Extra

## Exercise 4b.2:

According the the example, draw the logic circuit for the original expression and the last expression simplified.

Original expression:

$$\overline{A}BC + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C} + A\overline{B}C + ABC$$

Simplified expression: $BC + A\overline{B} + \overline{B}\overline{C}$

Extra

## Exercise 4b.3:

Convert each of the following Boolean expressions to SOP form:

(i) $AB + B(CD + EF)$

(ii) $(A + B)(B + C + D)$

(iii) $\overline{\overline{(\overline{A + B})} + C}$

Extra

Solution 4b.3:

(i) $AB + B(CD + EF) = AB + BCD + BEF$

(ii) $(A + B)(B + C + D)$

$= (A + B)B + (A + B)C + (A + B)D$

$= AB + BB + AC + BC + AD + BD$

(iii) $\overline{\overline{(A + B)} + C}$

$= (\overline{\overline{A + B}})\overline{C}$ (DeMorgan's Theorem II)

$= (A + B)\overline{C}$ (Apply rule 9)

$= A\overline{C} + B\overline{C}$

Extra

**Exercise 4b.4:** Define the variables of SOP expression $A\overline{C}+B\overline{C}$ and convert the expression to standard SOP form.

**Solution 4b.4:**

$$A\overline{C}+B\overline{C}=A\overline{C}(B+\overline{B})+B\overline{C}(A+\overline{A}) \quad \text{(Apply rule 6)}$$

$$=AB\overline{C}+A\overline{B}\overline{C}+AB\overline{C}+\overline{A}B\overline{C} \quad \text{(Apply rule 5)}$$

$$=AB\overline{C}+A\overline{B}\overline{C}+\overline{A}B\overline{C}$$

Extra

**Exercise 4b.5:** Convert the following Boolean expressions to standard POS form: $(A+B)(\overline{B}+C)$

**Rule 8: (A . Ā) = 0**

**Rule 12: (A + BC) = (A + B)(A + C)**

**Solution 4b.5:**

$$(A+B)(\overline{B}+C)$$

$$=(A+B+C\cdot\overline{C})(A\cdot\overline{A}+\overline{B}+C)$$

$$=(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+\overline{B}+C)$$

Extra

**Exercise 4b.6:** Represent the following Boolean expression:

(i) $AB\overline{C}+A\overline{B}\overline{C}+\overline{A}B\overline{C}$ as a sigma notation

(ii) $(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+B+C)$

as a PI notation.

## Solution 4b.6(i):

Expression: $AB\overline{C}+A\overline{B}\overline{C}+\overline{A}B\overline{C}$

110 100 010

Sigma notation: $\sum_{ABC}(6,4,2)$

## Solution 4b.6(ii):

Expression: $(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+B+C)$

000 001 010 100

PI notation: $\prod_{ABC}(0,1,2,4)$

Extra

## Exercise 4b.7:

A Boolean expression is written in sigma notation as $X = \sum_{ABC}(7,4,3)$. Determine the logic level (binary value) for each product term and write whole expression.

## Solution 4b.7:

$$\sum_{ABC}(7,4,3)$$

Logic level: 111 100 011

Expression: (SOP) $ABC + A\overline{B}\overline{C} + \overline{A}BC$

Extra

## Exercise 4b.8:

A Boolean expression is written in PI notation as $X=\prod_{ABC}(7,4,3)$ Determine the logic level (binary value) for each sum term and write whole expression.

## Solution 4b.8:

$$\prod_{ABC}(7,4,3)$$

Logic level: 111 100 011

Expression: (POS) $(\overline{A}+\overline{B}+\overline{C})(\overline{A}+B+C)(A+\overline{B}+\overline{C})$

