QUESTION 1 **20 MARKS**

a). An eight-person committee composed of A, B, C, D, E, F, G and H is to select a chairperson, vice of chairperson, secretary, and treasurer

i) In how many ways can this selection be done? (2 Marks)

ii) In how many ways can this be done if either A, B or C must be chairperson? (2Marks)

iii) In how many ways can this be done if D must hold one of the committee? (3 Marks)

iv) In how many ways can this be done if F and G must hold office? (3 Marks)

b). A committee of eight is to be made from 6 students and 10 lecturers. In how many ways can this be done?

i) If the committee contains exactly 4 students? (2 Marks)

ii) If the committee contains exactly 5 students? (2 Marks)

iii) If the committee contains at least 4 students? (3 Marks)

iv) If the committee contains at least 5 students? (3 Marks)

QUESTION 2 **10 MARKS**

c). Let H and T be the head and tail of a coin.
The coin is tossed 4 times and the result is recorded after each toss.

i) List all possible outcomes in the sample space. (2 Marks)

ii) Find the event E_1 that contains only the outcomes in which 2 tails appears. (2 Marks)

iii) Find the event E that contains only the outcomes in which 3 heads appears. (2 Marks)

iv) Suppose that the probability of P(HHHH)=2 P(HHHT) and P(TTTT)=3(PHHHT) and probability of the other outcomes are equal to P(HHHT). Find the probability of the all possible outcomes in the sample space.

(2 Marks)

v) Find Probability of the E_1. (1 Mark)

vi) Find Probability of the E_2. (1 Mark)

QUESTION 3 **23 MARKS**

a). Draw the graph (G) represented by the incidence matrix below:

$$\begin{array}{c} \\ v1 \\ v2 \\ v3 \\ v4 \\ v5 \end{array} \begin{array}{c} \begin{array}{ccccccc} e1 & e2 & e3 & e4 & e5 & e6 & e7 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

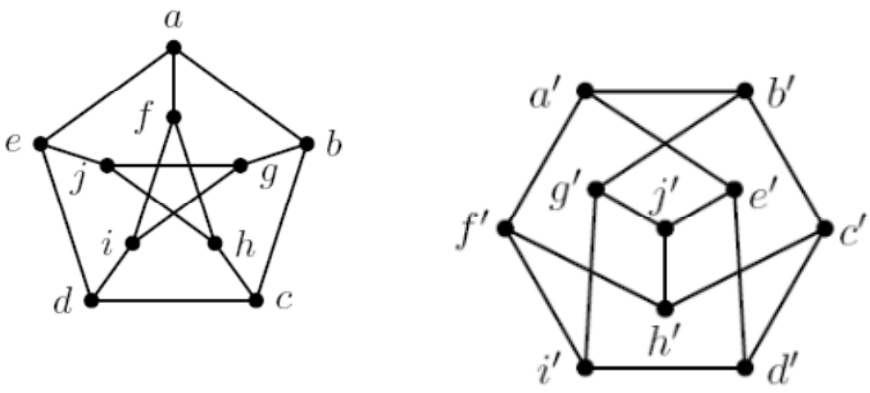
i) Is G is a simple graph?

ii) What is the degree for each vertex in G

iii) Determine whether G has an Euler cycle.

(9 marks)

b) Prove that the following two graphs in Figure 1 are isomorphic.



G R

Figure 1

(8 marks)

c). Find the shortest path from **a to z** using Djikstra's algorithm.

