UTM UNIVERSITI TEKNOLOGI MALAYSIA

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SKEE 1043

Three Phase Systems

06/07/2009

INSPIRING CREATIVE AND INNOVATIVE MINDS

Three Phase Systems

- A 3-Φ system is formed by 3*1-Φ systems, which are physically connected together for 3-Φ operation.
- To maintain constant power flow, the three single phase circuits must be operated with the same voltage and current magnitudes in each phase (circuit), with a relative phase angle of 120 electrical degrees apart.

Three Phase Systems

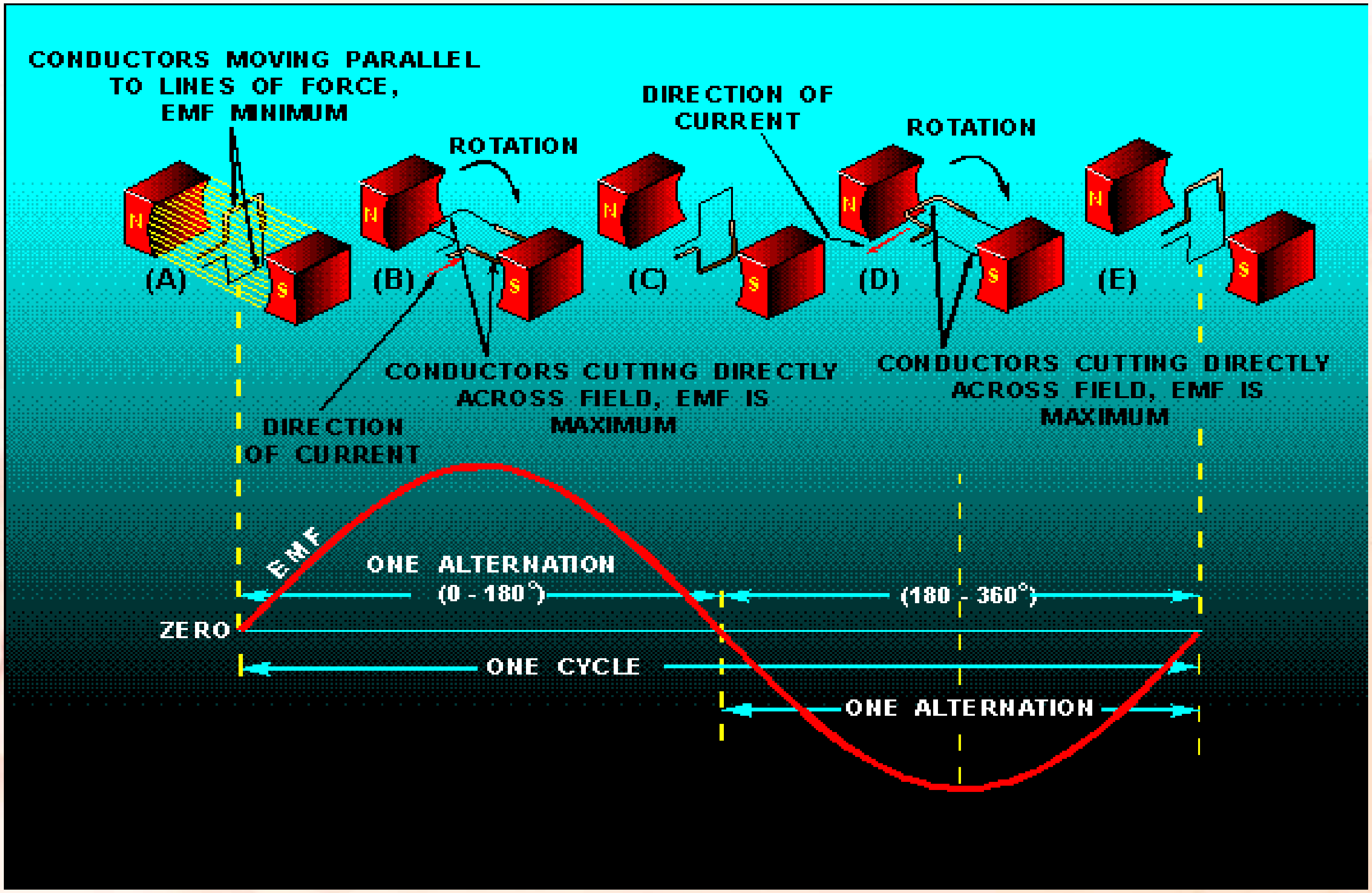
Advantages of 3- Φ compared to 1- Φ system

