SCSI1013: Discrete Structures

CHAPTER 5

FINITE AUTOMATA

SCSI1013: Discrete Structures

PART 1

DETERMINISTIC FINITE AUTOMATA (DFA)

Deterministic Finite Automata (DFA)

- In computer science, we study different types of computer languages, such as Basic, Pascal, and C++.
- We will discuss a type of a language that can be recognized by special types of machines.

Deterministic Finite Automata (DFA)

- A deterministic finite automaton (pl. automata) is a mathematical model of a machine that accepts languages of some alphabet.

Deterministic Finite Automata (DFA)

- Deterministic Finite Automaton is a quintuple M= { S, I, q_0, f_s, F}
  where,
  - S is a finite nonempty set of states
  - I is the input alphabet (a finite nonempty set of symbols)
  - q_0 is the initial state
  - f_s is the state transition function
  - F is the set of final states, subset of S.

example

- Let M={ $\{q_0,q_1,q_2\}$, {0,1}, q_0, f_s, $\{q_2\}$}
  where f_s is defined as follows:
  $f_s(q_0,0)= q_1$, $f_s(q_1,1)= q_2$
  $f_s(q_0,1)= q_0$, $f_s(q_2,0)= q_0$
  $f_s(q_1,0)= q_2$, $f_s(q_2,1)= q_1$

- Note that for M:
  $S=\{q_0,q_1,q_2\}$, I={0,1}, $F=\{q_2\}$
  q_0 is the initial state

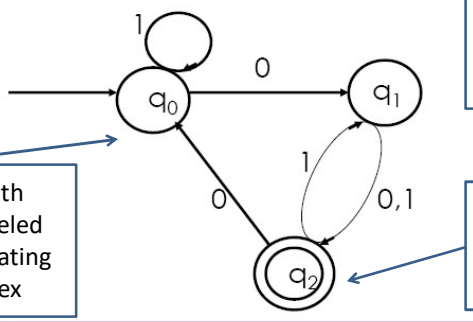
example

- The state transition function of a DFA is often described by means of a table, called a **transition table**.

f_s	0	1
q_0	q_1	q_0
q_1	q_2	q_2
q_2	q_0	q_1

example

- The transition diagram of this DFA is,



Each state represented by a small circle labeled with the state

Initial state with incoming unlabeled arrow not originating from any vertex

Final state with a double circle

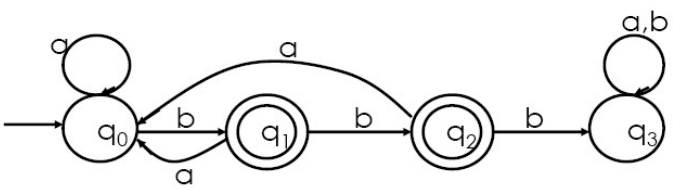
example

Let $M=(\{q_0,q_1,q_2,q_3\}, \{a,b\}, q_0, f_s, \{q_1,q_2\})$ where f_s is given by the table

f_s	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

example

- The transition diagram of this DFA is,



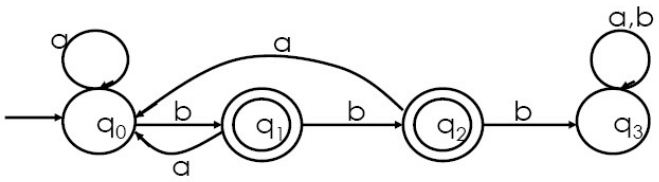
Deterministic Finite Automata (DFA)

- Let M= { S, I, q_0, f_s, F} be a DFA and *w* is an input string,
- *w* is said to be accepted by M if

