



**SCSI1013/SECI 1013: Discrete Structure**

**[2019/2020 - Semester 1]**

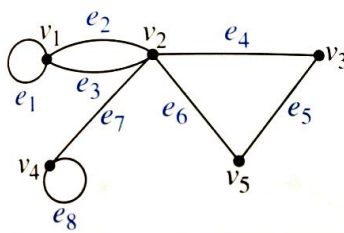
**Due Date: 15 December 2019**

**TUTORIAL 3**

1) A shuttle bus travels through eight residential colleges (C1, C2, C3, C4, C5, C6, C7, C8) to pick up students. The location of the colleges determines the bus route. There are two routes connecting C1 and C2. From C2, the shuttle can go to C3 through two routes, one route to C4 and one route to C5. There is only one route that connects C5, C6, and C7. In order to get to C7, the shuttle bus needs to pass through C6. C8 can only be access by a route through C7. During semester break; a shortcut route that connects C1, C4 and C7 can be used to shorten the journey. can be used to shorten the journey.

- a) Draw a graph that represents the route that connects the residential colleges.
- b) Decide whether the graph in 1a) is a simple graph? Justify your answer.
- c) List the degrees for each vertex in graph 1a).

2) Graph G in Figure 1 is not a simple graph.



**Figure 1: Graph G**

- a) Find the incidence matrix of the graph.
- b) Find a set of edges to remove to make it a simple graph.

- 3)  $H_1$  and  $H_2$  are adjacency matrices of two different graphs. Check whether these graphs are isomorphic? Justify your answer.

$$H_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}; H_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

- 4) Show whether the graphs in Figure 2 (G and H) are isomorphic.

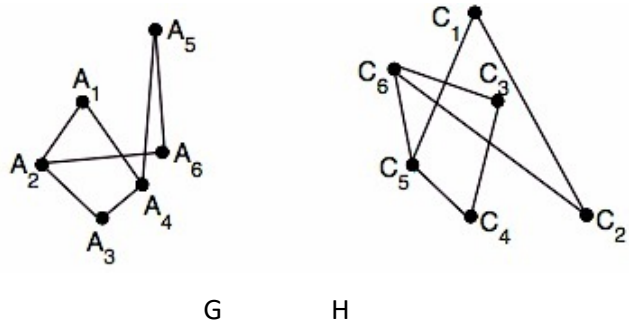


Figure 2

- 5) A tourist plans to visit seven places of attractions in a country, namely A, B, C, D, E, F, and G. These places are connected with inter city railway links. The route across these places is as shown in Figure 3.

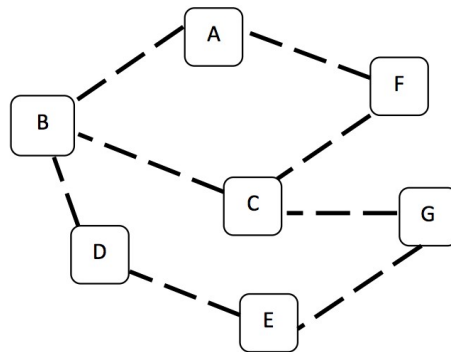
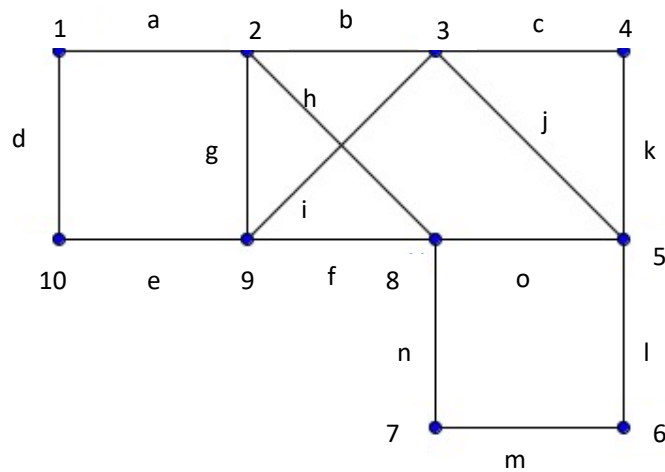