- High efficiency
- For same power transfer at the transmission line, less conductor and hence lighter in weight
- Construction and maintenance minimum and therefore cheaper
- Starting characteristic and operation of 3-Φ equipment better/more stable than 1- Φ

Basic AC Generation Principle

When a conductor is placed in a magnetic field, if one of them is moved, an electromotive force (emf) will be induced in the conductor. This effect is called electromagnetic induction.

Three Phase Systems



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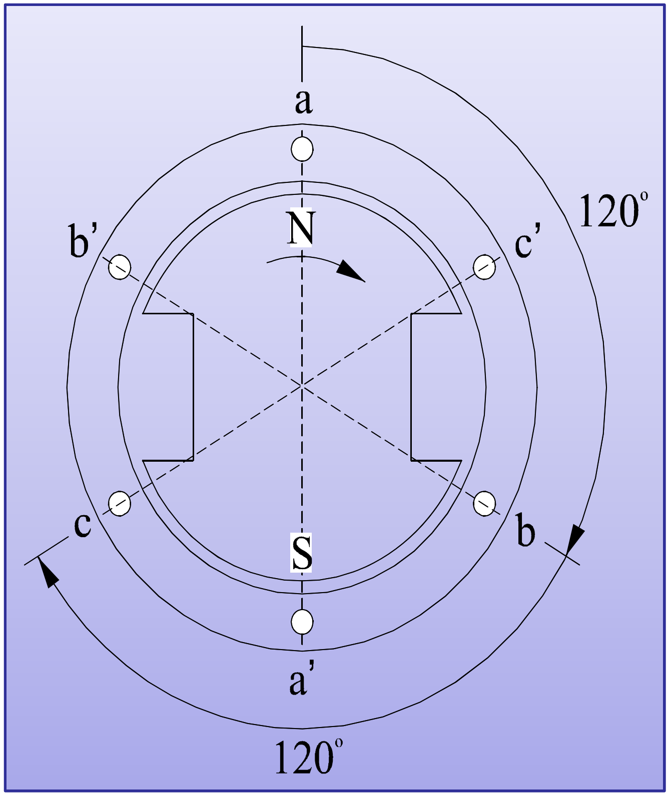
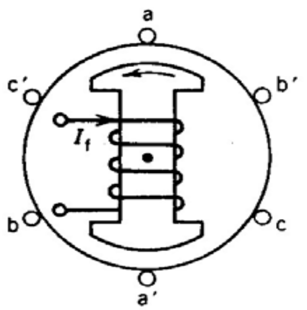
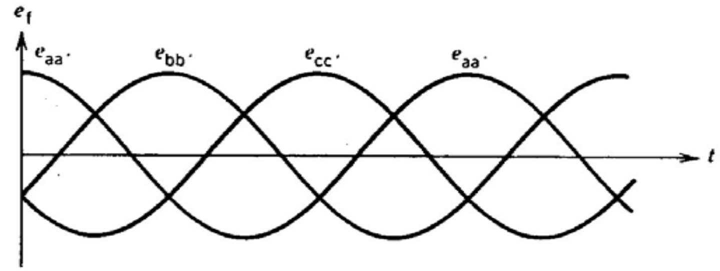
A simple three-phase two-pole generator