$$f_s^*(q_0, w) \in F$$

- f_s^* - extended transition function for M

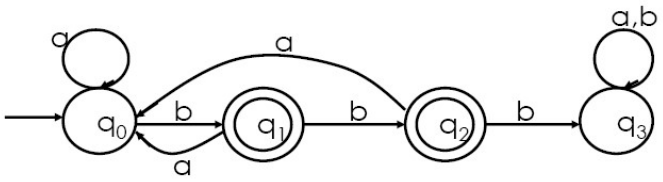
example



w= abb

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2$ accepted by M

example

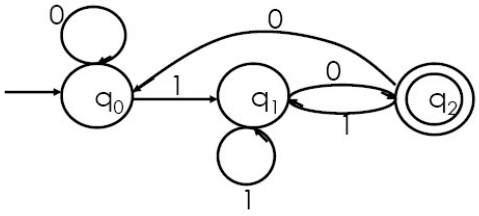


w= abba

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0$

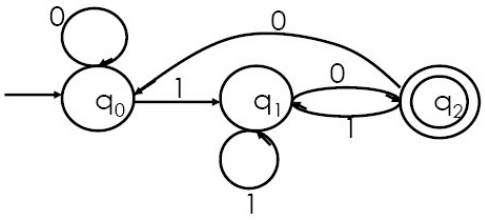
not accepted by M

example



- What are the states of M? q_0, q_1, q_2
- Write the set of input symbols. $I = \{0,1\}$
- Which is the initial state? q_0

example



- Write the set of final states. $F = \{q_2\}$
- Write the transition table for this DFA

example

The transition table, fs

f_s	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_1

example

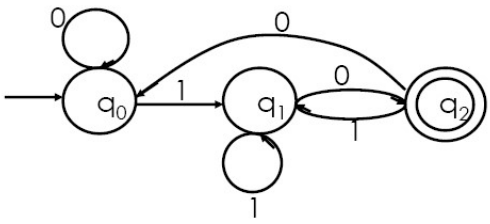
Which of the strings are accepted by M?

0111010, 00111, 111010,

0100, 1110

example

0111010

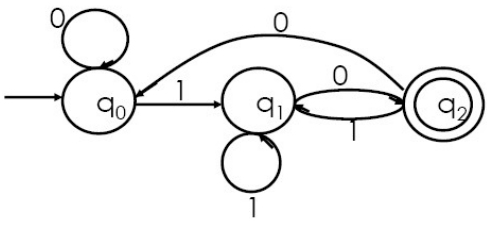


$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$

accepted by M

example

00111

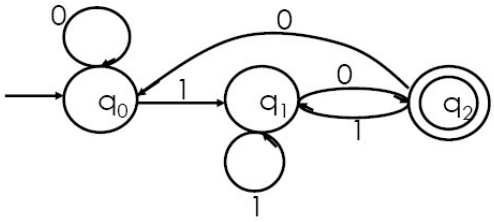


$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1$

not accepted by M

example

111010

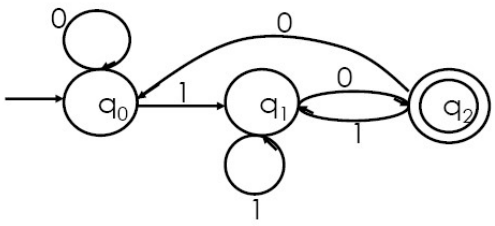


$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$

accepted by M

example

0100

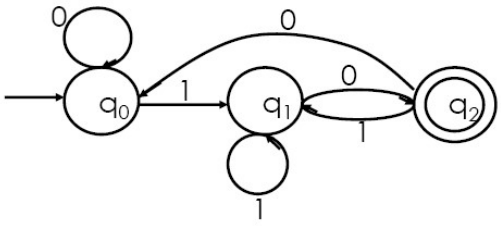


$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} q_0$

not accepted by M

example

1110



$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2$

accepted by M

example

Construct a state transition diagram of a DFA that accepts on {a,b} that contain an even number of a's and an odd number of b's.

Example of accepted strings:
aab, baa, baaabba

example

4 states,

q_0 even num. of a's & even num. of b's.
q_1 even num. of a's & odd num. of b's.
q_2 odd num. of a's & odd num. of b's.
q_3 odd num. of a's & even num. of b's.

$S = \{q_0, q_1, q_2, q_3\}$

example

set of states, $S = \{q_0, q_1, q_2, q_3\}$

set of input symbols, $I = \{a, b\}$

initial state, q_0

final state, q_1

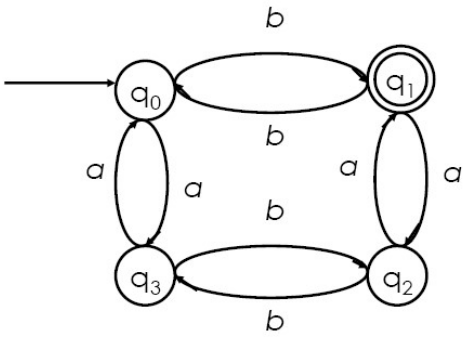
example

State transition function

f_s	a	b
q_0	q_3	q_1
q_1	q_2	q_0
q_2	q_1	q_3
q_3	q_0	q_2

example

State transition diagram



exercise

Let M=(S, I, q_0, f_s, F) be the DFA such that S={q_0,q_1,q_2}, I={a,b}, F={q_2}, q_0 =initial state, and f_s is given by,

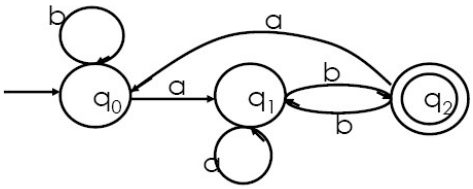
f_s	a	b
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

Draw the state diagram of M.