Figure 3

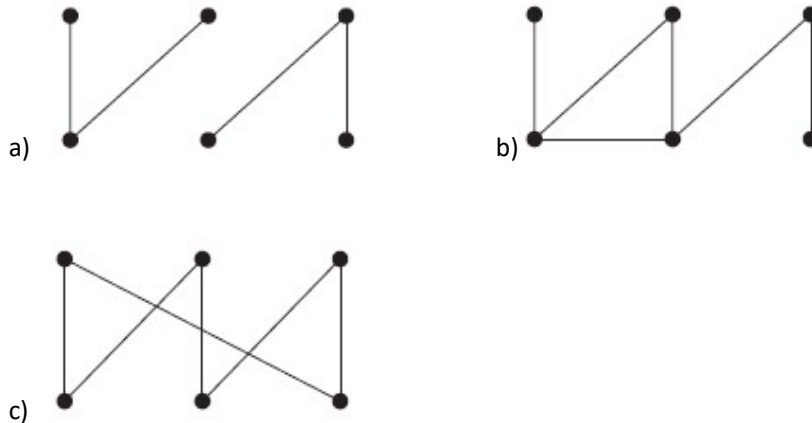
- Trace the journey of the tourist if he wanted to visit every place of attractions (A, B, C, D, E, F, G) exactly once starting and ending at E.
- Determine whether the tourist's journey in 5(a) is an Euler circuit or Hamiltonian circuit.

- c) If the intercity railway link from A to B is close due to maintenance work, plan the tourist's journey from A so he/she can still visit all the place of attractions and ends at any of the places following the Euler trail rules.
- 6) Tell if is possible to trace the diagram in Figure 4 beginning at a point and completion end at the same point but the pencil should not be lifted till the diagram is completely traced and a line in the diagram should not be retraced. Explain your reasoning and state the route by listing the vertices and edges in order visited.



**Figure 4**

- 7) Which of these graphs are trees? Justify your answer.



- 8) Construct a complete binary tree of height 4 and a full 3-ary tree of height 3.

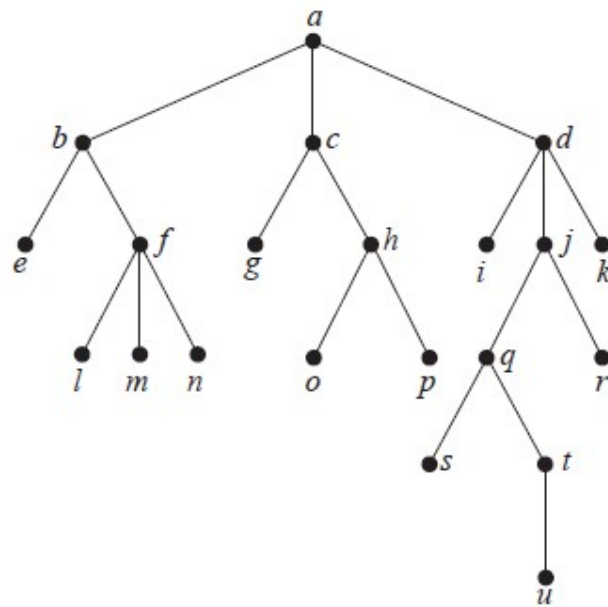
9) Represent the expression  $((x+2)^{\uparrow 3}) * (y - (3+x)) - 5$  using a binary tree.

Write this expression in pre order, in order and post order notation.

10) What is the value for the post order notation

$$521--14++*$$

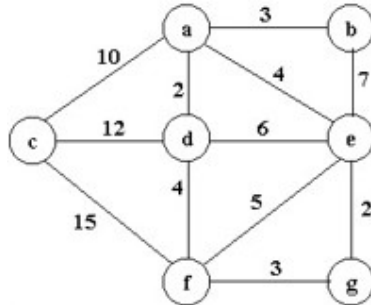
11) Given rooted tree in Figure 5



**Figure 5**

- Which vertex is the root?
- Which vertices are internal node?
- Which vertices are leaves?
- Which vertices are children of j?
- Which vertex is the parent of h?
- Which vertices are siblings of o?
- Which vertices are ancestors of m?
- Which vertices are descendants of b?

- 12) How many vertices does a full 5-ary tree with 100 internal vertices have?
- 13) How many leaves does a full 4-ary tree with 1000 vertices have?
- 14) Use Kruskal algorithm to find the minimum spanning tree for the following graph in Figure 6



**Figure 6**

- 15) A chain letter starts when a person sends a letter to 12 others. Each person receiving the letter either sends it to 12 others who have never received the letter before or does not send it to anyone. Suppose there are 20736 people received letter. How many people do not send the letter and how many people send out the letter?
- 16) A chain letter starts when a person sends a letter to five others. Each person who receives the letter either sends it to five other people who have never received it or does not send it to anyone. Suppose that 10,000 people send out the letter before the chain ends and that no one receives more than one letter. How many people receive the letter, and how many do not send it out?
- 17) A chain letter starts with a person sending a letter out to 6 others. Each person is asked to send the letter out to 6 others, and each letter contains a list of the previous four people in the chain. Unless there are fewer than four names in the list, each person sends one dollar to the first person in this list, removes the name of this person from the list, moves up each of the other three names one position, and inserts his or her name at the end of this list. If no person breaks the chain and no one receives more than one letter, how much money will a person in the chain ultimately receive?