SKMM3023

APPLIED NUMERICAL METHODS

REPORT (PROJECT 1)

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1.0 INTRODUCTION

1.1 BRIEF BACKGROUND OF PROJECT

This project contains four questions which are under ‘Chapter 5 : Curve Fitting and Interpolation’ and ‘Chapter 6 : Numerical Differentiation’. Students are needed to answer all the questions given to make sure students are fully understanding about the chapters involved. Curve fitting plays an important role in analysis, interpretation and correlation of the experimental data. In most general sense, curve fitting involves the determination of a continuous function to aid for data visualization, to infer values of a function where no data are available and to summarize he relationships among two or more variables [1]. Numerical differentiation in this project is used to determine the velocity, acceleration and jerk from the data given by using forward, central and backward difference approximations.

1.2 IMPORTANCE OF PROJECT

By completing this project, student will be able to fully understanding and able to solve the engineering problem. Students need to complete this project to get carry marks since this subject (Applied Numerical Methods) do not have final exam. So, students must make sure the project been completed respectively.

1.3 OBJECTIVE OF PROJECT

The objective of this project is to make sure students are able to solve the problems given. We must fully understand the topics to answer all the questions. In other words, the goals for this project is to abstract the ability of students to know plot and construct the curve that have best fit to a series of data points. Other than that, students should know why we need to approximate derivatives. We should also understand how to approximate the data given by using forward, central and backward difference approximation. Students will be able to know which finite difference approximation (forward, central and backward difference) are more accurate from this project. The finite difference approximations are one of the simplest and of the oldest methods to solve the differential equations. It is close to the numerical schemes used to solve ordinary differential equations [2].

2.0 METHODOLOGY
2.1 QUESTION 1
2.1.1 SAMPLE CALCULATION

For this sample calculation, take x = 2.00. So, we need to find the value of decrement and increment of x value. Below are the value obtained :

h = 0.05;	
x = 2.00;	f(x) = 7.3891;
x + h = 2.05;	f(x + h) = 7.7679;
x + 2h = 2.10;	(x + 2h) = 8.1662;
x + 3h = 2.15;	f(x + 3h) = 8.5849;
x – h = 1.95;	f(x – h) = 7.0287;
x – 2h = 1.90;	f(x – 2h) = 6.6859;
x – 3h = 1.85;	f(x – 3h) = 6.3598;

After obtain the value, calculate f'(t) and f''(t) by using backward, central and forward difference approximation. Then, calculate the error to compare the results between exact value and approximate value. Calculate for low accuracy and high accuracy.

Low Accuracy

Backward Difference Approximation

First Derivative

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

$$f'(x) = \frac{7.3891 - 7.0287}{0.05} = 7.2080$$

Error of first derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.2080}{7.3891}\right| \times 100\% = 2.4509\%$$

Second Derivative

$$f''(x) = \frac{f(x) - 2f(x-h) + f(x-2h)}{h^2}$$

$$f''(x) = \frac{7.3891 - 2(7.0287) + 6.6859}{(0.05)^2} = 7.0400$$

Error for second derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.040}{7.3891}\right| \times 100\% = 4.7245\%$$

Central Difference Approximation

First Derivative

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$f'(x) = \frac{7.7679 - 7.0287}{2(0.05)} = 7.3920$$

Error for first derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.3920}{7.3891}\right| \times 100\% = 0.0392\%$$

Second Derivative

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$f''(x) = \frac{7.7679 - 2(7.3891) + 7.0287}{(0.05)^2} = 7.3600$$

Error for second derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.360}{7.3891}\right| \times 100\% = 0.3938\%$$

Forward Difference Approximation

First Derivative

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \frac{7.7679 - 7.3891}{0.05} = 7.5760$$

Error for first derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.5760}{7.3891}\right| \times 100\% = 2.5294\%$$

Second Derivative

$$f''(x) = \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}$$

$$f''(x) = \frac{8.1662 - 2(7.7679) + 7.3891}{(0.05)^2} = 7.8000$$

Error for second derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.80}{7.3891}\right| \times 100\% = 5.5609\%$$

High Accuracy

Backward Difference Approximation

First Derivative

$$f'(x) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$$

$$f'(x) = \frac{3(7.3891) - 4(7.0287) + 6.6859}{2(0.05)} = 7.3840$$

Error for first derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.3840}{7.3891}\right| \times 100\% = 0.0690\%$$

Second Derivative

$$f''(x) = \frac{2f(x) - 5f(x-h) + 4f(x-2h) - f(x-3h)}{h^2}$$

$$f''(x) = \frac{2(7.3891) - 5(7.0287) + 4(6.6859) - 6.3598}{(0.05)^2} = 7.4000$$

Error for second derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.4000}{7.3891}\right| \times 100\% = 0.1475\%$$

Central Difference Approximation

First Derivative

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

$$f'(x) = \frac{-8.1662 + 8(7.7679) - 8(7.0287) + 6.6859}{12(0.05)} = 7.3888$$

Error for first derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.3888}{7.3891}\right| \times 100\% = 0.0041\%$$

Second Derivative

$$f''(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$$

$$f''(x) = \frac{-8.1662 + 16(7.7679) - 30(7.3891) + 16(7.0287) - 6.6859}{12(0.05)^2} = 7.3500$$

Error for second derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.3500}{7.3891}\right| \times 100\% = 0.5291\%$$

Forward Difference Approximation

First Derivative

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$$

$$f'(x) = \frac{-8.1662 + 4(7.7679) - 3(7.3891)}{2(0.05)} = 7.3810$$

Error for first derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.3810}{7.3891}\right| \times 100\% = 0.1096\%$$

Second Derivative

$$f''(x) = \frac{-f(x+3h) + 4f(x+2h) - 5f(x+h) + 2f(x)}{h^2}$$

$$f''(x) = \frac{-8.5849 + 4(8.1662) - 5(7.7679) + 2(7.3891)}{(0.05)^2} = 7.4400$$

Error of second derivative

$$\varepsilon_a = \left|\frac{7.3891 - 7.4400}{7.3891}\right| \times 100\% = 0.6889\%$$

2.1.2 RESULTS

		Backward Difference		Central Difference		Forward Difference	
x	$f(x)$	$f'(x)$	$f''(x)$	$f'(x)$	$f''(x)$	$f'(x)$	$f'(x)$
2.00	7.3891	7.2074	7.0302	7.3921	7.3906	7.5769	7.7695
2.05	7.7679	7.5769	7.3906	7.7711	7.7695	7.9654	8.1679
2.10	8.1662	7.9654	7.7695	8.1696	8.1679	8.3738	8.5866
2.15	8.5849	8.3738	8.1679	8.5884	8.5866	8.8031	9.0269
2.20	9.0250	8.8031	8.5866	9.0288	9.0269	9.2544	9.4897
2.25	9.4877	9.2544	9.0269	9.4917	9.4897	9.7289	9.9763
2.30	9.9742	9.7289	9.4897	9.9783	9.9763	10.2277	10.4878
2.35	10.486	10.2277	9.9763	10.4899	10.4878	10.7521	11.0255
2.40	11.023	10.7521	10.4878	11.0278	11.0255	11.3034	11.5908
2.45	11.588	11.3034	11.0255	11.5932	11.5908	11.8829	12.1850
2.50	12.182	11.8829	11.5908	12.1876	12.1850	12.4922	12.8098

TABLE 1 : RESULTS OF QUESTION 1(LOW ACCURACY)

		Backward Difference		Central Difference		Forward Difference	
x	$f(x)$	$f'(x)$	$f''(x)$	$f'(x)$	$f''(x)$	$f'(x)$	$f''(x)$
2.00	7.3891	7.3831	7.3730	7.3891	7.3891	7.3827	7.3712
2.05	7.7679	7.7617	7.7510	7.7679	7.7679	7.7612	7.7491
2.10	8.1662	8.1596	8.1484	8.1662	8.1662	8.1591	8.1464
2.15	8.5849	8.5780	8.5662	8.5849	8.5849	8.5774	8.5641
2.20	9.0250	9.0178	9.0054	9.0250	9.0250	9.0172	9.0032
2.25	9.4877	9.4801	9.4671	9.4877	9.4877	9.4795	9.4648
2.30	9.9742	9.9662	9.9525	9.9742	9.9742	9.9656	9.9500
2.35	10.486	10.4772	10.4628	10.4856	10.4856	10.4765	10.4602
2.40	11.023	11.0143	10.9992	11.0232	11.0232	11.0136	10.9965
2.45	11.588	11.5790	11.5632	11.5883	11.5883	11.5783	11.5603
2.50	12.182	12.1727	12.1560	12.1825	12.1825	12.1720	12.1530

TABLE 2 : RESULTS OF QUESTION 1(HIGH ACCURACY)

2.1.3 PSEUDOCODE

1. Start
2. Declare the value of x and h
   x = [1.85 1.90 1.95 2.00 2.05 2.10 2.15 2.20 2.25 2.30 2.35 2.40 2.45 2.50 2.55 2.60 2.65]
   h = 0.05
3. Set the equation
   fx = exp(x)
   fx1 = exp(x)
   fx2 = exp(x)
4. Set a = 4:1:14
5. Calculate the first and second derivatives for low and high accuracy using the formula of backward, central and forward difference approximation and find the error for each of the derivatives.