(6 marks)

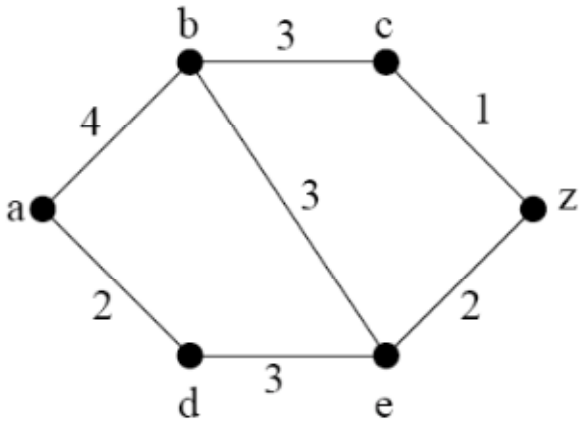


Figure 2

QUESTION 4 **12 Marks**

a). Find the disjunctive normal form and draw the combinatorial circuit corresponding to the disjunctive normal form. (Simplified your answer) (6 marks)

x	y	z	F(x,y,z)
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	0

b). Use Karnaugh maps to simplify the Boolean expression (6 marks)

i) WXY + WXZ' + WY'Z + X'Y'Z
ii) X'Y'Z + X'YZ + XY'Z'
iii) WXY + WXZ + WY'Z' + Y'Z'

QUESTION 5 **15 MARKS**

a) Let M=(S, I, q_0, f_s, F) be the deterministic finite automaton (DFA) such that S={q_0, q_1, q_2, q_3}, I={*a*, *b*, *c*}, F={q_2, q_3}, q_0 is the initial state, and f_s is defined as follows:

$f_s(q_0, a) = q_0$, $f_s(q_0, b) = q_1$, $f_s(q_0, c) = q_1$,
$f_s(q_1, a) = q_0$, $f_s(q_1, b) = q_2$, $f_s(q_1, c) = q_3$,
$f_s(q_2, a) = q_2$, $f_s(q_2, b) = q_1$, $f_s(q_2, c) = q_3$,
$f_s(q_3, a) = q_0$, $f_s(q_3, b) = q_0$, $f_s(q_3, c) = q_0$,

i. Draw the state diagram of M.

ii. Which of the strings *abc*, *bac*, and *acb* are accepted by M?

(6 marks)

b) Let M= (S, I, O, q_0, f_s, f_o) be a finite state machine (FSM). The transition diagram of M is shown in Figure 3.

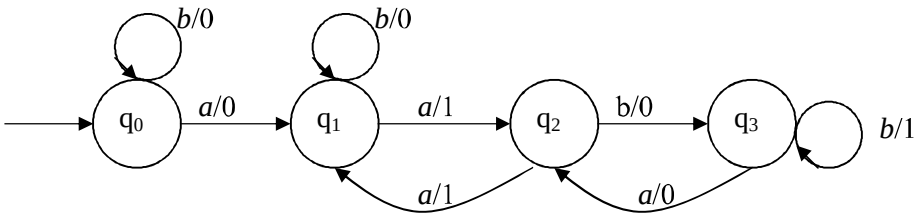


Figure 3

i. Write the transition table of M.

ii. What is the output string if the input string is *aabb*?

iii. What is the output if the input string is *baba*?

iv. Is the string *abba* accepted by M?

(5 marks)

c) Design a finite state machine (FSM) that accepts all strings over {*a*, *b*} that contain at least two *a*'s.

(4 marks)

QUESTION 6 **20 MARKS**

a) Consider the (2,6) encoding function f: $B^2 \rightarrow B^6$.

f(00) = 000000
f(01) = 010101
f(10) = 101010
f(11) = 111111

i. Show that f is a group code.
ii. Find the minimum distance.
iii. How many errors will f detect?

(6 marks)

b) Let

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Determine the (3,6) group code f_H: $B^3 \rightarrow B^6$.

(8 marks)

c) Consider the (3, 5) group encoding function f: $B^3 \rightarrow B^5$ defined by

f(000) = 00000
f(001) = 00101
f(010) = 01010
f(011) = 01101
f(100) = 10001
f(101) = 10100
f(110) = 11011
f(111) = 11111

Decode the following words relative to a maximum likelihood decoding function.

i. 10011
ii. 11101
iii. 01000

(6 marks)