Which of the strings
abaa, bbbabb, bbbaa dan bababa
are accepted by M?

exercise

The transition diagram of M is,



Construct the transition table of M.
Which of the strings
baba, baab, abab dan abaab
are accepted by M?

exercise

- Construct a state transition diagram of a DFA M with the input set {0,1} such that M accepts only the string 101.

Exercise
Past Year 2015/2016

(a) Let $B = \{S, I, q_o, f_s, F\}$ be the Deterministic Finite Automata (DFA) machine as depicted in Figure 7.

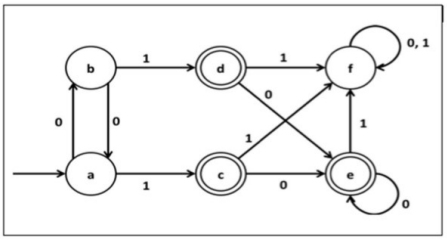


Figure 7

i. List all the components of S, I, q_o, F. (4 marks)

ii. Construct a transition table for the above machine. (4 marks)

iii. Is the string 111010 accepted by the machine? (3 marks)

iv. Find the sequence of configurations and state if the string 01011101 is accepted by the machine. (3 Marks)

Exercise
Past Year 2015/2016

(b) Let $H = \{S, I, O, q_0, f_s, f_0\}$ be the finite state machine of equipment Y. Table 4 shows the transition table for the machine.

Table 4

State	f_s		f_o	
	0	1	0	1
A	B	C	0	0
B	D	B	1	0
C	E	C	0	0
D	B	F	0	0
E	D	B	0	1
F	E	C	0	1

i. Draw the transition diagram with $q_0 = \{A\}$ for the above machine. (5 marks)

ii. What is the output string if the input string is 01110001. (2 marks)

iii. Is input string 11101010 accepted by the machine? Explain using the sequence of configurations and its output. (4 marks)

PART 2

FINITE STATE MACHINE (FSM)

Finite State Machines (FSM)

- Automata with input as well as output.
- Every state has an input and corresponding to the input the state also has an output.
- These types of automata are commonly called **finite state machines**.

Finite State Machines (FSM)

- A finite state machine is a sextuple,
  $M = \{ S, I, O, q_0, f_s, f_o \}$
  where,
  S is a finite nonempty set of states
  I is the input alphabet
  O is the output alphabet
  q_0 is the initial state
  f_s is the state transition function
  f_0 is the output function.

example

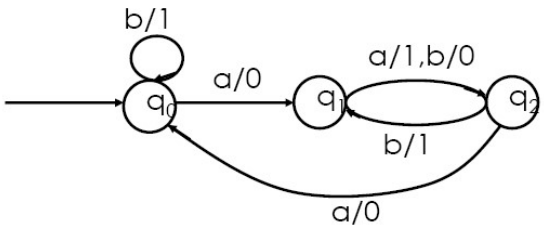
- Let $M = \{ S, I, O, q_0, f_s, f_o \}$ be the FSM
- where,
  $S = \{q_0, q_1, q_2\}$,
  $I = \{a, b\}$,
  $O = \{0, 1\}$,
  q_0= initial state,

example

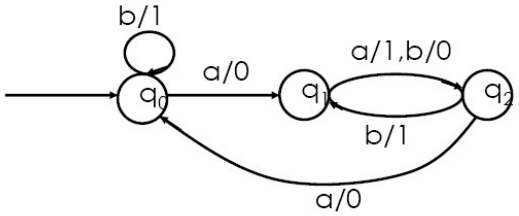
f_s and f_0

	f_s a	b	f_o a	b
q_0	q_1	q_0	0	1
q_1	q_2	q_2	1	0
q_2	q_0	q_1	0	1

example



example

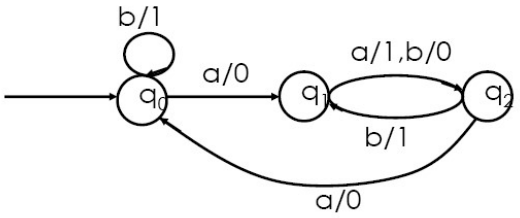


Input string: bbab

$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2$

Output string: 1100 Output: 0

example



Input string: bababaa

$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2 \xrightarrow[0]{a} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[1]{a} q_2$