Low Accuracy

Backward Difference

First Derivative :

$$fx1_{backward} = \frac{fx(1,a) - fx(1,a-1)}{h}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx1_{backward}}{fx}\right| \times 100\%$$

Second Derivative :

$$fx2_{backward} = \frac{fx(1,a) - 2fx(1,a-1) + fx(1,a-2)}{h^2}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx2_{backward}}{fx}\right| \times 100\%$$

Central Difference

First Derivative :

$$fx1_{central} = \frac{fx(1,a+1) - fx(1,a-1)}{2h}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx1_{central}}{fx}\right| \times 100\%$$

Second Derivative :

$$fx2_{central} = \frac{fx(1,a+1) - 2fx(1,a) + fx(1,a-1)}{h^2}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx2_{central}}{fx}\right| \times 100\%$$

Forward Derivative

First Derivative

$$fx1_{forward} = \frac{fx(1,a+1) - fx(1,a)}{h}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx1_{forward}}{fx}\right| \times 100\%$$

Second Derivative

$$fx2_{forward} = \frac{fx(1, a + 2) - 2fx(1, a + 1) + fx(1, a)}{h^2}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx2_{forward}}{fx}\right| \times 100\%$$

High Accuracy

Backward Difference

First Derivative

$$fx1h_{backward} = \frac{3fx(1, a) - 4fx(1, a - 1) + fx(1, a - 2)}{2h}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx1h_{backward}}{fx}\right| \times 100\%$$

Second Derivative

$$fx2h_{backward} = \frac{2fx(1, a) - 5fx(1, a - 1) + 4fx(1, a - 2) - fx(1, a - 3)}{h^2}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx2h_{backward}}{fx}\right| \times 100\%$$

Central Difference

First Derivative

$$fx1h_{central} = \frac{-fx(1, a + 2) + 8fx(1, a + 1) - 8fx(1, a - 1) + fx(1, a - 2)}{12h}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx1h_{central}}{fx}\right| \times 100\%$$

Second Derivative

$$fx2h_{central} = \frac{-fx(1, a + 2) + 16fx(1, a + 1) - 30fx(1, a) + 16fx(1, a - 1) - fx(1, a - 2)}{12h^2}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx2h_{central}}{fx}\right| \times 100\%$$

Forward Difference

First Derivative

$$fx1h_{forward} = \frac{-fx(1, a + 2) + 4fx(1, a + 1) - 3fx(1, a)}{2h}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx1h_{forward}}{fx}\right| \times 100\%$$

Second Derivative

$$fx2h_{forward} = \frac{-fx(1, a+3) + 4fx(1, a+2) - 5fx(1, a-1) + 2fx(1, a)}{h^2}$$

Error :

$$\varepsilon_a = \left|\frac{fx - fx2h_{forward}}{fx}\right| \times 100\%$$

6. Display the value of backward, central and forward difference approximation
7. Plot the graph
8. End

2.1.4 GRAPH

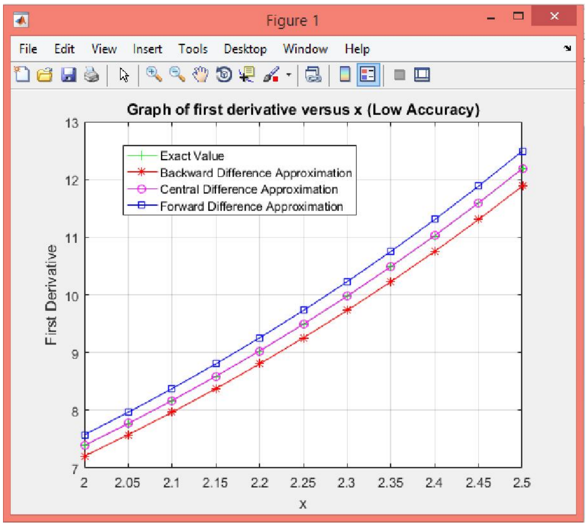


Figure 1.1

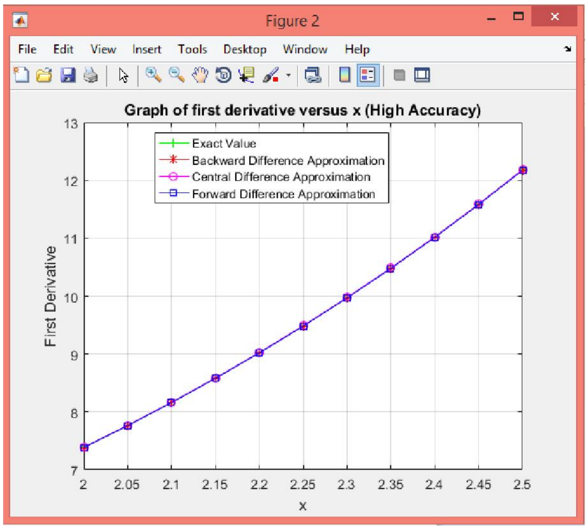


Figure 1.2

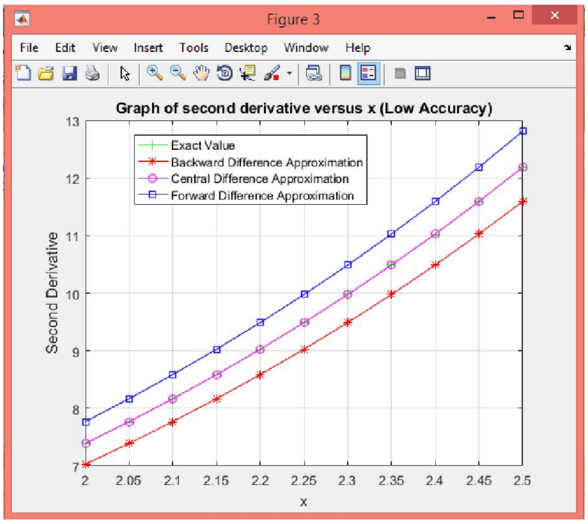


Figure 1.3

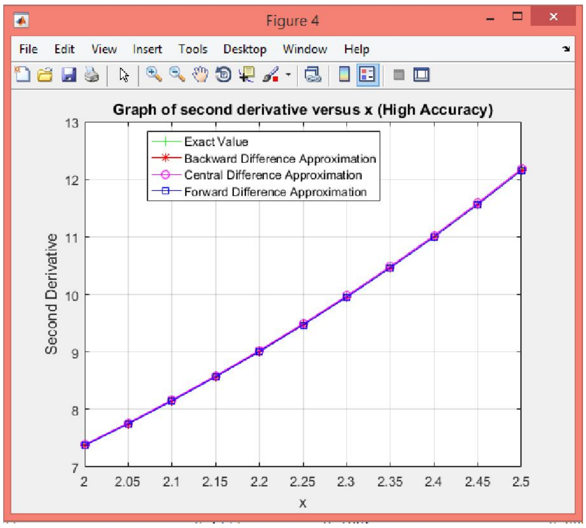


Figure 1.4

2.2 QUESTION 2

2.2.1 SAMPLE CALCULATION

Time, ti (sec)	t1 = 0.20	t2 = 0.30	t3 = 0.41
Altitude, hi (ft)	h1 = 445.98	h2 = 471.85	h3 = 503.46

In this question, there are two step size which is h = 0.10 and h = 0.11. Choose only h = 0.10 for this sample of calculation. Calculate $\frac{dh}{dt}$ for t=0.30 by using the formula of backward, central and forward difference approximation.

Step size, h = 0.1; t = 0.30; t + h = 0.41; t – h = 0.20;

Backward Difference Approximation

$$h'(t) = \frac{h(t) - h(t-h)}{h}$$

$$h'(t) = \frac{471.85 - 445.98}{0.10} = 258.70$$

Central Difference Approximation

$$h'(t) = \frac{h(t+h) - h(t-h)}{2h}$$

$$h'(t) = \frac{503.46 - 445.98}{2(0.10)} = 287.40$$

Forward Difference Approximation

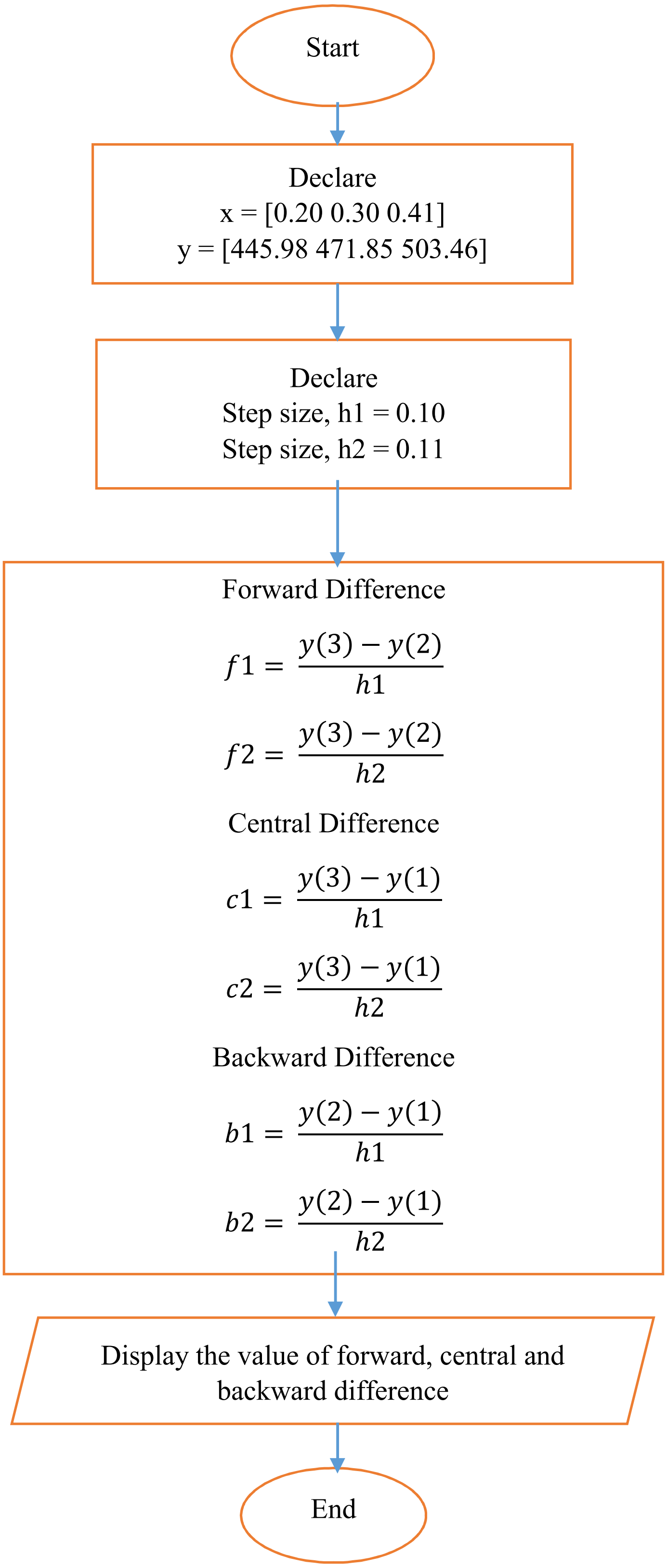
$$h'(t) = \frac{h(t+h) - h(t)}{h}$$

$$h'(t) = \frac{503.46 - 471.85}{0.10} = 316.10$$

2.2.2 RESULTS

		$h'(t)$		
t	$h(t)$	Backward Difference	Central Difference	Forward Difference
0.30	471.85	258.70	287.40	316.10