Extra

**Exercise 4b.9:** Convert the following SOP expressions to an equivalent POS expression: $A\overline{C}+B\overline{C}$

**Solution 4b.9:**

- **Step 1**: Need to convert the expression into standard SOP (refer Exercise 4b.7)

$$AB\overline{C} + A\overline{B}\overline{C} + \overline{A}B\overline{C}$$

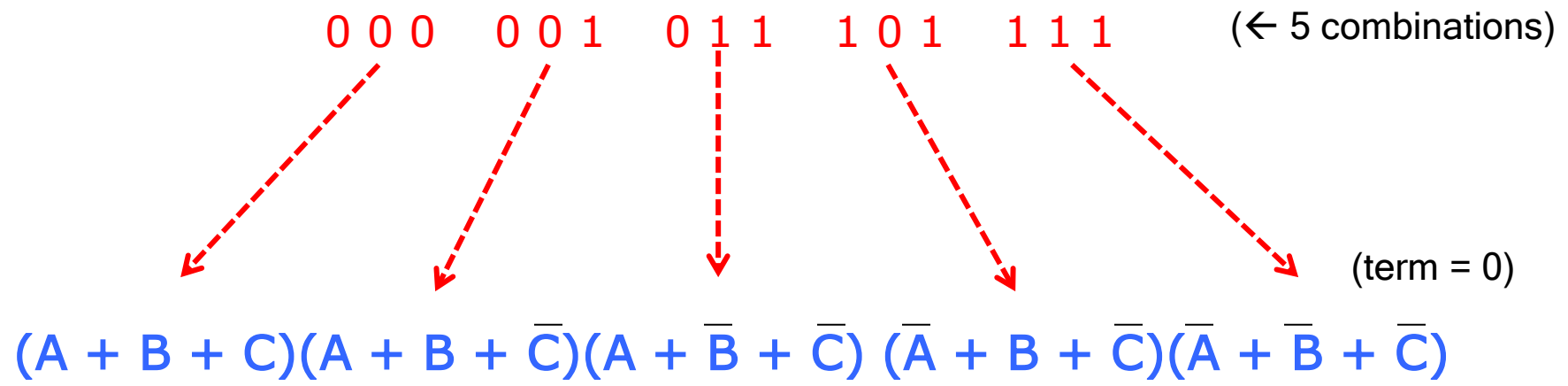
- **Step 2**: Binary number for each SOP term.
  Variables = 3 (A, B, C); $2^3$ = 8 possible combinations.

$$AB\overline{C} + A\overline{B}\overline{C} + \overline{A}B\overline{C}$$

1 1 0  1 0 0  0 1 0 (← 3 combinations)

Extra

- **Step 3**: Equivalent sum term for each binary number.



# Exercise 4b.11:

Extra

From the truth table, determine the:

(i) standard SOP expression; and

(ii) equivalent standard POS expression

| INPUT | | | OUTPUT |
|---|---|---|---|
| A | B | C | X |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

## Solution 4b.11(i):

Convert the binary values to product terms (SOP).

| INPUT | | | OUTPUT | PRODUCT TERMS |
|---|---|---|---|---|
| A | B | C | X | |
| 0 | 0 | 0 | 0 | |
| 0 | 0 | 1 | 0 | |
| 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | 1 | $\bar{A}BC$ |
| 1 | 0 | 0 | 1 | $A\bar{B}\bar{C}$ |
| 1 | 0 | 1 | 0 | |
| 1 | 1 | 0 | 1 | $AB\bar{C}$ |
| 1 | 1 | 1 | 1 | $ABC$ |

Standard SOP expression:

$$\bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

## Solution 4b.11(ii):

Convert the binary values to sum terms (POS).