Output string: 1000101 Output: 1

example

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM
- where,
  S ={q_0,q_1,q_2 ,q_3},
  I ={a,b},
  O ={0,1},
  q_0 = initial state,

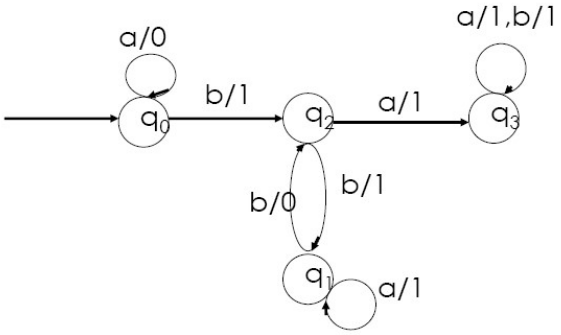
example

- $f_s$and f_0

	f_s a	b	f_o a	b
q_0	q_0	q_2	0	1
q_1	q_1	q_2	1	0
q_2	q_3	q_1	1	1
q_3	q_3	q_3	1	1

example

- Draw the transition diagram of M.



example

- What is the output string if the input string is *abbabab*?

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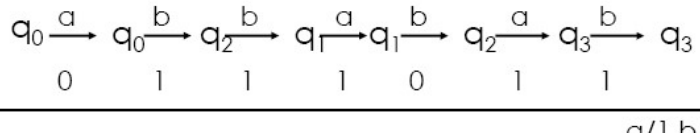
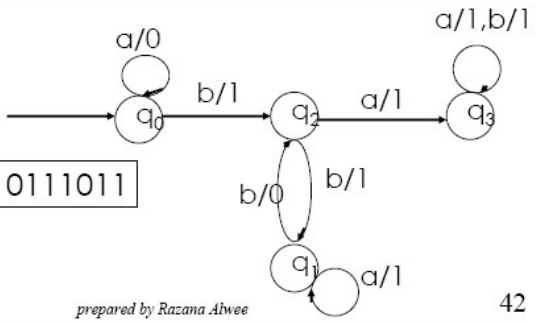
abbabab





Output string: 0111011

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example

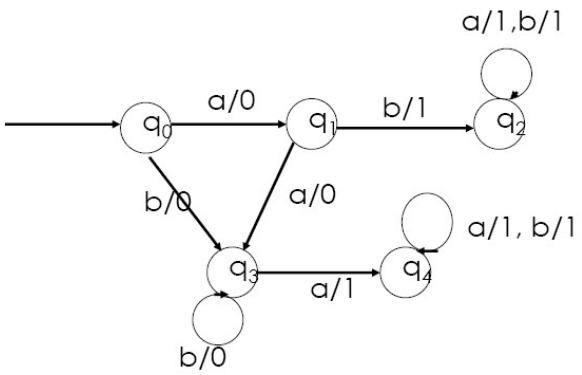
- What is the output of *abbabab*?

  Output: 1

Finite State Machines (FSM)

- Let M be a FSM.
- Let x be a nonempty string in M.
- We say that x is accepted by M if and only if the output of x is 1.

example



example

- Write the transition table of M.
- What is the output string if the input string is *aaabbbb*?
- What is the output if the input string is *bbbaaaa*?

example

- Is the string *aaa* accepted by M?
- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

example

- The transition table of M.

	f_s a	b	f_o a	b
q_0	q_1	q_3	0	0
q_1	q_3	q_2	0	1
q_2	q_2	q_2	1	1
q_3	q_4	q_3	1	0
q_4	q_4	q_4	1	1

example

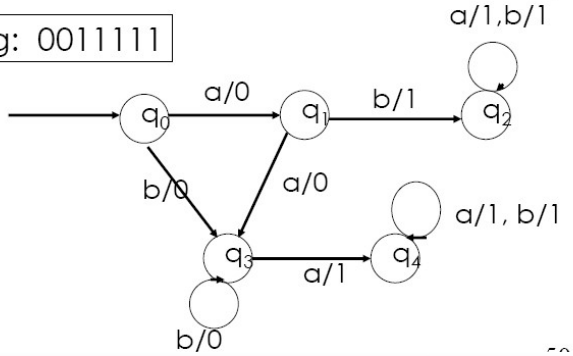
- What is the output string if the input string is *aaabbbb*?