2.2.3 FLOWCHART



2.2.4 GRAPH

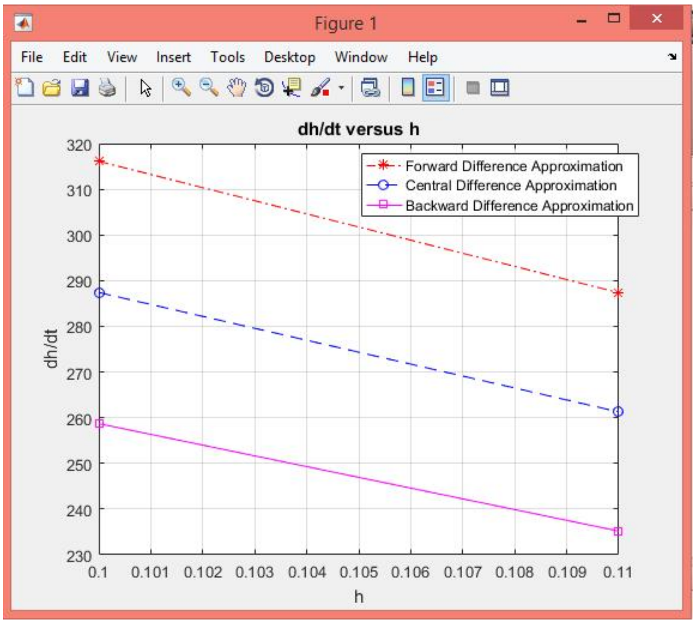


Figure 2.1

2.3 QUESTION 3

2.3.1 EXACT SOLUTION

$$s(\theta) = L\,\{\,1.0 - 2.65\left(\frac{\theta}{\beta}\right)^2 + 2.80\left(\frac{\theta}{\beta}\right)^5 + 3.25\left(\frac{\theta}{\beta}\right)^6 - 6.85\left(\frac{\theta}{\beta}\right)^7 + 2.60\left(\frac{\theta}{\beta}\right)^8\,\}$$

$$\theta(t) = \sin 120\pi t\,; \quad L = 1\,inch\,; \quad \beta = {}^{\pi}/_{2}\,;$$

Substitute $\theta(t), L$ and β into the equation :

$$s(t) = \{\,1.0 - 2.65\left(\frac{\sin 120\pi t}{\pi/2}\right)^2 + 2.80\left(\frac{\sin 120\pi t}{\pi/2}\right)^5 + 3.25\left(\frac{\sin 120\pi t}{\pi/2}\right)^6 - 6.85\left(\frac{\sin 120\pi t}{\pi/2}\right)^7 + 2.60\left(\frac{\sin 120\pi t}{\pi/2}\right)^8\,\}$$

Differentiate $s(t)$ to $s'(t)$:

$$s'(t) = -\frac{2544\cos(120\pi t)\sin(120\pi t)}{\pi} + \frac{53760\cos(120\pi t)\,sin^4(120\pi t)}{\pi^4} + \frac{149760\cos(120\pi t)\,sin^5(120\pi t)}{\pi^5} - \frac{736512\cos(120\pi t)\,sin^6(120\pi t)}{\pi^6} + \frac{638976\cos(120\pi t)\,sin^7(120\pi t)}{\pi^7}$$

Differentiate $s'(t)$ to $s''(t)$:

$$s''(t) = -305280\,cos^2(120\pi t) + 305280\,sin^2(120\pi t) + \frac{258048000\,cos^2(120\pi t)\,sin^3(120\pi t) - 6451200\,sin^5(120\pi t)}{\pi^3} + \frac{89856000\,cos^2(120\pi t)\,sin^4(120\pi t) - 17971200\,sin^6(120\pi t)}{\pi^4} - \frac{530288640\,cos^2(120\pi t)\,sin^5(120\pi t) + 88381440\,sin^7(120\pi t)}{\pi^5} + \frac{536739840\,cos^2(120\pi t)\,sin^6(120\pi t) - 76677120\,sin^8(120\pi t)}{\pi^6}$$

Differentiate $s''(t)$ to $s'''(t)$

$$s'''(t) = 146534400\pi\cos(120\pi t)\sin(120\pi t) + \frac{9289728000\,cos^3(120\pi t)\,sin^2(120\pi t) - 10063872000\cos(120\pi t)\,sin^4(120\pi t)}{\pi^2} + \frac{43130880000\,cos^3(120\pi t)\,sin^3(120\pi t) - 34504704000\cos(120\pi t)\,sin^5(120\pi t)}{\pi^3} - \frac{318173184000\,cos^3(120\pi t)\,sin^4(120\pi t) + 201509683200\cos(120\pi t)\,sin^6(120\pi t)}{\pi^4} + \frac{386452684800\,cos^3(120\pi t)\,sin^5(120\pi t) - 202427596800\cos(120\pi t)\,sin^7(120\pi t)}{\pi^5}$$

Then, substitute t = 1/240 into $s'(t), s''(t)$ and $s'''(t)$, we will get :

$$s'(t) = 0.0; \qquad s''(t) = 121779.9510; \qquad s'''(t) = 0.0;$$

2.3.2 NUMERICAL METHOD

$$s(\theta) = L\,\{\,1.0 - 2.65\left(\frac{\theta}{\beta}\right)^2 + 2.80\left(\frac{\theta}{\beta}\right)^5 + 3.25\left(\frac{\theta}{\beta}\right)^6 - 6.85\left(\frac{\theta}{\beta}\right)^7 + 2.60\left(\frac{\theta}{\beta}\right)^8\}$$

$\theta(t) = \sin 120\pi t\,;\quad L = 1\ inch\,;\quad \beta = {}^{\pi}/_{2}\,;$

Substitute $\theta(t), L$ and β into the equation :

$$s(t) = \{\,1.0 - 2.65\left(\frac{\sin 120\pi t}{\pi/2}\right)^2 + 2.80\left(\frac{\sin 120\pi t}{\pi/2}\right)^5 + 3.25\left(\frac{\sin 120\pi t}{\pi/2}\right)^6 - 6.85\left(\frac{\sin 120\pi t}{\pi/2}\right)^7 + 2.60\left(\frac{\sin 120\pi t}{\pi/2}\right)^8\}$$

Calculate the velocity, acceleration and jerk for low and high accuracy by using forward, central and backward difference approximation.

Let h = 1/20000

t = 1/240;	s(t) = 0.2150;
t + h = 253/60000;	s(t + h) = 0.2151;
t + 2h = 8/1875;	s(t + 2h) = 0.2156;
t + 3h = 259/60000;	s(t + 3h) = 0.2164;
t + 4h = 131/30000;	s(t + 4h) = 0.2174;
t – h = 247/60000;	s(t – h) = 0.2151;
t – 2h = 61/15000;	s(t – 2h) = 0.2156;
t – 3h = 241/60000;	s(t – 3h) = 0.2164;
t – 4h = 119/30000;	s(t – 4h) = 0.2174;

Central Difference

Low Accuracy

$$Velocity = \frac{s(t+h) - s(t-h)}{2h}$$

$$Velocity = \frac{0.2151 - 0.2151}{2(^1/_{20000})} = 0.0$$

$$Acceleration = \frac{s(t+h) - 2s(t) + s(t-h)}{h^2}$$

$$Acceleration = \frac{0.2151 - 2(0.2150) + 0.2151}{(^1/_{20000})^2} = 121800.7543$$

$$Jerk = \frac{s(t+2h) - s(t+h) + 2s(t-h) - s(t-2h)}{2h^3}$$

$$Jerk = \frac{0.2156 - 0.2151 + 2(0.2151) - 0.2156}{0\left(2(^1/_{20000})^3\right)} = 0.0$$

High Accuracy

$$Velocity = \frac{-s(t+2h) + 8s(t+h) - 8s(t-h) + s(t-2h)}{12h}$$

$$Velocity = \frac{-0.2156 + 8(0.2151) - 8(0.2151) + 0.2156}{12(^1/_{20000})} = 0.0$$

$$Acceleration = \frac{-s(t+2h) + 16s(t+h) - 30s(t) + 16s(t-h) - s(t-2h)}{12h^2}$$

$$Acceleration = \frac{-0.2156 + 16(0.2151) - 30(0.2150) + 16(0.2151) - 0.2156}{12(^1/_{20000})^2}$$

$$= 121779.9750$$

$$Jerk = \frac{-s(t+3h) + 8s(t+2h) - 13s(t+h) + 13s(t-h) - 8s(t-2h) + s(t-3h)}{8h^3}$$

$$Jerk = \frac{-0.2164 + 8(0.2156) - 13(0.2151) + 13(0.2151) - 8(0.2156) + 0.2164}{8(^1/_{20000})^3} = 0.0$$

2.3.3 RESULTS

		Central Difference	
	Exact Value	Low Accuracy	High Accuracy
Velocity	0.0	0.0	0.0
Acceleration	121779.9510	121800.7543	121779.9750
Jerk	0.0	0.0	0.0