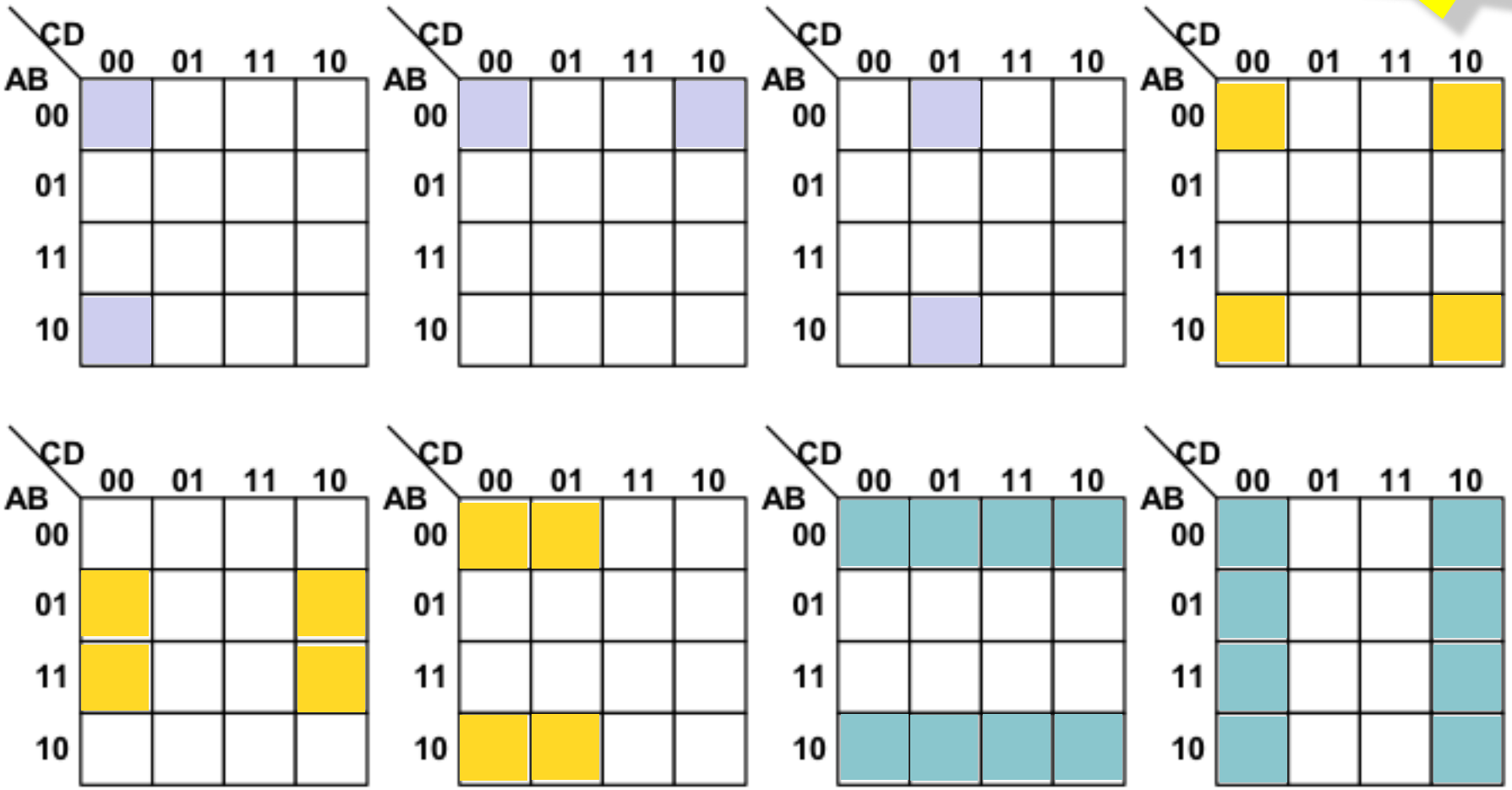
| INPUT | | | OUTPUT | SUM TERMS |
|---|---|---|---|---|
| A | B | C | X | |
| 0 | 0 | 0 | 0 | ---> $(A + B + C)$ |
| 0 | 0 | 1 | 0 | ---> $(A + B + \bar{C})$ |
| 0 | 1 | 0 | 0 | ---> $(A + \bar{B} + C)$ |
| 0 | 1 | 1 | 1 | |
| 1 | 0 | 0 | 1 | |
| 1 | 0 | 1 | 0 | ---> $(\bar{A} + B + \bar{C})$ |
| 1 | 1 | 0 | 1 | |
| 1 | 1 | 1 | 1 | |

Standard POS expression:

$$(A + B + C)\,(A + B + \bar{C})\,(A + \bar{B} + C)\,(\bar{A} + B + \bar{C})$$

- In a K-map with 4-variable or more, the top-most & bottom-most cells of a column (and row) are adjacent.