aaabbbb

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4$

0 0 1 1 1 1 1

Output string: 0011111



example

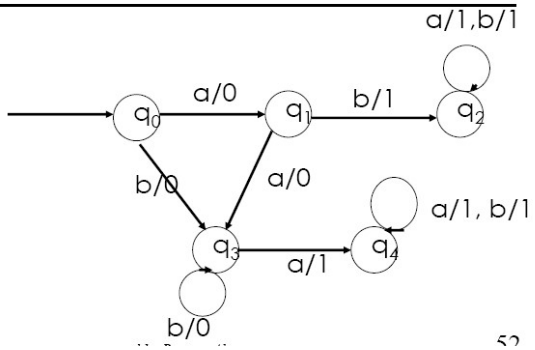
- What is the output if the input string is *bbbaaaa*?

bbbaaaa

$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{a} q_4 \xrightarrow{a} q_4 \xrightarrow{a} q_4$

0 0 0 1 1 1 1

Output: 1



example

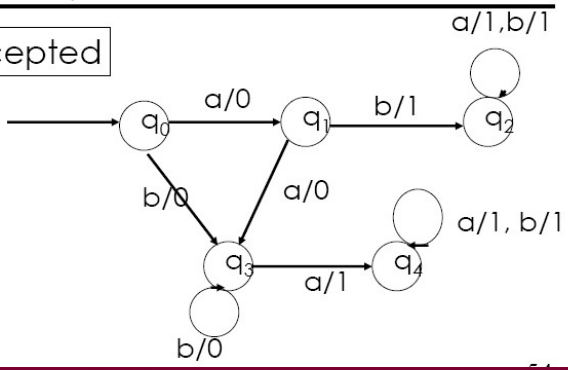
- Is the string *aaa* accepted by M?

aaa

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4$

0 0 1

Output: 1,accepted



example

- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

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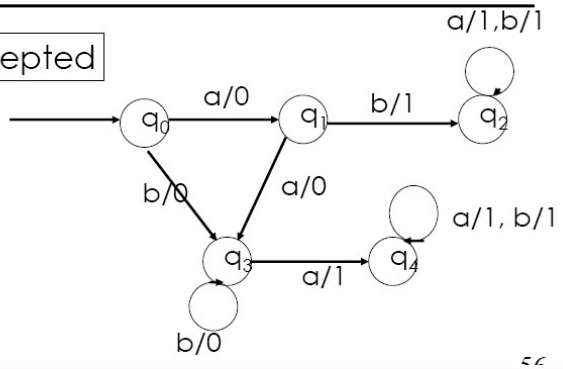
ba

$q_0 \xrightarrow{b} q_3 \xrightarrow{a} q_4$

0 1

Output: 1,accepted



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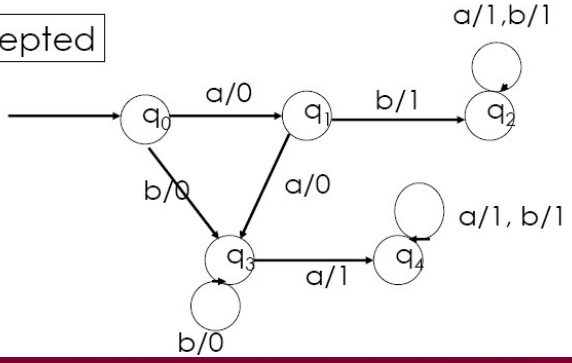
aabbba

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4$

0 0 0 0 0 1

Output: 1,accepted



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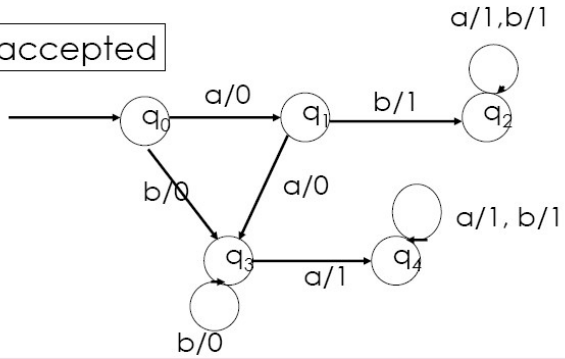
bbbb

