



**UNIVERSITI TEKNOLOGI MALAYSIA**

# **TEST 1**

**SEMESTER I - 2017/2018**

**SUBJECT CODE : SCS11013**

**SUBJECT : DISCRETE STRUCTURE**

**TIME : 2 HOURS (8.00 P.M - 10.00 P.M)**

**DATE : 23<sup>rd</sup> OCTOBER 2017**

**INSTRUCTIONS TO THE STUDENTS:**

**Please answer all questions in the answer booklet.**

**QUESTION 1****[10 MARKS]**

- a) Sketch Venn diagrams that show the universal set,  $U$ , the sets  $A$  and  $B$ , and a single element  $x$  in each of the following cases:

(i)  $x \in A; x \notin B; B \subset A$  (2 marks)

(ii)  $x \in A; x \in B; A$  is not a subset of  $B; B$  is not a subset of  $A$  (2 marks)

- b) Prove the following identity, stating carefully which of the set laws you are using at each stage of the proof.

$$B \cup (\emptyset \cap A) = B \quad (4 \text{ marks})$$

- c) In a certain programming language, all variable names have to be 3 characters long. The first character must be a letter from 'a' to 'z'; the others can be letters or digits from 0 to 9. If  $L = \{a, b, c, \dots, z\}$ ,  $D = \{0, 1, 2, \dots, 9\}$ , and  $V = \{\text{permissible variable names}\}$ , use a Cartesian product to complete:

$$V = \{pqr \mid (p, q, r) \in \dots\} \quad (2 \text{ marks})$$

**QUESTION 2****[20 MARKS]**

- a) Suppose that the variable  $x$  represents student,  $F(x)$  means " $x$  is freshman," and  $M(x)$  means " $x$  is a math major". Determine the symbolic statement equivalent to the following statement.

i) No math major is a freshman. (1 mark)

ii) Some freshmen are math major. (1 mark)

- b) Let  $P(x, y): 2x + y = 1$ , where the domain of discourse is the set of all integers. What are the truth values of the following:

i)  $\forall x \exists y P(x, y)$  (2 marks)

ii)  $\forall x \forall y P(x, y)$  (2 marks)

c) Evaluate the following logical equivalent using truth table.

$$(\neg p \rightarrow (q \vee r)) \equiv (q \rightarrow (p \vee r)) \quad (5 \text{ marks})$$

e) Prove the following compound proposition using logical law.

$$(P \vee \neg Q) \vee (R \wedge (Q \vee \neg Q)) \equiv R \vee (P \vee \neg Q) \quad (4 \text{ marks})$$

f) Proof directly that,

“If  $m$  is an even integer and  $n$  is an odd integer then  $m+n$  is an odd”  
(5 marks)

### QUESTION 3

[25 MARKS]

a)  $A = \{(1,3), (2,4), (-4,-8), (3,9), (1,5), (3,6)\}$ . For all  $(a,b), (c,d) \in A$ . Determine  $R$ , a relation defined on  $A$  as

$$(a,b)R(c,d) \Leftrightarrow ad = bc \quad (5 \text{ marks})$$

b) Determine whether the following relation is reflexive, symmetric, antisymmetric and/or transitive. (Note: Use diagram to assist your justification and you can define an appropriate set if necessary).

i)  $R$  is relation on the set of people where  $(a,b) \in R$ , if  $a$  is taller than  $b$ .  
(4 marks)

ii)  $R$  is a relation on the set of all web pages where  $(a,b) \in R$ , if everyone who has visited web page  $a$  also visited web page  $b$ .  
(3 marks)

iii)  $R$  is a relation on the set of people in the Facebook friend list, where  $(a,b) \in R$ , if  $a$  is a friend of  $b$ .  
(3 marks)

c) Determine whether the relation presented by the following matrices is equivalence relation.

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

(10 marks)

**QUESTION 4****[14 MARKS]**

a) Which of the following relations are functions from  $A$  to  $B$ . Write their domain and range. If it is not a function, give a reason.

i)  $R = \{(1, -2), (3, 7), (4, -6), (8, 1)\}$ ,  $A = \{1, 3, 4, 8\}$ ,  $B = \{-2, 7, -6, 1, 2\}$

(2 marks)

ii)  $R = \{(1, 0), (1, -1), (2, 3), (4, 1)\}$ ,  $A = \{1, 2, 4\}$ ,  $B = \{0, -1, 3, 1, 0\}$

(2 marks)

i)  $R = \{(1, -1), (2, -2), (3, -3), (4, -4), (5, -5)\}$ ,  $A = \{0, 1, 2, 3, 4, 5\}$ ,  $B = \{-1, -2, -3, -4, -5\}$

(2 marks)

b) Prove that

$f : X \rightarrow Y$  defined by  $f(x) = 3x - 7$  is a one-to-one function.

(4 marks)

c) If  $f(x) = \sqrt{x+1}$  and  $g(x) = x^2 + 2$

i) Calculate  $f \circ g$

(2 marks)

ii) Calculate  $g \circ f$

(2 marks)

**QUESTION 5****[16 MARKS]**

a) Determine  $s_5$  if  $s_0, s_1, s_2, \dots$  is a sequence satisfying the given recurrence relation and initial conditions.

$$s_n = 2s_{n-1} + s_{n-2} - s_{n-3} \quad \text{for } n \geq 3, \quad s_0 = 2, \quad s_1 = -1, \quad s_2 = 4$$

(3 marks)

b) A consumer purchased items costing RM280 with a department store credit card that charges 1.5% interest per month compounded monthly. Write a recurrence relation and initial condition for  $b_n$ , the balance of the consumer's account after  $n$  months if no further charges occur and the minimum monthly payment of RM25 is made.

(3 marks)

c) Write a recursive algorithm to find the  $n$  term of sequence defined by

$$a_1 = 2, \quad a_2 = 3, \quad \text{and} \quad a_n = a_{n-1}a_{n-2} \quad \text{for } n \geq 3. \quad \text{Then, use the algorithm to trace } a_7.$$

(10 marks)

### Theorem for Logic

|                      |                                                                                                                          |
|----------------------|--------------------------------------------------------------------------------------------------------------------------|
| Indempotent Laws     | $p \wedge p \equiv p$<br>$p \vee p \equiv p$                                                                             |
| Double negation laws | $\neg \neg p \equiv p$                                                                                                   |
| Commutative law      | $p \wedge q \equiv q \wedge p$<br>$p \vee q \equiv q \vee p$                                                             |
| Associative laws     | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$<br>$(p \vee q) \vee r \equiv p \vee (q \vee r)$                     |
| Distributive laws    | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$<br>$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ |
| Absorption Laws      | $p \wedge (p \vee q) \equiv p$<br>$p \vee (p \wedge q) \equiv p$                                                         |
| De Morgan's Laws     | $\neg (p \wedge q) \equiv (\neg p) \vee (\neg q)$<br>$\neg (p \vee q) \equiv (\neg p) \wedge (\neg q)$                   |