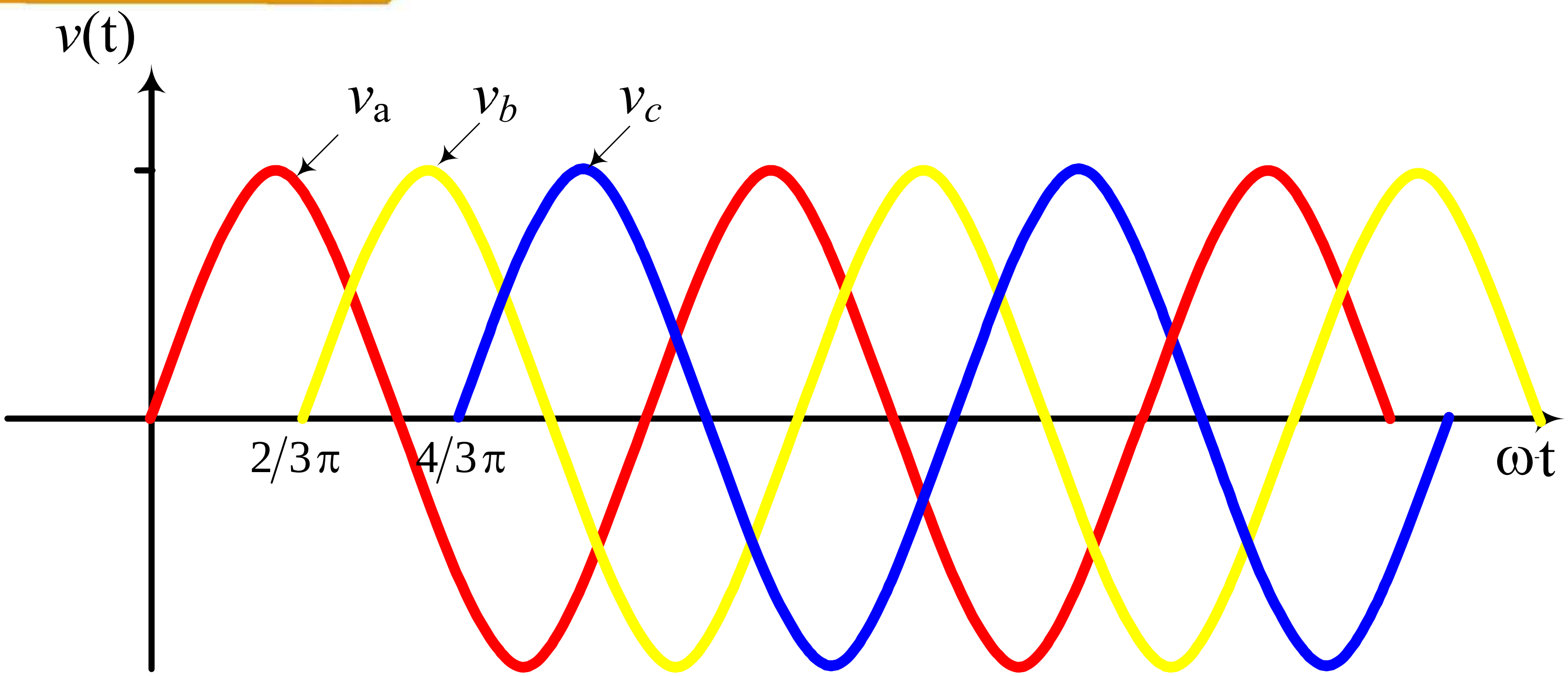


Three separate windings or coils with terminals a-a', b-b' and c-c' are physically placed 120° apart around the stator.





