SCSI1013: Discrete Structures

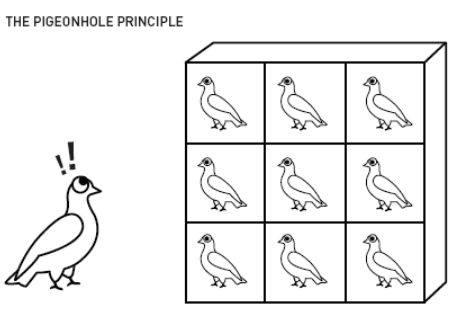
COUNTING METHODS (Part 3)

Pigeonhole Principle

Introduction

- The Pigeonhole Principle is a really simple concept
- Discovered in the 1800s
- Peter Gustav Lejeune Dirichlet was the youngest member of the Prussian Academy of Sciences, he worked at number theory and analysis
- He also came up with a simple little thing that he called The Dirichlet Drawer Principle (or Shoe Box Principle), but that we now call The Pigeonhole Principle.

Pigeonhole Principle



- Imagine 9 pigeonholes and 10 pigeons. A storm comes along, and all of the pigeons take shelter inside the pigeonholes.
- They could be arranged any number of ways. For instance, all 10 pigeons could be inside one hole, and the rest of the holes could be empty.
- What we know for sure, no matter what, is that there is at least one hole that contains more than one pigeon.

The principle works no matter what the particular number of pigeons and pigeonholes. As long as there are **(N - 1) number of pigeonholes, and (N) number of pigeons**, we know there will always be at least two pigeons in one hole.

Pigeonhole Principle (1st Form)

Pigeonhole Principle
(First Form)

If n pigeons fly into k pigeonholes and $k < n$, some pigeonhole contains at least two pigeons

Pigeonhole Principle (1st Form)

- The principle tells nothing about how to locate the pigeonhole that contains 2 or more pigeons
- It only asserts the existence of a pigeonhole containing 2 or more pigeons
- To apply this principle, one must decide
  - which objects are the pigeons
  - Which objects are the pigeonholes

Example (PP - 1st Form)

1. Among 8 people there are at least two persons who have the same birthday.
   - Pigeonholes : Days (7) – Monday to Sunday
   - Pigeons: People (8)

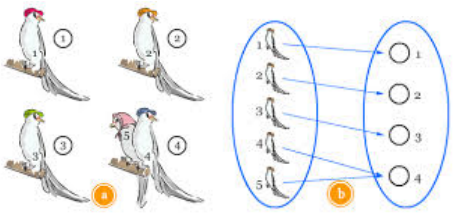
2. Among 13 people there are at least two persons whose month of birth is same.
   - Pigeonholes : Months(12) – January to December
   - Pigeons: People (13)

Example (PP - 1st Form)

3. In a party there are *n* people. Prove that there are at least two persons who know exactly the same number of people.
   - Pigeonholes : How many people a person can know which is at most *n*-1 (need to exclude him/herself)
   - Pigeons: People (*n*)
   - If a person knows *i* people then the person is put in the *i*-th box. There are *n* people. So there must be one box which contains 2 persons.

Pigeonhole Principle (2nd Form)

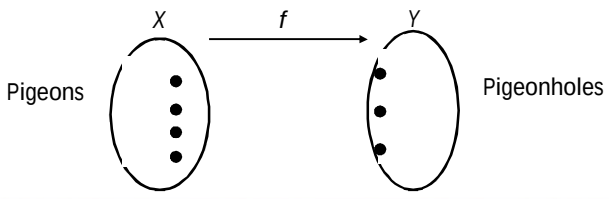
Pigeonhole Principle
(Second Form)



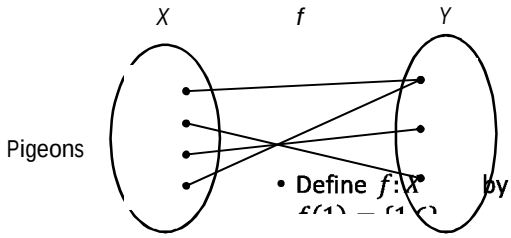
Pigeonhole Principle (2nd Form)

- The 2nd form can be redu

Example 1 (PP – 2nd Form)



Example 1 (PP – 2nd Form) continue



- Define $f: X$ by $f(a) = y_i$ if $a \in y_i$.

Example 2 (PP – 2nd Form)

Example 2 (PP – 2nd Form) continue

- If we take $Y = \{0,1,2,\ldots,9\}$, then we can

Pigeonhole Principle (3rd Form)

Pigeonhole Principle

(Third Form)

The Generalized Pigeonhole Principle

Ceiling function that takes as input a **real number** x and gives as output the least **integer** ceiling (x) = $\lceil x \rceil$ that is greater than or equal to x

Pigeonhole Principle (3rd Form)

- To prove – argue by contr

Pigeonhole Principle (3rd Form)

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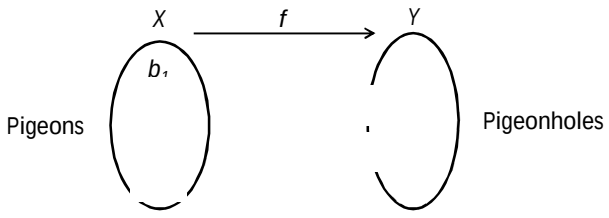
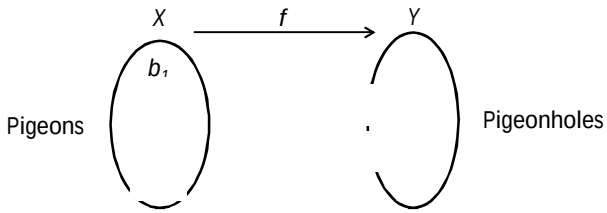
Example 1 (PP – 3rd Form)

Suppose that there are 50 people in

More Examples on Pigeonhole



More Examples on Pigeonhole continue



Solution:

Exercise