TABLE 5 : COMPARISON BETWEEN EXACT AND APPROXIMATE VALUE

2.3.4 ERROR ANALYSIS

Formula to calculate error :

$$Error = \left|\frac{Exact\ Value - Approximate\ Value}{Exact\ Value}\right| \times 100\%$$

For velocity and jerk (low and high accuracy)

$$Error = \left|\frac{0.0 - 0.0}{0.0}\right| \times 100\% = 0\%$$

For acceleration

Low accuracy

$$Error = \left|\frac{121779.9510 - 121800.7543}{121779.9510}\right| \times 100\% = 0.0171\%$$

High accuracy

$$Error = \left|\frac{121779.9510 - 121779.9750}{121779.9510}\right| \times 100\% = 0.0$$

2.3.5 PSEUDOCODE

1. Start
2. Declare the value of t and h
   t = 1/240;   h = 1/20000
3. Obtain the equation after all value been substitute
4. $$s(t) = \{\, 1.0 - 2.65\left(\frac{\sin 120\pi t}{\pi/2}\right)^2 + 2.80\left(\frac{\sin 120\pi t}{\pi/2}\right)^5 + 3.25\left(\frac{\sin 120\pi t}{\pi/2}\right)^6 - 6.85\left(\frac{\sin 120\pi t}{\pi/2}\right)^7 + 2.60\left(\frac{\sin 120\pi t}{\pi/2}\right)^8 \,\}$$
5. Calculate the velocity, acceleration and jerk for low and high accuracy by using central difference approximation

$$Velocity = \frac{s(t+h)-s(t-h)}{2h}$$ (low accuracy)

$$Velocity = \frac{-s(t+2h)+8s(t+h)-8s(t-h)+s(t-2h)}{12h}$$ (high accuracy)

$$Acceleration = \frac{s(t+h)-2s(t)+s(t-h)}{h^2}$$ (low accuracy)

$$Acceleration = \frac{-s(t+2h)+16s(t+h)-30s(t)+16s(t-h)-s(t-2h)}{12h^2}$$ (high accuracy)

$$Jerk = \frac{s(t+2h)-s(t+h)+2s(t-h)-s(t-2h)}{2h^3}$$ (low accuracy)

$$Jerk = \frac{-s(t+3h)+8s(t+2h)-13s(t+h)+13s(t-h)-8s(t-2h)+s(t-3h)}{8h^3}$$ (high accuracy)

6. Display the output of central difference approximation for low and high accuracy
7. End

2.4 QUESTION 4

2.4.1 SAMPLE CALCULATION

Find the velocity (y'(t)), acceleration (y''(t)) and jerk (y'''(t)) for the following time.
t = 0.04, 0.16, 0.24, 0.32, 0.44, 0.52, 0.60;   h = 0.04;

Low Accuracy

For t = 0.04, use forward difference approximation

$$Velocity = \frac{y(t+h) - y(t)}{h}$$

$$Velocity = \frac{0.235 - 0.156}{0.04} = 1.9750$$

$$Acceleration = \frac{y(t+2h) - 2y(t+h) + y(t)}{h^2}$$

$$Acceleration = \frac{0.248 - 2(0.235) + 0.156}{(0.04)^2} = -41.2500$$

$$Jerk = \frac{y(t+3h) - 3y(t+2h) + 3y(t+h) - y(t)}{h^3}$$

$$Jerk = \frac{0.0125 - 3(0.248) + 3(0.235) - 0.156}{(0.04)^3} = -2851.5625$$

For t=0.16, 0.24, 0.32, 0.44, 0.52, use central difference approximation
t = 0.16;

$$Velocity = \frac{y(t+h) - y(t-h)}{2h}$$

$$Velocity = \frac{0.072 - 0.248}{2(0.04)} = -2.200$$

$$Acceleration = \frac{y(t+h) - 2y(t) + y(t-h)}{h^2}$$

$$Acceleration = \frac{0.072 - 2(0.0125) + 0.248}{(0.04)^2} = 184.3750$$

$$Jerk = \frac{y(t+2h) - y(t+h) + 2y(t-h) - y(t-2h)}{2h^3}$$

$$Jerk = \frac{0.085 - 0.072 + 2(0.248) - 0.235}{(2(0.04)^3)} = 2140.6250$$

For t = 0.60, use backward difference approximation

$$Velocity = \frac{y(t) - y(t-h)}{h}$$

$$Velocity = \frac{0.561 - 0.048}{0.04} = 12.8250$$

$$Acceleration = \frac{y(t) - 2y(t-h) + y(t-2h)}{h^2}$$

$$Acceleration = \frac{0.561 - 2(0.048) + 0.035}{(0.04)^2} = 312.5000$$

$$Jerk = \frac{y(t) - 3y(t-h) + 3y(t-2h) - y(t-3h)}{h^3}$$

$$Jerk = \frac{0.561 - 3(0.048) + 3(0.035) - 0.245}{(0.04)^3} = 4328.1250$$

High Accuracy

For t = 0.04, use forward difference approximation

$$Velocity = \frac{-y(t+2h) + 4y(t+h) - 3y(t)}{2h}$$

$$Velocity = \frac{-0.248 + 4(0.235) - 3(0.156)}{2(0.04)} = 2.8000$$

$$Acceleration = \frac{-y(t+3h) + 4y(t+2h) - 5y(t+h) + 2y(t)}{h^2}$$

$$Acceleration = \frac{-0.0125 + 4(0.248) - 5(0.235) + 2(0.156)}{(0.04)^2} = 72.8125$$

$$Jerk = \frac{-3y(t+4h) + 14y(t+3h) - 24y(t+2h) + 18y(t+h) - 5y(t)}{2h^3}$$

$$Jerk = \frac{-3(0.072) + 14(0.0125) - 24(0.248) + 18(0.235) - 5(0.156)}{2(0.04)^3} = -19867.1875$$

For t = 0.16, 0.24, 0.32, and 0.44, use central difference approximation

Take t = 0.16

$$Velocity = \frac{-y(t+2h) + 8y(t+h) - 8y(t-h) + y(t-2h)}{12h}$$

$$Velocity = \frac{-0.085 + 8(0.072) - 8(0.248) + 0.235}{12(0.04)} = -2.6208$$

$$Acceleration = \frac{-y(t+2h) + 16y(t+h) - 30y(t) + 16y(t-h) - y(t-2h)}{12h^2}$$

$$Acceleration = \frac{-0.085 + 16(0.072) - 30(0.0125) + 16(0.248) - 0.235}{12(0.04)^2} = 230.4688$$

$$Jerk = \frac{-y(t+3h) + 8y(t+2h) - 13y(t+h) + 13y(t-h) - 8y(t-2h) + y(t-3h)}{8h^3}$$

$$Jerk = \frac{-0.524 + 8(0.085) - 13(0.072) + 13(0.248) - 8(0.235) + 0.156}{8(0.04)^3} = 1406.2500$$

For t = 0.52 and 0.60, use backward difference approximation

Take t = 0.52

$$Velocity = \frac{3y(t) - 4y(t-h) + y(t-2h)}{2h}$$

$$Velocity = \frac{3(0.035) - 4(0.245) + 0.245}{2(0.04)} = -7.8750$$

$$Acceleration = \frac{2y(t) - 5y(t-h) + 4y(t-2h) - y(t-3h)}{h^2}$$

$$Acceleration = \frac{2(0.035) - 5(0.245) + 4(0.245) - 0.123}{(0.04)^2} = -186.2500$$

$$Jerk = \frac{5y(t) - 18y(t-h) + 24y(t-2h) - 14y(t-3h) + 3y(t-4h)}{2h^3}$$

$$Jerk = \frac{5(0.035) - 18(0.245) + 24(0.245) - 14(0.123) + 3(0.456)}{2(0.04)^3} = 10085.9375$$

Fit polynomial 4^{th} order

$$a_0(n) + a_1\sum t + a_2\sum t^2 + a_3\sum t^3 + a_4\sum t^4 = \sum y$$
$$a_0\sum t + a_1\sum t^2 + a_2\sum t^3 + a_3\sum t^4 + a_4\sum t^5 = \sum ty$$
$$a_0\sum t^2 + a_1\sum t^3 + a_2\sum t^4 + a_3\sum t^5 + a_4\sum t^6 = \sum t^2y$$
$$a_0\sum t^3 + a_1\sum t^4 + a_2\sum t^5 + a_3\sum t^6 + a_4\sum t^7 = \sum t^3y$$
$$a_0\sum t^4 + a_1\sum t^5 + a_2\sum t^6 + a_3\sum t^7 + a_4\sum t^8 = \sum t^4y$$

$n = 15$;
$\sum t = 4.8$; $\sum t^2 = 1.984$; $\sum t^3 = 0.9216$; $\sum t^4 = 0.4565$
$\sum t^5 = 0.2354$; $\sum t^6 = 0.1249$; $\sum t^7 = 0.0676$; $\sum t^8 = 0.0371$;
$\sum y = 3.2815$; $\sum ty = 1.1343$; $\sum t^2y = 0.4878$; $\sum t^3y = 0.2332$;
$\sum t^4y = 0.1191$;

$$15a_0 + 4.8a_1 + 1.984a_2 + 0.9216a_3 + 0.4565a_4 = 3.2815$$
$$4.8a_0 + 1.984a_1 + 0.9216a_2 + 0.4565a_3 + 0.2354a_4 = 1.1343$$
$$1.984a_0 + 0.9216a_1 + 0.4565a_2 + 0.2354a_3 + 0.1249a_4 = 0.4878$$
$$0.9216a_0 + 0.4565a_1 + 0.2354a_2 + 0.1249a_3 + 0.0676a_4 = 0.2332$$
$$0.4565a_0 + 0.2354a_1 + 0.1249a_2 + 0.0676a_3 + 0.0371a_4 = 0.1191$$

Solving all the simultaneous equations, we get :
$a_0 = 0.6235$; $a_1 = -11.8590$; $a_3 = 82.6079$; $a_4 = -203.7637$; $a_5 = 163.7460$
$y = 0.6235 - 11.8590t + 82.6079t^2 - 203.7637t^3 + 163.7460t^4$

So, plot the graph based on the above equation.