$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3$

0 0 0 0

Output: 0,not accepted



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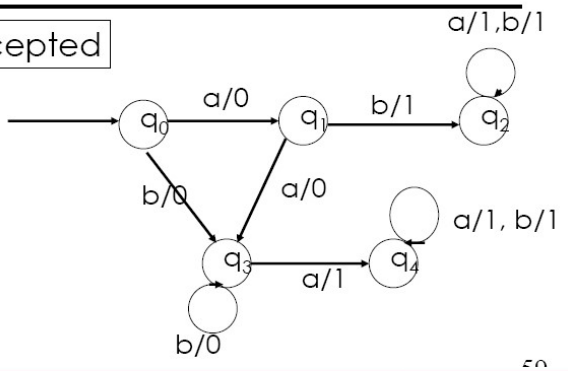
aaabbbb

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4$

0 0 1 1 1 1 1

Output: 1,accepted



example

- Consider a vending machine that sells candy and the cost of a candy is 50 cents.
- The machine accepts any sequence of 10-, 20-, or 50 cent coins.
- After inserting at least 50 cents, the customer can press the button to release the candy.

example

- If the customer inputs more than 50 cents, the machine does not return the change.
- After selling the candy, the machine returns to initial state.
- Construct a finite state machine that models this vending machine.

example

States,

q_0, initial state (0)

q_1, 10 cents

q_2, 20 cents

q_3, 30 cents

q_4, 40 cents

q_5, ≥ 50 cents

example

$S = \{q_0, q_1, q_2, q_3, q_4, q_5\}$,

$I = \{10, 20, 50, B\}$,

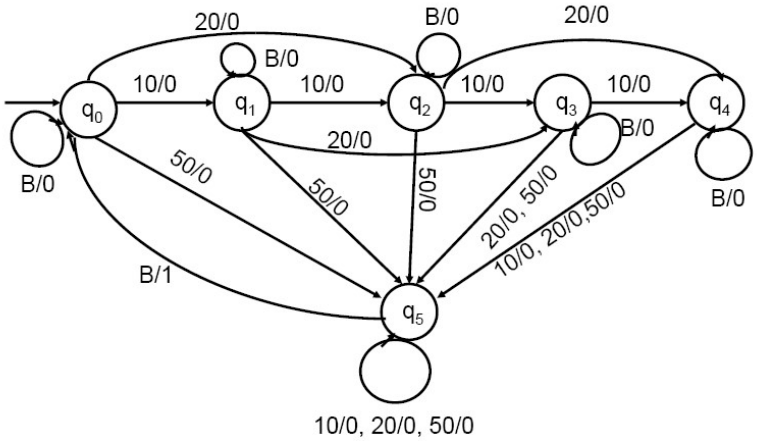
$O = \{0, 1\}$,

q_0 = initial state,

example

	f_s				f_o			
	10	20	50	B	10	20	50	B
q_0	q_1	q_2	q_5	q_0	0	0	0	0
q_1	q_2	q_3	q_5	q_1	0	0	0	0
q_2	q_3	q_4	q_5	q_2	0	0	0	0
q_3	q_4	q_5	q_5	q_3	0	0	0	0
q_4	q_5	q_5	q_5	q_4	0	0	0	0
q_5	q_5	q_5	q_5	q_0	0	0	0	1

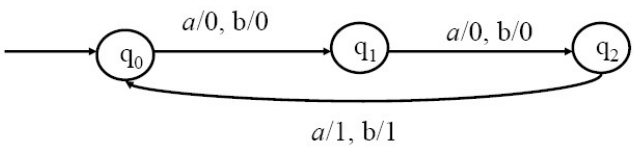
example



example

- Design a FSM, with input alphabet I={a, b}, that outputs a 1 if the number of input symbols read so far is divisible by 3.

example



exercise

Let M= { S, I, O, q_0, f_s, f_o} be a FSM where,

S ={q_0,q_1,q_2},

I ={a,b},

O ={0,1},

q_0 = initial state,

exercise

f_s and f_o

	f_s		f_o	
	a	b	a	b
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

- Draw the transition diagram of M.
- What is the output string if the input string is *aabbb*?
- What is the output string if the input string is *ababab*?
- What is the output if the input string is *abbbaba*?
- What is the output if the input string is *bbbababa*?

