

CONFIDENTIAL



UNIVERSITI TEKNOLOGI MALAYSIA

TEST 1

SEMESTER I 2016/2017

SUBJECT CODE : SCS11013
SUBJECT NAME : DISCRETE STRUCTURE
YEAR/COURSE : 1SCSJ/SCSR/SCSV/SCSB/SCSD
TIME : 2 HOURS (8.00 PM – 10.00 PM)
DATE : 13 OKTOBER 2016
VENUE : BK1-6, N28

INSTRUCTIONS TO THE STUDENTS:

Answer all questions in the answer booklet.

NAME	
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SECTION	
LECTURER	

(This question paper consist of 5 pages including this pages)

Question 1**[10 marks]**

- a) Let $A = \{a, \{a\}, b, \{a, b\}\}$. State whether the following statement is TRUE or FALSE.
- i. $\{a\} \subseteq A$ (1 mark)
 - ii. $\{a, b\} \in A$ (1 mark)
- b) Given set $B = \{x | x \in Z, x^2 + 6 \leq 10\}$, where Z is the set of integers. List the elements of set B . (1 mark)
- c) A group of 35 students attended English class for one semester. In order to sit for final exam, they need to have at least 80% attendance. 10 students have 100% attendance. 15 students have $\geq 90\%$ attendance. 30 students have $\geq 75\%$ attendance. 5 students have lower than 75% attendance and 8 are not allowed to sit for final exam.
- i. How many students are not allowed to sit for final exam with attendance $< 80\%$ but $\geq 75\%$? (1 mark)
 - ii. Find the number of students with attendance $< 90\%$ but $\geq 80\%$? (1 mark)
- d) The following Venn Diagram in Figure 1 shows set A , B and C . Find $(A \cap B \cap C) \times (B - C)$. (2 marks)

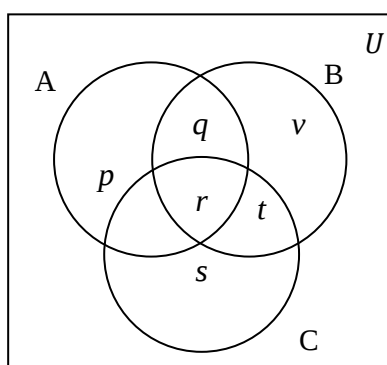


Figure 1

- e) Let A and B be sets. Use properties of sets to simplify $(A \cup B') \cap (B \cap C)'$. (3 marks)

Question 2

[25 marks]

- a) Base on the propositions ***p***, ***q*** and ***r***, rewrite the given statements using logic symbols.

p : Power button is on

q : Computer shuts down

r : Computer displays start screen

- i. Computer displays start screen only with the condition that the power button is on.

(1 mark)

- ii. If the power button is on then the computer does not shuts down or displays start screen.

(2 marks)

- iii. Once the power button is off, it is either the computer shuts down or does not displays start screen.

(2 marks)

- b) State the truth value for the given compound propositions below provided the truth values for ***p*** and ***r*** is **FALSE** and ***q*** is **TRUE**.

- i. $(p \rightarrow \neg q) \wedge \neg(r \vee q)$

(2 marks)

- ii. $(\neg p \wedge \neg q) \rightarrow (p \vee \neg r)$

(2 marks)

- iii. $\neg(\neg p \leftrightarrow \neg q) \wedge r$

(2 marks)

- c) Let $P(x, y) = (x * y)^2 \geq 1$. Given the domain of discourse for x and y is set of integer, ***Z***. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

- i. $\exists x \exists y P(x, y)$

(2 marks)

- ii. $\forall x \forall y P(x, y)$

(2 marks)

- d) Use truth table to check if the compound propositions ***A*** and ***B*** are logically equivalent.

$$A = (\neg p \vee q) \rightarrow r$$

$$B = \neg q \leftrightarrow (\neg p \vee r)$$

(6 marks)

- d) Simplify the following compound propositions using Laws of Logic.

$$(a \vee b) \wedge \neg(\neg a \wedge b)$$

(3 marks)

Question 3**[25 marks]**

- a) Let R be the relation from $X = \{1, 2, 3, 4\}$ to $Y = \{1, 3, 5, 7\}$ defined by xRy if and only if $x \in X, y \in Y$ and $x + 3y \leq 12$.
- List the elements of the set R . (3 marks)
 - Find the domain of R . (1 mark)
 - Find the range of R . (1 mark)
- b) Let $A = \{a, b, c\}$ and $R: A \rightarrow A$. Draw the digraph representing the following properties:
- Symmetric and reflexive but not transitive (5 marks)
 - Reflexive but not anti-symmetric. (3 marks)
 - Write a matrix representing an irreflexive relation R on a set A . (3 Marks)
- c) Let $B = \{d, e, f, g\}$ and R be the relation on B that has the matrix M_R

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

- List in-degrees and out-degrees of all vertices. (4 marks)
- Determine whether the relation R on the set B represented by M_R is an equivalence relation. Explain. (5 marks)

Question 4**[10 marks]**

Here are two functions $f: \{1, 2, 3\} \rightarrow \{10, 11, 12, 13\}$ and $g: \{10, 11, 12, 13\} \rightarrow \{4, 5, 6\}$ whose rules are given in Table 1.

Table 1

x	1	2	3
$f(x)$	11	13	10

x	10	11	12	13
$g(x)$	4	5	4	6

- What is $f(1)$? (1 mark)
- What is $g(11)$? (1 mark)
- Draw arrow diagrams for f and g . (3 marks)
- Which of these compositions can be defined: gof , gog , fog , or fof ? (1 mark)
- For any of the compositions above that are defined, give the domain and codomain, and draw the arrow diagram. (4 marks)

Question 5**[10 marks]**

a) Let, $f: \mathbf{R} \rightarrow \mathbf{R}$ be the function with rule $f(x) = 5x-7$

i. Show that f is one-to-one.

(2 marks)

ii. Find the inverse of f .

(2 marks)

(note: $\mathbf{R}$ is the set of real numbers)

b) Let $X = \{-1, 0, 1\}$ and $Y = \{-2, 0, 2\}$. For each $x \in X$, define functions $f: X \rightarrow Y$ and $g: X \rightarrow Y$ by:

$$f(x) = x^2 - x$$

$$g(x) = 2x$$

Determine if f and g are one-to-one, onto Y , and/or bijection.

(6 marks)

Question 6**[10 marks]**

A robot can take steps of 1, 2 or 3 meters. As examples (Table 2):

Table 2

Distance	Sequence of Steps	Number of Ways to Walk
1	1	1
2	1, 1 or 2	2
3	1, 1, 1 or 1, 2 or 2, 1 or 3	4
4	1, 1, 1, 1 or 1, 1, 2 or 1, 2, 1 or 2, 1, 1 or 2, 2, or 1, 3 or 3, 1	7

Let $walk(n)$ denote the number of ways the robot can walk n meters. We have observed that,

$$walk(1) = 1, \quad walk(2) = 2, \quad walk(3) = 4.$$

Now suppose that $n > 3$. Since the walk must begin with either a 1-meter, 2-meter or a 3-meter step, all of the ways to walk n meters are accounted for. We obtain the formula,

$$walk(n) = walk(n-1) + walk(n-2) + walk(n-3)$$

For example,

$$walk(4) = walk(3) + walk(2) + walk(1) = 4 + 2 + 1 = 7$$

a) Write the recursive algorithm to calculate the number of ways the robot can walk n meters.

(6 marks)

b) Find $walk(6)$.

(4 marks)

*** end of questions ***