2.4.2 RESULTS

Time	Velocity	Acceleration	Jerk
0.04	1.9750	-41.2500	-2851.5625
0.16	-2.2000	184.3750	2140.6250
0.24	5.6500	266.2500	-1222.6563
0.32	-0.8500	317.5000	4921.8750
0.44	1.5250	-76.2500	-3281.2500
0.52	-2.4625	139.3750	5921.8750
0.60	12.8250	312.5000	4328.1250

TABLE 3 : LOW ACCURACY

Time	Velocity	Acceleration	Jerk
0.04	2.8000	72.8125	-19867.1875
0.16	-2.6208	230.4688	1406.2500
0.24	7.0677	350.9115	-8390.6250
0.32	-1.2125	437.0833	1982.4219
0.44	2.9104	-101.7188	-9308.5937
0.52	-7.8750	-186.2500	10085.9375
0.60	19.0750	485.6250	-1414.0625

TABLE 4 : HIGH ACCURACY

2.4.3 ERROR ANALYSIS

Time (sec)	Displacement (m)	Exact Displacement (m)	Error (%)
0.04	0.156	0.2687	41.94
0.16	0.0125	0.1135	88.99
0.24	0.085	0.2620	67.56
0.32	0.236	0.3277	27.98
0.44	0.245	0.1784	37.33
0.52	0.035	0.1157	69.75
0.60	0.561	0.4555	23.16

2.4.4 PSEUDOCODE (PART A)

1. Start
2. Declare value of t, y and h
   t = 0.04, 0.08, 0.12, 0.16, 0.20, 0.24, 0.28, 0.32, 0.36, 0.40, 0.44, 0.48, 0.52, 0.56, 0.60
   y = 0.156, 0.235, 0.248, 0.0125, 0.072, 0.085, 0.524, 0.236, 0.456, 0.123, 0.245, 0.245, 0.035, 0.048, 0.561
   h = 0.04
3. Calculate the forward, central and backward difference approximation for low and high accuracy

Low Accuracy

For t = 0.04, use forward difference approximation

$$Velocity = \frac{y(t+h) - y(t)}{h}$$

$$Acceleration = \frac{y(t+2h) + 4y(t+h) + y(t)}{h^2}$$

$$Jerk = \frac{y(t+3h) - 3y(t+2h) + 3y(t+h) - y(t)}{h^3}$$

For t = 0.16, 0.24, 0.32, 0.44, and 0.52, use central difference approximation

$$Velocity = \frac{y(t+h) - y(t-h)}{2h}$$

$$Acceleration = \frac{y(t+h) - 2y(t) + y(t-h)}{h^2}$$

$$Jerk = \frac{y(t+2h) - y(t+h) + 2y(t-h) - y(t-2h)}{2h^3}$$

For t = 0.60, use backward difference approximation

$$Velocity = \frac{y(t) - y(t-h)}{h}$$

$$Acceleration = \frac{y(t) - 2y(t-h) + y(t-2h)}{h^2}$$

$$Jerk = \frac{y(t) - 3y(t-h) + 3y(t-2h) - y(t-3h)}{h^3}$$

High Accuracy

For t = 0.04, use forward difference approximation

$$Velocity = \frac{-y(t+2h) + 4y(t+h) - 3y(t)}{2h}$$

$$Acceleration = \frac{-y(t+3h) + 4y(t+2h) - 5y(t+h) + 2y(t)}{h^2}$$

$$Jerk = \frac{-3y(t+4h) + 14y(t+3h) - 24y(t+2h) + 18y(t+h) - 5y(t)}{2h^3}$$

For t = 0.16, 0.24, 0.32, and 0.44, use central difference approximation

$$Velocity = \frac{-y(t+2h) + 8y(t+h) - 8y(t-h) + y(t-2h)}{12h}$$

$$Acceleration = \frac{-y(t+2h) + 16y(t+h) - 30y(t) + 16y(t-h) - y(t-2h)}{12h^2}$$

$$Jerk = \frac{-y(t+3h) + 8y(t+2h) - 13y(t+h) + 13y(t-h) - 8y(t-2h) + y(t-3h)}{8h^3}$$

For t = 0.52 and 0.60, use backward difference approximation

$$Velocity = \frac{3y(t) - 4y(t-h) + y(t-2h)}{2h}$$

$$Acceleration = \frac{2y(t) - 5y(t-h) + 4y(t-2h) - y(t-3h)}{h^2}$$

$$Jerk = \frac{5y(t) - 18y(t-h) + 24y(t-2h) - 14y(t-3h) + 3y(t-4h)}{2h^3}$$

4. Display the value of forward, central and backward difference approximation for low and high accuracy
5. Plot the graph
6. End

2.4.5 GRAPH (PART A)

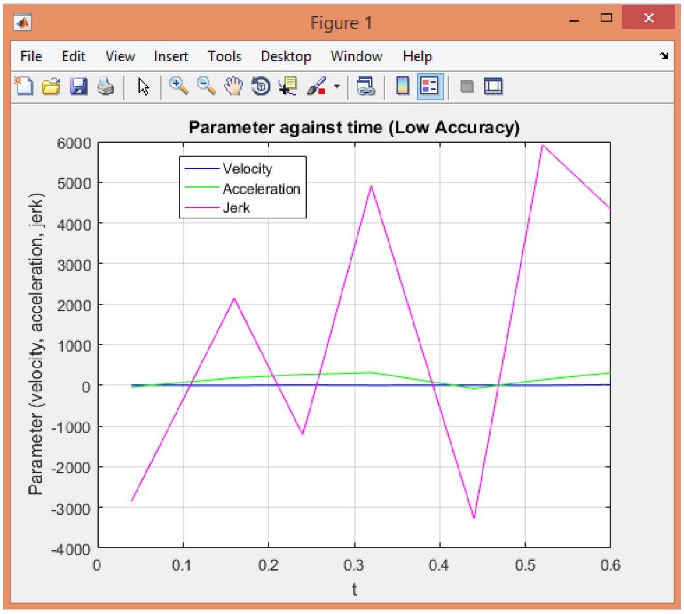


Figure 4.1

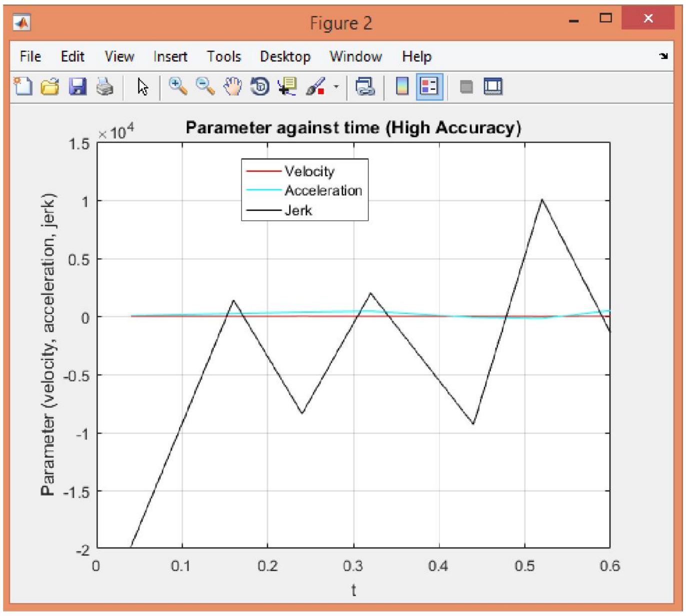


Figure 4.2

2.4.6 GRAPH (PART B)

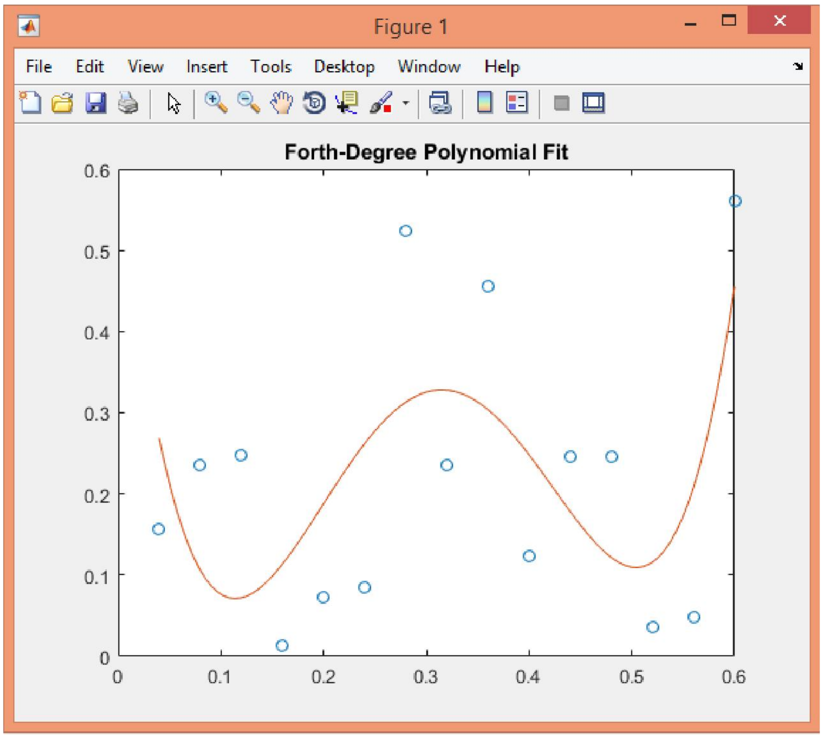


Figure 4.3

3.0 DISCUSSION OF RESULTS

3.1 QUESTION 1

Both plotted graph (Figure 1.1 and Figure 1.2) for first derivatives shows that the graph increasing exponentially which means the approximation value increases as the value of x increases.