Extra



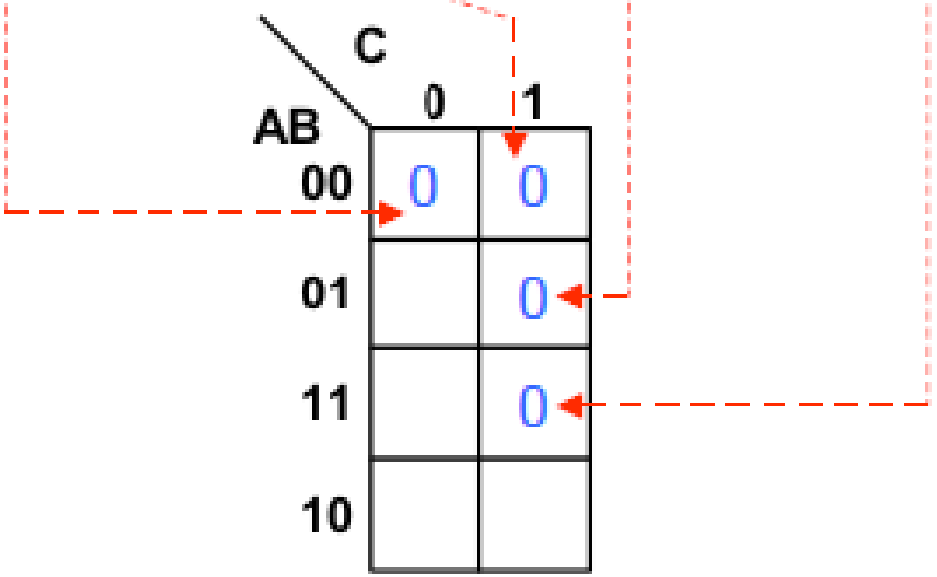
POS

Extra

**Example:** 3 variables

$$(A+B+C)(A+\overline{B}+\overline{C})(A+B+\overline{C})(\overline{A}+\overline{B}+\overline{C})$$

0 0 0 0 1 1 0 0 1 1 1 1



Exercise 4b.12: Map the following expression on a K-map.


$$(A+B+C+D)(A+\overline{B}+\overline{C}+D)(\overline{A}+B+\overline{C}+D)$$

Extra

**Exercise 4b.12:** Map the following SOP expression on a K-Map:

$$\bar{B}\bar{C} + A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD$$

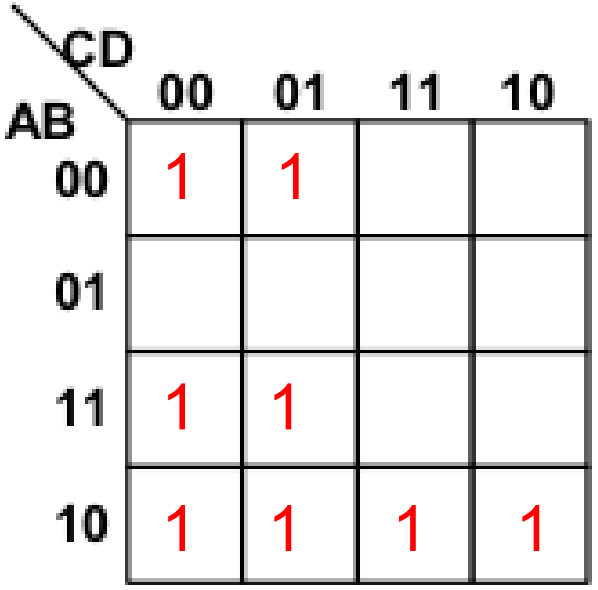
**Solution 4b.12:** Expand the terms by including all combinations of the missing variables numerically as follow:

$\bar{B}\bar{C}$ + $A\bar{B}$ + $AB\bar{C}$ + $A\bar{B}C\bar{D}$ + $\bar{A}\bar{B}\bar{C}D$ + $A\bar{B}CD$

| | | | | | |
|---|---|---|---|---|---|
| 0000 | 1000 | 1100 | 1010 | 0001 | 1011 |
| 0001 | 1001 | 1101 | | | |
| 1000 | 1010 | | | | |
| 1001 | 1011 | | | | |

Extra

Map each binary values by placing 1 in the right cell of the 4-variables K-map




$\bar{B}\bar{C} + A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD$

| | | | | | |
|---|---|---|---|---|---|
| 0000 | 1000 | 1100 | 1010 | 0001 | 1011 |
| 0001 | 1001 | 1101 | | | |
| 1000 | 1010 | | | | |
| 1001 | 1011 | | | | |

