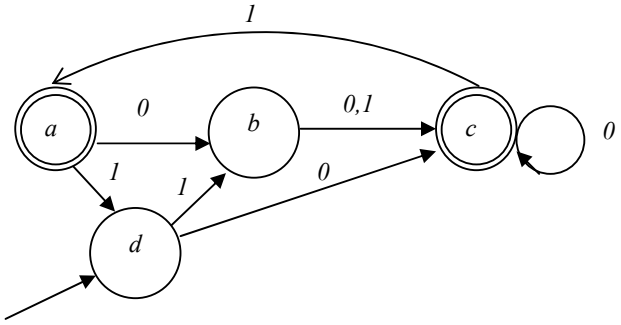


Figure 3: Deterministic Finite Automaton *M*

i) Find the initial state, set of input symbols and set of final states. (1½ marks)

ii) Write the transition table for this DFA. (2 marks)

iii) Determine the state that the machine ends for the input string 0101011. (1½ marks)

iv) Is the input string 1101 accepted by the DFA? (1½ marks)

b) Let $A = (S, I, O, A, f, g)$ be a finite state machine (FSM) defined by the transition table shown in Table 2.

Table 2: Transition table of FSM *A*

	f			*g*		
Input / State	*x*	*y*	*z*	*x*	*y*	*z*
A	*A*	*B*	*C*	*0*	*1*	*0*
B	*B*	*B*	*A*	*1*	*1*	*1*
C	*C*	*B*	*A*	*1*	*0*	*0*

i) Draw the transition diagram of the finite state machine *A*. (2 marks)

ii) Find the output string for the input string *xyxzx*. (1½ marks)

iii) Find the output generated from the input string *zxyxz*. (1½ marks)

iv) Determine whether the input string *zxyxz* is accepted by the FSM? (1 mark)

c) Construct a deterministic finite automata $M = \{(s_0, s_1, s_2), (0,1), s_0, f_s, s_2))\}$, that recognize the set of bit strings that end with two 0s by using the transition diagram in Figure 4 as the starting point. (2½ marks)

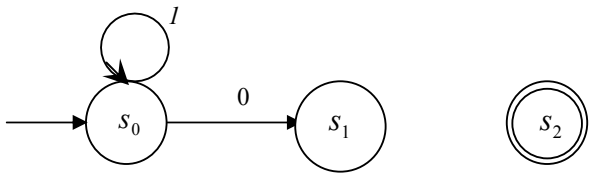


Figure 4: Transition diagram

Question 6 **[20 Marks]**

a) Consider the (3,7) encoding function $f: B^3 \rightarrow B^7$ which is given by

$f(000) = 0000000$

$f(001) = 0010110$

$f(010) = 0101000$

$f(011) = 0111110$

$f(100) = 1000101$

$f(101) = 1010011$

$f(110) = 1101101$

$f(111) = 1111011$

i) Find the weight of the above encoding function. (2 marks)

ii) Find the minimum distance. (7 marks)

iii) How many errors can f detect? (1 mark)

b) Let H be a parity check matrix. Determine the (3,6) group code $f_H: B^3 \to B^6$. (4 marks)

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

c) Consider the (3,5) group encoding function $f: B^3 \to B^5$ defined by

$f(000) = 00000$

$f(001) = 00110$

$f(010) = 01001$

$f(011) = 01111$

$f(100) = 10011$

$f(101) = 10101$

$f(110) = 11010$

$f(111) = 11100$

Decode the word 11011 relative to a maximum likelihood decoding function. (6 marks)