Emf Generated



$v_a = V_a \sin \omega t$

$v_b = V_b \sin (\omega t - 120^o)$

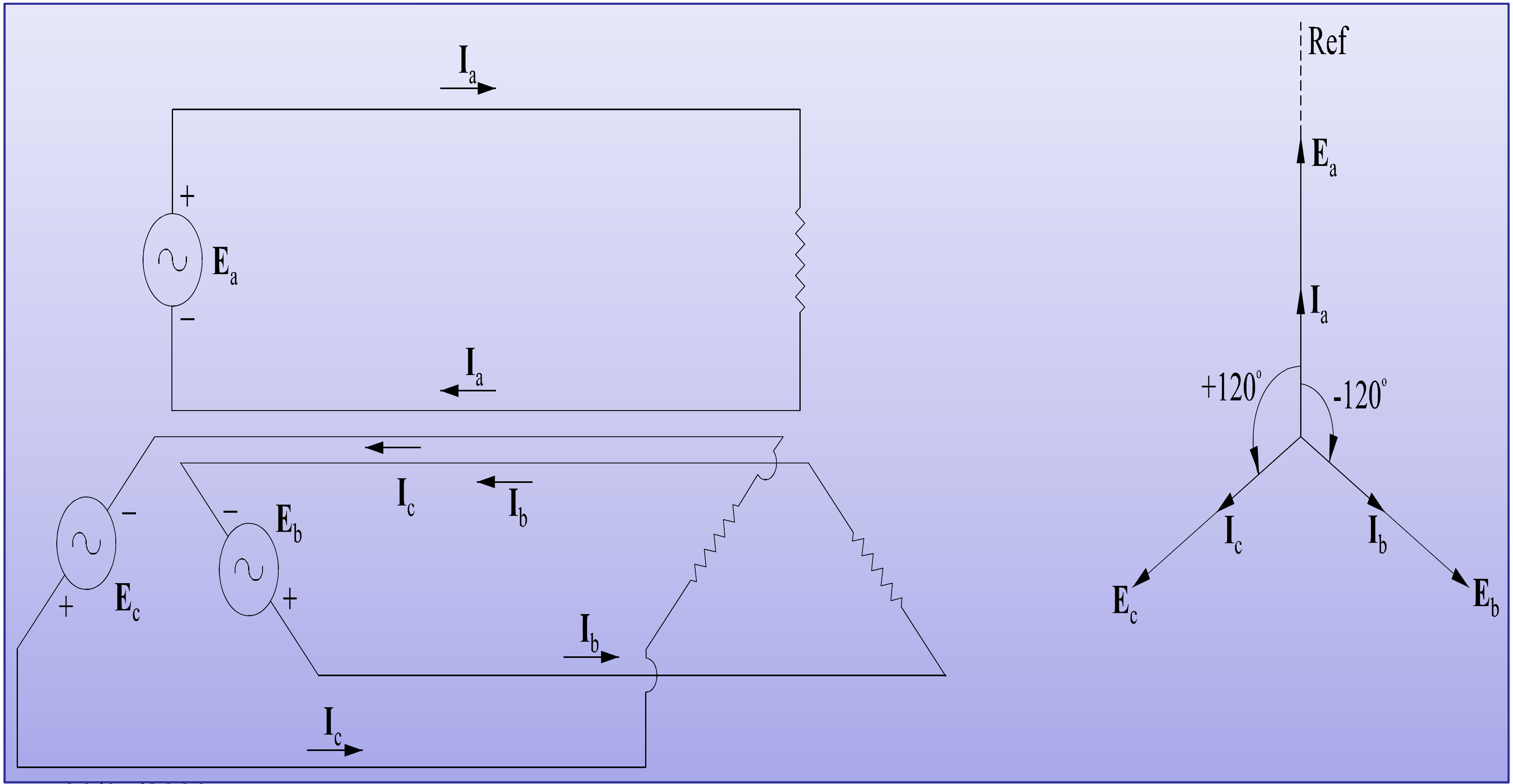
$v_c = V_c \sin (\omega t - 240^o) = V_c \sin (\omega t + 120^o)$