Extra

**Exercise 4b.13:** Use a Karnaugh map to minimize the following standard SOP.

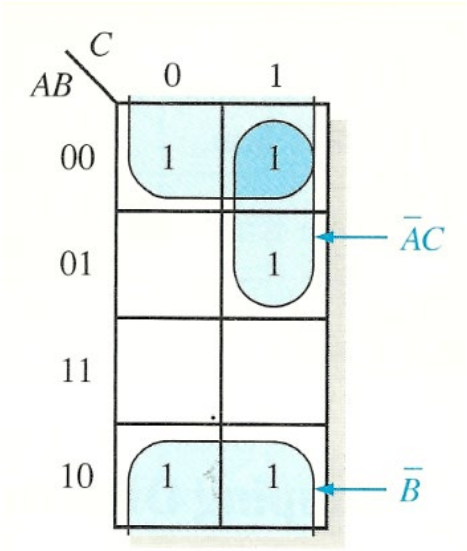
$$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

**Solution 4b.13:**

The binary values of the expression.

1 0 1  0 1 1  0 0 1  0 0 0  1 0 0

Map the expression and group the cells.




The resulting minimum SOP Expression is = $\bar{B} + \bar{A}C$


**Exercise 4b.13b:** Transform the expression $\overline{B} + \overline{A}C$ into the K-Map by expanding numerically to standard form. Then, generate the simplified expression from the K-Map.

Extra

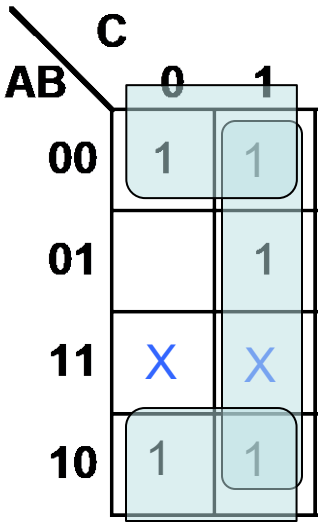
Exercise 4b.14: Use a Karnaugh map to minimize the following expression by expanding numerically to standard form. Then, generate the simplified expression.

$$(A + B + D)(\overline{A} + \overline{B})(\overline{A} + C + \overline{D})$$

Extra

**Exercise 4b.14:** Use a Karnaugh map to minimize a standard SOP expression $A\overline{B}C+\overline{A}BC+\overline{A}\,\overline{B}C+\overline{A}\,\overline{B}\,\overline{C}+A\overline{B}\,\overline{C}$ (Use "*don't care*" terms, $d(6,7)$ into the K-maps to produce better results)

**Solution 4b.14:**




The binary values of the expression.

1 0 1  0 1 1  0 0 1  0 0 0  1 0 0

The binary value of "don't care" notation.

1 1 0  1 1 1

Map the expression and group the cells.

Expression: $\overline{B}+C$

Extra

**Exercise 4b.15:** Use a Karnaugh map to minimize an expression

$$\sum_{ABCD}(0,5,8,9,11)$$

(Use "*don't care*" terms, $d(1,2,10)$ into the K-maps to produce better results)

Extra

**Exercise 4b.16:** Transform the following expression into the K-Map. Then, generate the simplified expression from the K-Map.

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

**Solution 4b.16:** Expand the expression to SOP form.

$$= (\overline{AB})(\overline{AC}) + \overline{A}\,\overline{B}C$$ .....apply DeMorgan Theorem

$$= (\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C$$ .....apply distributive law

$$= \overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$ .....apply Rule 7

$$= \overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

Extra

Make the standard form expression.

$$\overline{A}+\overline{A}\,\overline{C}+\overline{A}\,\overline{B}+\overline{B}\,\overline{C}+\overline{A}\,\overline{B}C$$