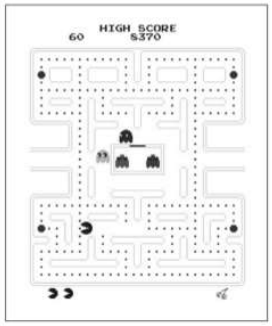
exercise

- Design a FSM that accepts all string over {a,b} that begin with aa.
- For example: aaab, aabba, aababab

Exercise
Past Year 2015/2016

Pac-Man is one of the few games to have been consistently published for over three decades, having been remade on numerous platforms and spawned many sequels. The typical version of Pac-Man is a one player game where he/she manoeuvres the Pac-Man around the maze, attempting to avoid four 'ghosts' characters while eating dots that distributed throughout the maze. Among the dots, there are four super dots that located at four corners of the maze. If the Pac-Man collides with the ghost, he loses one of his three lives and play resumes with the ghosts reassigned to their initial starting location. When Pac-Man eats a super dot, he is able to chase the ghosts for a few seconds of time before the super dot expires. The game ends when Pac-Man has lost all his three lives. Figure 8 shows a screenshot of the Pac-Man game.

Exercise
Past Year 2015/2016



Label:

Super dot

Ghost

Pac-Man

Noted that the **ghosts** in Pac-Man have four behaviours:

S_1 : randomly wander the maze
S_2 : chase Pac-Man when he is within the line of sight
S_3 : avoid Pac-Man when has consumed a super dot
S_4 : return to the initial position to restart the game

The inputs are:

A : spot Pac-Man (Pac-Man is within the line of sight)
B : lose Pac-Man (Pac-Man is not within the line of sight)
C : Pac-Man eats super dot
D : super dot expires
E : collides with Pac-Man
F : reach the initial position

The outputs are:

0 : nothing happened
1 : Pac-Man loses his life
2 : number of ghosts reduces by 1

Exercise
Past Year 2015/2016

Complete the transition table below. (Note: Copy Table 5 in your answer booklet and complete the unshaded cells only). (10 Marks)

Table 5

State	Input, f_z						Output, f_o					
	A	*B*	*C*	*D*	*E*	*F*	*A*	*B*	*C*	*D*	*E*	*F*
S_1												
S_2												
S_3												
S_4												

Exercise
Past Year 2016/2017

ATM is a computerized machine that provides bank customers to gain access to their accounts using magnetic encoded plastic card and code number. It enables the customer to perform online transactions without involving cashier, clerk and bank teller. The customer make cash withdrawal, check account balances, transfer money as well as purchase prepaid mobile phone credit by using ATM card.

Typical PIN based ATM has following processes:

• Insert ATM card to establish interface. The card will be validated to ensure the correct ATM card is inserted. If the card is unsuccessfully validated, the card is ejected and ATM session is terminated.

• For valid ATM card, user is asked to enter the PIN and press the execution key for the system to match the PIN, if PIN does not matches, then user access is denied to the next stage and he or she is requested to repeat the operation with the correct PIN for a fixed two retries.

Exercise
Past Year 2016/2017

• If after the third time the PIN is unmatched, the card will be locked and ATM session is terminated.

• If the PIN matches then the transaction interface is displayed and user has to select the transaction that they wish to perform.

• After the machine performs the chosen transaction, it will prompt user if a new transaction is to be performed. If the response is 'Yes', the transaction interface is again displayed and if 'No' the transaction is terminated and card ejected and ATM session is terminated.

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Exercise
Past Year 2016/2017

Based on the above description, the states, inputs and outputs are as follow:

States:

S_1: Welcome screen
S_2: Validating card
S_3: 1st PIN entering screen
S_4: 2nd PIN entering screen
S_5: 3rd PIN entering screen
S_6: Choosing transaction screen
S_7: Performing transaction
S_8: Asking for other transaction screen
S_9: Terminate ATM session

Inputs:

A: Insert ATM card
B: Valid ATM card
C: Invalid ATM card
D: Correct PIN
E: Incorrect PIN
F: Transaction is selected
G: Complete transaction
H: Yes for other transaction
I: No for other transaction screen

States:

0: Nothing happened
1: Eject ATM card
2: Lock ATM card

Exercise
Past Year 2016/2017

Redraw the transition diagram for the above system in your answer booklet and complete it.
(10 Marks)