$V_a = V\angle 0°$

$V_b = V\angle -120^o$

$V_c = V\angle -240^o = V\angle 120^o$

A three phase system and its vector diagrams



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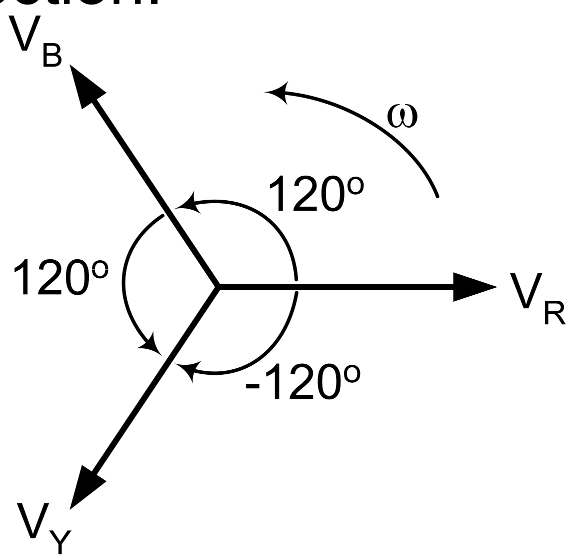
Three Phase System-Phase Sequence

- RYB or positive sequence, we describe the phase sequence as Red-Yellow-Blue
- V_R leads V_Y, which in turn leads V_B. This sequence is produced when the rotor rotates in the counterclockwise direction.



$$\mathbf{V_R} = \mathbf{V_{R(rms)}} \angle 0^o$$

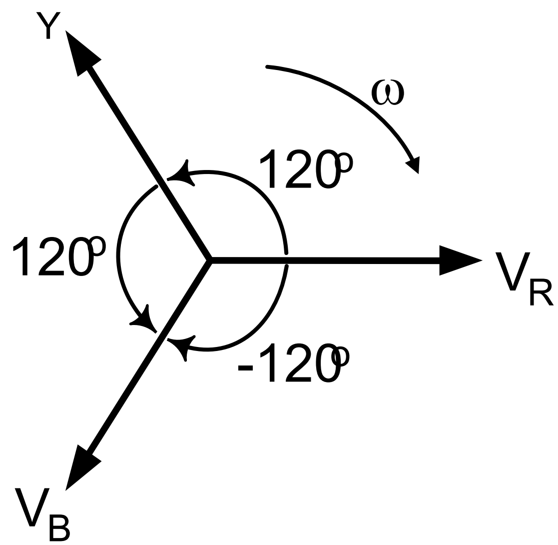
$$\mathbf{V_Y} = \mathbf{V_{Y(rms)}} \angle -120^o$$

$$\mathbf{V_B} = \mathbf{V_{B(rms)}} \angle -240^o$$

$$= \mathbf{V_{B(rms)}} \angle 120^o$$

Three Phase System-Phase Sequence

- RBY or negative sequence, we describe the phase sequence as Red-Blue-Yellow
- V_R leads V_B, which in turn leads V_Y. This sequence is produced when the rotor rotates in the clockwise direction.



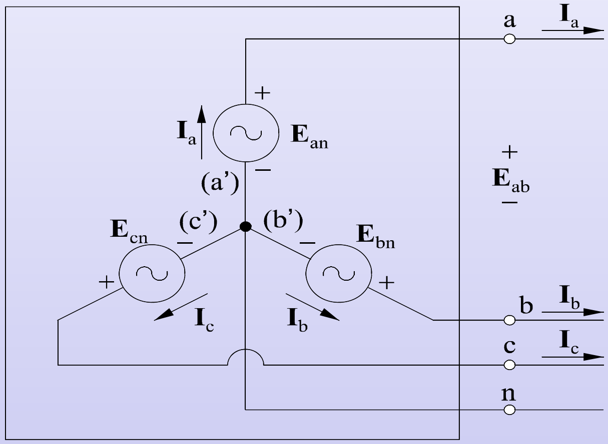
$$\mathbf{V_R} = \mathbf{V_{R(rms)}} \angle \mathbf{0^o}$$