.....apply Rule 6

$$=\overline{A}(B+\overline{B})+\overline{A}\,\overline{C}(B+\overline{B})+\overline{A}\,\overline{B}(C+\overline{C})+\overline{B}\,\overline{C}(A+\overline{A})+\overline{A}\,\overline{B}C$$

$$=\overline{A}B+\overline{A}\,\overline{B}+\overline{A}B\overline{C}+\overline{A}\,\overline{B}\,\overline{C}+\overline{A}\,\overline{B}C+\overline{A}\,\overline{B}\,\overline{C}+A\overline{B}\,\overline{C}+\overline{A}\,\overline{B}\,\overline{C}+\overline{A}\,\overline{B}C \quad \text{Rule 5}$$

$$=\overline{A}B(C+\overline{C})+\overline{A}\,\overline{B}(C+\overline{C})+\overline{A}B\overline{C}+\overline{A}B\overline{C}+\overline{A}\,\overline{B}C+A\overline{B}\,\overline{C}$$

$$=\overline{A}BC+\overline{A}B\overline{C}+\overline{A}\,\overline{B}C+\overline{A}\,\overline{B}\,\overline{C}+\overline{A}B\overline{C}+\overline{A}\,\overline{B}\,\overline{C}+\overline{A}\,\overline{B}C+A\overline{B}\,\overline{C} \quad \text{Rule 5}$$

$$=\overline{A}BC+\overline{A}B\overline{C}+\overline{A}\,\overline{B}C+\overline{A}\,\overline{B}\,\overline{C}+A\overline{B}\,\overline{C}$$

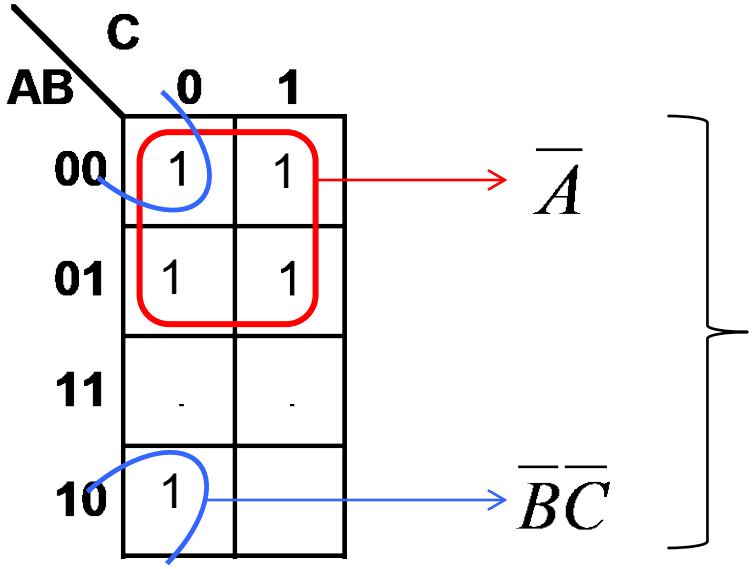
Extra

Get the binary value for each term.

$$\overline{A}BC + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\,\overline{B}\,\overline{C}$$

011 010 001 000 100

Fill in the K-Map and grouping cells.




$$\overline{A} + \overline{B}\overline{C}$$

Extra

**Exercise 4b.17:** Using the same expression in exercise 4b.16, simplify the expression using the Boolean algebra and laws.

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

**Solution 4b.17:** Expand the expression to SOP form.

$$= (\overline{AB})(\overline{AC}) + \overline{A}\,\overline{B}C$$ .....apply DeMorgan Theorem

$$= (\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C$$ .....apply distributive law

$$= \overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$ .....apply Rule 7

$$= \overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

Extra

Simplify the expression.

$$\overline{A}+\overline{A}\,\overline{C}+\overline{A}\,\overline{B}+\overline{B}\,\overline{C}+\overline{A}\,\overline{B}C$$

$$=(\overline{A}+\overline{A}\,\overline{C})+(\overline{A}\,\overline{B}+\overline{A}\,\overline{B}C)+\overline{B}\,\overline{C}$$ .....apply Rule 10

$$=\overline{A}+\overline{A}\,\overline{B}+\overline{B}\,\overline{C}$$

$$=(\overline{A}+\overline{A}\,\overline{B})+\overline{B}\,\overline{C}$$ .....apply Rule 10

$$=\overline{A}+\overline{B}\,\overline{C}$$

Compare your answer with Exercise 4b.16