From Figure 1.1, only central difference approximation is equal to the exact value. Small deviation occurs between the central and the exact value which the error is 0.0417%. Error of forward and backward difference between exact value are 2.5422% and 2.4588. This shows that central difference is more accurate compared to forward and backward difference.

From Figure 1.2 which is for high accuracy, all the value of forward and backward difference seems to be equal to the exact value. The errors for both difference approximations are 0.0865% and 0.0803% respectively. While the central difference is completely same with the exact value because the error is 0.0%. This prove that central difference is more accurate. The slope of the graph for all approximations are quite similar.

For the second derivative, the plotted graph (Figure 1.3 and Figure 1.4) are the same with the plotted graph for the first derivative. The patterns and trends of the graphs also similar. The only difference is the error occurs between the data.

The errors obtained from the graph in Figure 1.3 (low accuracy) for forward, central and backward difference are 5.1490%, 0.0208% and 4.8572%. Only central difference value is near to the exact value. For graph in Figure 1.4, the errors for forward, central and backward are 0.2421%, 0.0% and 0.2171% respectively.

From all the plotted graph, mostly central difference approximations meet the expectations which is equal to the exact value. Based on all the errors obtained for low and high accuracy, the errors for high accuracy is much smaller compared to the low accuracy. This prove that the accuracy level of high accuracy is greater than low accuracy.

3.2 QUESTION 2

For this question, there are two step size which are 0.10 and 0.11. So, we need to make comparison between them. Based on the graph in Figure 2.1, the value of forward, central and backward difference approximation for step size equal to 0.10 is greater than the step size equal to 0.11.

From the graph obtained, the finite difference approximations are inversely proportional to the step size. The trend for the graph is decreasing linearly which means when the step size increases, the finite difference approximations (forward, central and backward difference) becomes smaller. This shows that smaller step size is more accurate compare to the larger step size. No error analysis for this question since the exact value is not given.

3.3 QUESTION 3

From previous questions, we know that the central difference is more accurate compare to the forward and backward difference. In this question, we use both methods which are exact solution and numerical methods to compare the accuracy of the output of velocity, acceleration and jerk.

Based on the error analysis, for velocity and jerk, the error is 0.0% which means the values obtained from the numerical methods are equal to the exact solution. The acquired value of acceleration for low and high accuracy are 121800.7543m/s^2 and 121779.9750m/s^2 respectively that is slightly different from the exact value. The error exist between the exact and approximation value for low accuracy is 0.0171% meanwhile for high accuracy is 0.0 (actual value near to zero). This prove that the accuracy level of low accuracy is smaller than the high accuracy.

3.4 QUESTION 4

Students are needed to find the velocity, acceleration and jerk for t equal to 0.04, 0.16, 0.24, 0.32, 0.44, 0.52 and 0.60 by using the suitable finite difference approximation formulas for low and high accuracy to determine the accuracy level.

For t equal to 0.04, we need to use forward difference since the value before do not given in the questions. Use backward difference for t equal to 0.60 for low accuracy while for high accuracy, t equal to 0.52 also need to use this finite difference. The value of approximations for other t are determined by using the central difference approximations because the value before and after are given. As we already know, central difference is more accurate compared to the forward and backward difference.

In part A, the plotted graph (Figure 4.1 and Figure 4.2) shows the fluctuation of the data (velocity, acceleration and jerk) against the time from 0.04 to 0.60 seconds. The graph presents both the increase and decline of velocity, acceleration and jerk over the time. Although there is decline in the data output, the data still gradually increases over the time. From both graph, we can see that the fluctuation for high accuracy is smaller compared to the low accuracy which means the value obtained from calculation for high accuracy is lower than low accuracy and more accurate.

For part B, students are needed to fit the polynomial until 4^{th} order from the data given. Before that, we need to form five equations with five unknowns to form the general equation for the graph. The, we get the equation which is $y = 0.6235 - 11.8590t + 82.6079t^2 - 203.7637t^3 + 163.7460t^4$. After doing this, we get the graph in Figure 4.3 which shows all the plotted points and the line that have been curved fit. It also shows the fluctuation of the data where there is a decline and increase of data over the time. But mostly, the data still increase throughout the time given.

Based on the error analysis, the range of error obtained from the needed time is from 20% to 90%. At t equal to 0.60 seconds, the error occurs is the lowest while at t equal to 0.16 seconds, the error exists is the highest. The error acquired from this part is high become we have done repetitive calculation to obtain the data output.

In a nutshell, for this questions, students should be able know the suitable finite difference approximation formulas for every value of t given. From this, we able to solve the question easily.

4.0 CONCLUSION

As a conclusion, the objective of this project have been achieved. Students able to solve all the question given as we are fully understanding the topic of curve fitting and numerical differentiation. We also able to know the suitable solution to use to answer the question correctly.

From graph in Figure 1.1, Figure 1.2, Figure 1.3 and Figure 1.4, it shows that the finite different approximation is increasing exponentially against the value of x. From this, it is proven that the central difference is more accurate compared to the forward and backward different. From the graph in Figure 2.1, the finite difference is inversely proportional to the step size. This shows that when the step size increase, the value of finite difference decrease. Furthermore, the smaller step size is more accurate compared to the larger step size. Based on graph in Figure 4.1 and 4.2, the data obtained are fluctuated. It shows the decline and increase of the data over the time. But mostly, the data will gradually increase over the time. From this question, we also able to learn how to curve fit and construct the line.

Basically, we able to learn why we need to approximation the derivatives using forward, central and backward difference and able to which finite difference is more accurate. We also can use this to solve the engineering problem. This project also prove that the accuracy level of high accuracy is greater than the low accuracy. We also get to learn to find the error when there is an exact value given. For certain, we need to do error analysis to find the error exists between the exact value and approximate value. In order to reduce the error, we need to reduce the step size so that the values obtained are more accurate.

5.0 APPENDIX

Matlab coding for question 1

```
%Question 1
x = [ 1.85 1.90 1.95 2.00 2.05 2.10 2.15 2.20 2.25 2.30 2.35
2.40 2.45 2.50 2.55 2.60 2.65 ];
h = 0.05;

fx = exp(x);
fx1 = exp(x);
fx2 = exp(x);

a = 4:1:14;

%Backward difference approximation
%Low Accuracy
%First derivative
fx1_backward = (fx(1,a)-fx(1,a-1))/h;
error1_back = (fx1(1,a) - fx1_backward)./(fx1(1,a))*100;
%Second derivative
fx2_backward = (fx(1,a)-2*fx(1,a-1)+fx(1,a-2))/(h*h);
error2_back = (fx2(1,a) - fx2_backward)./(fx2(1,a))*100;

%High Accuracy
%First derivative
fx1h_backward = ((3*fx(1,a))-(4*fx(1,a-1))+fx(1,a-2))/(2*h);
error1h_back = (fx1(1,a) - fx1h_backward)./(fx1(1,a))*100;
%Second derivative
fx2h_backward = ((2*fx(1,a))-(5*fx(1,a-1))+(4*fx(1,a-2))-
fx(1,a-3))/(h*h);
error2h_back = (fx1(1,a) - fx2h_backward)./(fx1(1,a))*100;

%Central difference approximation
%Low Accuracy
%First derivative
fx1_central = (fx(1,a+1)-fx(1,a-1))/(2*h);
error1_cent = abs((fx1(1,a) - fx1_central)./(fx1(1,a)))*100;
%Second derivative
fx2_central = (fx(1,a+1)-2*fx(1,a)+fx(1,a-1))/(h*h);
error2_cent = abs((fx2(1,a) - fx2_central)./(fx2(1,a)))*100;

%High Accuracy
%First derivative
fx1h_central = (-fx(1,a+2)+(8*fx(1,a+1))-(8*fx(1,a-1))+fx(1,a-
2))/(12*h);
error1h_cent = abs((fx1(1,a) - fx1h_central)./(fx1(1,a)))*100;
%Second derivative
fx2h_central = (-fx(1,a+2)+(16*fx(1,a+1))-
(30*fx(1,a))+(16*fx(1,a-1))-fx(1,a-2))/(12*h*h);
error2h_cent = abs((fx1(1,a) - fx2h_central)./(fx1(1,a)))*100;
```

```
%Forward difference approximation
%Low Accuracy
%First derivative
fx1_forward = (fx(1,a+1)-fx(1,a))/h;
error1_forw = abs((fx1(1,a) - fx1_forward)./(fx1(1,a)))*100;
%Second derivative
fx2_forward = (fx(1,a+2)-2*fx(1,a+1)+fx(1,a))/(h*h);
error2_forw = abs((fx1(1,a) - fx2_forward)./(fx2(1,a)))*100;

%High Accuracy
%First derivative
fx1h_forward = (-fx(1,a+2)+(4*fx(1,a+1))-(3*fx(1,a)))/(2*h);
error1h_forw = abs((fx1(1,a) - fx1h_forward)./(fx1(1,a)))*100;
%Second derivative
fx2h_forward = (-fx(1,a+3)+(4*fx(1,a+2))-
(5*fx(1,a+1))+(2*fx(1,a)))/(h*h);
error2h_forw = abs((fx1(1,a) - fx2h_forward)./(fx1(1,a)))*100;

%display the output
fprintf ('Low Accuracy\n');
fprintf ('\nFirst Derivative\n');
fprintf ('\nx\t\t\tfx\t\t\tBackward
Difference\t\t\tError\t\t\tCentral
Difference\t\t\tError\t\t\tForward Difference\t\t\tError\n');
fprintf
('%.4f\t\t%.4f\t\t%.4f\t\t\t\t\t\t%.4f\t\t\t%.4f\t\t\t\t\t\t%.
4f\t\t\t%.4f\t\t\t\t\t\t%.4f\n',[x(1,a);fx(1,a);fx1_backward;e
rror1_back;fx1_central;error1_cent;fx1_forward;error1_forw]);
fprintf ('\nSecond Derivative\n');
fprintf ('\nx\t\t\tfx\t\t\tBackward
Difference\t\t\tError\t\t\tCentral
Difference\t\t\tError\t\t\tForward Difference\t\t\tError\n');
fprintf
('%.4f\t\t%.4f\t\t%.4f\t\t\t\t\t\t%.4f\t\t\t%.4f\t\t\t\t\t\t%.
4f\t\t\t%.4f\t\t\t\t\t\t%.4f\n',[x(1,a);fx(1,a);fx2_backward;e
rror2_back;fx2_central;error2_cent;fx2_forward;error2_forw]);
fprintf ('\nHigh Accuracy\n');
fprintf ('\nFirst Derivative\n');
fprintf ('\nx\t\t\tfx\t\t\tBackward
Difference\t\t\tError\t\t\tCentral
Difference\t\t\tError\t\t\tForward Difference\t\t\tError\n');
fprintf
('%.4f\t\t%.4f\t\t%.4f\t\t\t\t\t\t%.4f\t\t\t%.4f\t\t\t\t\t\t%.
4f\t\t\t%.4f\t\t\t\t\t\t%.4f\n',[x(1,a);fx(1,a);fx1h_backward;
error1h_back;fx1h_central;error1h_cent;fx1h_forward;error1h_fo
rw]);
fprintf ('\nSecond Derivative\n');
fprintf ('\nx\t\t\tfx\t\t\tBackward
Difference\t\t\tError\t\t\tCentral
Difference\t\t\tError\t\t\tForward Difference\t\t\tError\n');
fprintf
('%.4f\t\t%.4f\t\t%.4f\t\t\t\t\t\t%.4f\t\t\t%.4f\t\t\t\t\t\t%.
4f\t\t\t%.4f\t\t\t\t\t\t%.4f\n',[x(1,a);fx(1,a);fx2h_backward;
```

```
error2h_back;fx2h_central;error2h_cent;fx2h_forward;error2h_fo
rw]);


%plot the graph
x1 = [2.00 2.05 2.10 2.15 2.20 2.25 2.30 2.35 2.40 2.45 2.50];
fxi = exp (x1);
figure
plot(x1,fxi,'-g+',x1,fx1_backward,'-r*',x1,fx1_central,'-
mo',x1,fx1_forward,'-bs');
title('Graph of first derivative versus x (Low Accuracy)');
xlabel('x'); ylabel('First Derivative');
legend('Exact Value','Backward Difference
Approximation','Central Difference Approximation','Forward
Difference Approximation')
grid on
figure
plot(x1,fxi,'-g+',x1,fx1h_backward,'-r*',x1,fx1h_central,'-
mo',x1,fx1h_forward,'-bs');
title('Graph of first derivative versus x (High Accuracy)');
xlabel('x'); ylabel('First Derivative');
legend('Exact Value','Backward Difference
Approximation','Central Difference Approximation','Forward
Difference Approximation')
grid on
figure
plot(x1,fxi,'-g+',x1,fx2_backward,'-r*',x1,fx2_central,'-
mo',x1,fx2_forward,'-bs');
title('Graph of second derivative versus x (Low Accuracy)');
xlabel('x'); ylabel('Second Derivative');
legend('Exact Value','Backward Difference
Approximation','Central Difference Approximation','Forward
Difference Approximation')
grid on
figure
plot(x1,fxi,'-g+',x1,fx2h_backward,'-r*',x1,fx2h_central,'-
mo',x1,fx2h_forward,'-bs');
title('Graph of second derivative versus x (High Accuracy)');
xlabel('x'); ylabel('Second Derivative');
legend('Exact Value','Backward Difference
Approximation','Central Difference Approximation','Forward
Difference Approximation')
grid on;
```