$$\mathbf{V_B} = \mathbf{V_{B(rms)}} \angle -\mathbf{120^o}$$

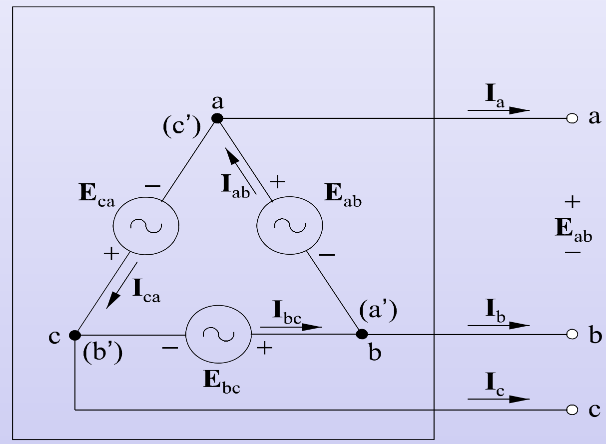
$$\mathbf{V}_Y = V_{Y(rms)} \angle -240^o$$
$$= V_{Y(rms)} \angle 120^o$$

- Remember in this subject all phasor systems rotate counter clockwise

Three Phase System-Connections



(a) Star



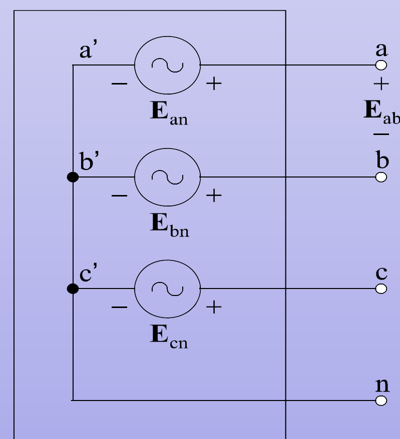
(b) Delta

Star Connection (Y)

- Usually is being used in system that need high voltage and low current
- There are two type of Y connection, i.e:

i) 3 wire star connection

ii) 4 wire star connection (3 live conductors and one neutral)