Matlab coding for question 2

```
x = [0.20 0.30 0.41];
y = [445.98 471.85 503.46];
h1 = 0.10;
h2 = 0.11;

%Forward Difference Approximation
Forward1 = (y(3) - y(2))/h1;
Forward2 = (y(3) - y(2))/h2;

%Backward Difference Approximation
Backward1 = (y(2) - y(1))/h1;
Backward2 = (y(2) - y(1))/h2;

%Central Difference Approximation
Central1 = (y(3) - y(1))/(2*h1);
Central2 = (y(3) - y(1))/(2*h2);

%display the output
fprintf ('\nFor step size, h = 0.10\n');
fprintf ('\nForward : ds/dt = %.4f\n', Forward1);
fprintf ('Backward : ds/dt = %.4f\n', Backward1);
fprintf ('Central : ds/dt = %.4f\n', Central1);
fprintf ('\nFor step size, h = 0.11\n');
fprintf ('\nForward : ds/dt = %.4f\n', Forward2);
fprintf ('Backward : ds/dt = %.4f\n', Backward2);
fprintf ('Central : ds/dt = %.4f\n\t', Central2);

%plot the graph
figure
x1 = [h1 h2];
y1 = [Forward1 Forward2];
y2 = [Central1 Central2];
y3 = [Backward1 Backward2];
plot(x1,y1,'-.r*',x1,y2,'--bo',x1,y3,'-ms');
title ('dh/dt versus h');
xlabel ('h'); ylabel ('dh/dt');
legend('Forward Difference Approximation','Central Difference
Approximation','Backward Difference Approximation')
grid on;
```

Matlab coding for question 3

Exact Solution

```
syms t
s = 1.0 - 2.65*((sin(120*pi*t))/(pi/2))^2 +
2.80*((sin(120*pi*t))/(pi/2))^5 +
3.25*((sin(120*pi*t))/(pi/2))^6 -
6.85*((sin(120*pi*t))/(pi/2))^7 +
2.60*((sin(120*pi*t))/(pi/2))^8;

s1 = diff(s)
s2 = diff(s1);
s3 = diff(s2);

t = 1/240;

s1new = vpa(subs (s1));
s2new = vpa(subs (s2));
s3new = vpa(subs (s3));

fprintf ('\nThe velocity is %.4f\n',s1new);
fprintf ('\nThe acceleration is %.4f\n',s2new);
fprintf ('\nThe jerk is %.4f\n',s3new);
```

Numerical Methods

```
t = 1/240;
s = @(t) 1.0 - 2.65*((sin(120*pi*t))/(pi/2))^2 +
2.80*((sin(120*pi*t))/(pi/2))^5 +
3.25*((sin(120*pi*t))/(pi/2))^6 -
6.85*((sin(120*pi*t))/(pi/2))^7 +
2.60*((sin(120*pi*t))/(pi/2))^8;

h = 1/20000;          %let h = 1/1000

%Low Accuracy
%forward difference
sfl1 = (s(t+h)-s(t))/h;
sfl2 = (s(t+2*h)-(2*s(t+h))+s(t))/(h*h);
sfl3 = (s(t+3*h)-(3*s(t+2*h))+(3*s(t+h))-s(t))/(h*h*h);

%central difference
scl1 = (s(t+h)-s(t-h))/(2*h);
scl2 = (s(t+h)-(2*s(t))+s(t-h))/(h*h);
scl3 = (s(t+2*h)-s(t+h)+(2*s(t-h))-s(t-2*h))/(2*h*h*h);

%backward difference
sbl1 = (s(t)-s(t-h))/h;
sbl2 = (s(t)-(2*s(t-h))+s(t-2*h))/(h*h);
sbl3 = (s(t)-(3*s(t-h))+(3*s(t-2*h))-s(t-3*h))/(h*h*h);
```

```
%High accuracy
%forward difference
sfh1 = (-s(t+2*h)+(4*s(t+h))-(3*s(t)))/(2*h);
sfh2 = (-s(t+3*h)+(4*s(t+2*h))-(5*s(t+h))+(2*s(t)))/(h*h);
sfh3 = ((-3*s(t+4*h))+(14*s(t+3*h))-(24*s(t+2*h))+(18*s(t+h))-
(5*s(t)))/(2*h*h*h);

%central difference
sch1 = (-s(t+2*h)+(8*s(t+h))-(8*s(t-h))+s(t-2*h))/(12*h);
sch2 = (-s(t+2*h)+(16*s(t+h))-(30*s(t))+(16*s(t-h))-s(t-
2*h))/(12*h*h);
sch3 = (-s(t+3*h)+(8*s(t+2*h))-(13*s(t+h))+(13*s(t-h))-(8*s(t-
2*h))+s(t-3*h))/(8*h*h*h);

%backward difference
sbh1 = ((3*s(t))-(4*s(t-h))+s(t-2*h))/(2*h);
sbh2 = ((2*s(t))-(5*s(t-h))+(4*s(t-2*h))-s(t-3*h))/(h*h);
sbh3 = ((5*s(t))-(18*s(t-h))+(24*s(t-2*h))-(14*s(t-
3*h))+(3*s(t-4*h)))/(2*h*h*h);

fprintf ('Low Accuracy\n');
fprintf ('\n\t\t\t\t\tForward Difference\t\t\tCentral
Difference\t\t\tBackward Difference\n')
fprintf
('Velocity\t\t\t%.4f\t\t\t\t\t\t%.4f\t\t\t\t\t\t%.4f\n',sfl1,s
cl1,sbl1);
fprintf
('Acceleration\t\t%.4f\t\t\t\t\t%.4f\t\t\t\t\t%.4f\n',sfl2,scl
2,sbl2);
fprintf
('Jerk\t\t\t\t%.4f\t\t\t\t%.4f\t\t\t%.4f\n',sfl3,scl3,sbl3);
fprintf ('\nHigh Accuracy\n');
fprintf ('\n\t\t\t\t\tForward Difference\t\t\tCentral
Difference\t\t\tBackward Difference\n')
fprintf
('Velocity\t\t\t%.4f\t\t\t\t\t\t%.4f\t\t\t\t\t\t%.4f\n',sfh1,s
ch1,sbh1);
fprintf
('Acceleration\t\t%.4f\t\t\t\t\t%.4f\t\t\t\t\t%.4f\n',sfh2,sch
2,sbh2);
fprintf
('Jerk\t\t\t\t%.4f\t\t\t\t\t%.4f\t\t\t\t\t\t%.4f\n',sfh3,sch3,
sbh3);
```

Matlab coding for question 4

```
t = [0.04 0.08 0.12 0.16 0.20 0.24 0.28 0.32 0.36 0.40 0.44
0.48 0.52 0.56 0.60];
y = [0.156 0.235 0.248 0.0125 0.072 0.085 0.524 0.236 0.456
0.123 0.245 0.245 0.035 0.048 0.561];

h = 0.04;

%Low Accuracy
%For t = 0.04, use forward difference approximation
velocity1 = (y(1,2)-y(1,1))/h;
acceleration1 = (y(1,3)-(2*y(1,2))+y(1,1))/(h*h);
jerk1 = (y(1,4)-(3*y(1,3))+(3*y(1,2))-y(1,1))/(h*h*h);

velocity11 = (-y(3)+(4*y(2))-(3*y(1)))/(2*h);
acceleration11 = (-y(4)+(4*y(3))-(5*y(2))+(2*y(1)))/(h*h);
jerk11 = ((-3*y(5))+(14*y(4))-(24*y(3))+(18*y(2))-
(5*y(1)))/(2*h*h*h);

%For t = 0.16, use central difference approximation
velocity2 = (y(1,5)-y(1,3))/(2*h);
acceleration2 = (y(1,5)-(2*y(1,4))+y(1,3))/(h*h);
jerk2 = (y(1,6)-y(1,5)+(2*y(1,3))-y(1,2))/(2*h*h*h);

velocity21 = (-y(6)+(8*y(5))-(8*y(3))+y(2))/(12*h);
acceleration21 = (-y(6)+(16*y(5))-(30*y(4))+(16*y(3))-
y(2))/(12*h*h);
jerk21 = (-y(7)+(8*y(6))-(13*y(5))+(13*y(3))-
(8*y(2))+y(1))/(8*h*h*h);

%For t = 0.24, use central difference approximation
velocity3 = (y(1,7)-y(1,5))/(2*h);
acceleration3 = (y(1,7)-(2*y(1,6))+y(1,5))/(h*h);
jerk3 = (y(1,8)-y(1,7)+(2*y(1,5))-y(1,4))/(2*h*h*h);

velocity31 = (-y(8)+(8*y(7))-(8*y(5))+y(4))/(12*h);
acceleration31  = (-y(8)+(16*y(7))-(30*y(6))+(16*y(5))-
y(4))/(12*h*h);
jerk31 = (-y(9)+(8*y(8))-(13*y(7))+(13*y(5))-
(8*y(4))+y(3))/(8*h*h*h);

%For t = 0.32, use central difference approximation
velocity4 = (y(1,9)-y(1,7))/(2*h);
acceleration4 = (y(1,9)-(2*y(1,8))+y(1,7))/(h*h);
jerk4 = (y(1,10)-y(1,9)+(2*y(1,7))-y(1,6))/(2*h*h*h);

velocity41 = (-y(10)+(8*y(9))-(8*y(7))+y(6))/(12*h);
acceleration41 = (-y(10)+(16*y(9))-(30*y(8))+(16*y(7))-
y(6))/(12*h*h);
jerk41 = (-y(11)+(8*y(10))-(13*y(9))+(13*y(7))-
(8*y(6))+y(5))/(8*h*h*h);
```

```
%For t = 0.44, use central difference approximation
velocity5 = (y(1,12)-y(1,10))/(2*h);
acceleration5 = (y(1,12)-(2*y(1,11))+y(1,10))/(h*h);
jerk5 = (y(1,13)-y(1,12)+(2*y(1,10))-y(1,9))/(2*h*h*h);

velocity51 = (-y(13)+(8*y(12))-(8*y(10))+y(9))/(12*h);
acceleration51 = (-y(13)+(16*y(12))-(30*y(11))+(16*y(10))-
y(9))/(12*h*h);
jerk51 = (-y(14)+(8*y(13))-(13*y(12))+(13*y(10))-
(8*y(9))+y(8))/(8*h*h*h);

%For t = 0.52, use central difference approximation
velocity6 = (y(1,14)-y(1,12))/(2*h);
acceleration6 = (y(1,14)-(2*y(1,13))+y(1,12))/(h*h);
jerk6 = (y(1,15)-y(1,14)+(2*y(1,12))-y(1,11))/(2*h*h*h);

%For t = 0.52, use backward difference approximation
velocity61 = ((3*y(13))-(4*y(12))+y(11))/(2*h);
acceleration61 = ((2*y(13))-(5*y(12))+(4*y(11))-y(10))/(h*h);
jerk61 = ((5*y(13))-(18*y(12))+(24*y(11))-
(14*y(10))+(3*y(9)))/(2*h*h*h);

%For t = 0.60, use backward difference approximation
velocity7 = (y(1,15)-y(1,14))/h;
acceleration7 = (y(1,15)-(2*y(1,14))+y(1,13))/(h*h);
jerk7 = (y(1,15)-(3*y(1,14))+(3*y(1,13))-y(1,12))/(h*h*h);

velocity71 = ((3*y(15))-(4*y(14))+y(13))/(2*h);
acceleration71 = ((2*y(15))-(5*y(14))+(4*y(13))-y(12))/(h*h);
jerk71 = ((5*y(15))-(18*y(14))+(24*y(13))-
(14*y(12))+(3*y(1)))/(2*h*h*h);

fprintf('Low Accuracy\n');
fprintf('\n\t\tTime\t\t\tVelocity\t\tAcceleration\t\t\tJerk\t\
t\n');
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,1), velocity1, acceleration1, jerk1);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,4), velocity2, acceleration2, jerk2);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,6), velocity3, acceleration3, jerk3);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,8), velocity4, acceleration4, jerk4);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,11), velocity5, acceleration5, jerk5);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,13), velocity6, acceleration6, jerk6);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,15), velocity7, acceleration7, jerk7);

fprintf('\nHigh Accuracy\n');
```

```
fprintf('\n\t\tTime\t\t\tVelocity\t\tAcceleration\t\t\tJerk\t\
t\n');
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,1), velocity11, acceleration11, jerk11);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,4), velocity21, acceleration21, jerk21);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,6), velocity31, acceleration31, jerk31);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,8), velocity41, acceleration41, jerk41);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,11), velocity51, acceleration51, jerk51);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,13), velocity61, acceleration61, jerk61);
fprintf('\n\t\t%.4f\t\t\t%.4f\t\t\t%.4f\t\t\t\t%.4f\t\t\n',
t(1,15), velocity71, acceleration71, jerk71);

%Fit polynomial forth order

%Equation 1 = a11 + a12 + a13 + a14 + a15 = b11
%Equation 2 = a21 + a22 + a23 + a24 + a25 = b21
%Equation 3 = a31 + a32 + a33 + a34 + a35 = b31
%Equation 4 = a41 + a42 + a43 + a44 + a45 = b41
%Equation 5 = a51 + a52 + a53 + a54 + a55 = b51

a11 = sum(power(t,0)); a12 = sum(power(t,1)); a13 =
sum(power(t,2)); a14 = sum(power(t,3)); a15 = sum(power(t,4));
a21 = sum(power(t,1)); a22 = sum(power(t,2)); a23 =
sum(power(t,3)); a24 = sum(power(t,4)); a25 = sum(power(t,5));
a31 = sum(power(t,2)); a32 = sum(power(t,3)); a33 =
sum(power(t,4)); a34 = sum(power(t,5)); a35 = sum(power(t,6));
a41 = sum(power(t,3)); a42 = sum(power(t,4)); a43 =
sum(power(t,5)); a44 = sum(power(t,6)); a45 = sum(power(t,7));
a51 = sum(power(t,4)); a52 = sum(power(t,5)); a53 =
sum(power(t,6)); a54 = sum(power(t,7)); a55 = sum(power(t,8));
b11 = sum(y); b21 = sum(power(t,1).*y); b31 =
sum(power(t,2).*y); b41 = sum(power(t,3).*y); b51 =
sum(power(t,4).*y);

A = [a11, a12, a13, a14, a15; a21, a22, a23, a24, a25; a31,
a32, a33, a34, a35; a41, a42, a43, a44, a45; a51, a52, a53,
a54, a55];
B = [b11; b21; b31; b41; b51];
X = linsolve(A,B);

fprintf('\ny = %.4f %.4fx + %.4fx^2 %.4fx^3 +
%.4fx^4\n',X(1,1),X(2,1),X(3,1),X(4,1),X(5,1));

%plot the graph
t1 = [0.04 0.16 0.24 0.32 0.44 0.52 0.60];
y2 = [velocity1 velocity2 velocity3 velocity4 velocity5
velocity6 velocity7];
```

```
y3 = [acceleration1 acceleration2 acceleration3 acceleration4
acceleration5 acceleration6 acceleration7];
y4 = [jerk1 jerk2 jerk3 jerk4 jerk5 jerk6 jerk7 ];
y5 = [velocity11 velocity21 velocity31 velocity41 velocity51
velocity61 velocity71];
y6 = [acceleration11 acceleration21 acceleration31
acceleration41 acceleration51 acceleration61 acceleration71];
y7 = [jerk11 jerk21 jerk31 jerk41 jerk51 jerk61 jerk71 ];

figure
plot (t1,y2,'b-',t1,y3,'g-',t1,y4,'m-');
xlabel ('t'); ylabel('Parameter (velocity, acceleration,
jerk)')
title ('Parameter against time (Low Accuracy)');
legend('Velocity','Acceleration','Jerk');
grid on
figure
plot (t1,y5,'r-',t1,y6,'c-',t1,y7,'k-');
xlabel ('t'); ylabel('Parameter (velocity, acceleration,
jerk)')
title ('Parameter against time (High Accuracy)');
legend('Velocity','Acceleration','Jerk');
grid on

p = polyfit(t,y,4);
t8 = linspace (0.04,0.6);
y8 = polyval(p,t8);
figure
plot(t,y,'o')
hold on
plot(t8,y8)
hold off
title('Forth-Degree Polynomial Fit')
```

6.0 REFERENCES

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