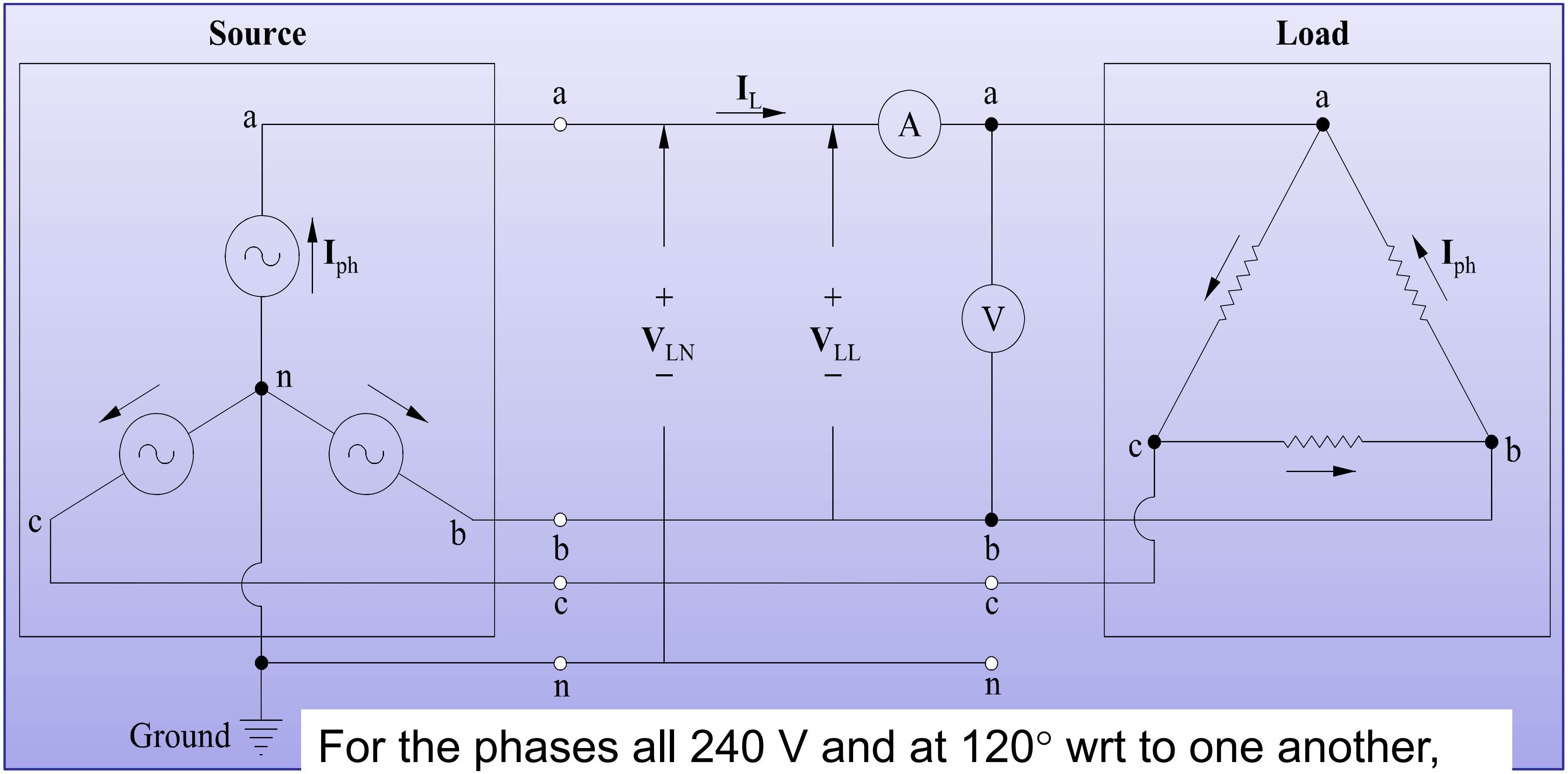


(c) Star

Delta Connection (△)

- Usually is being used in system that need high line current and low phase current.
- Connect all the conductor in series.
- No neutral line

An example of three phase connections



For the phases all 240 V and at 120° wrt to one another, the system is balanced if the 3 loads are equal

Definitions

Commonly AC power is a three-phase (3-Φ) supply system as shown

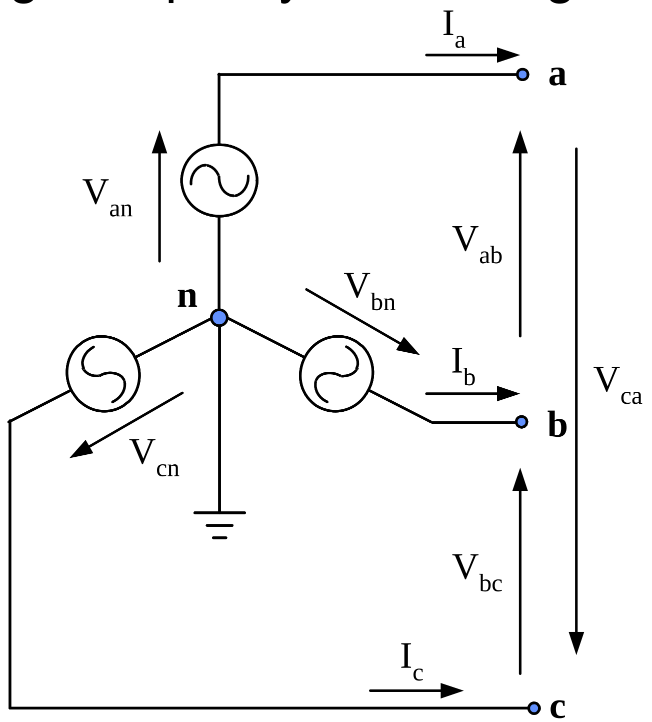
The 240 Volts single phase supply is just one part of such a multiphase system

It is a common practice to use the following to specify the voltage and current ratings of the three phase devices:

- Voltage between external terminals (**line-to-line voltage**): V_{LL}
- Current flowing in external lines (**line current**): I_L

V_{LL} and I_L are called 'line' quantities.

The voltages and currents internal to the device are referred to as 'Phase' quantities (e.g. phase-to-neutral voltage).



Relationship between voltages and currents

For balanced systems the relationships are:

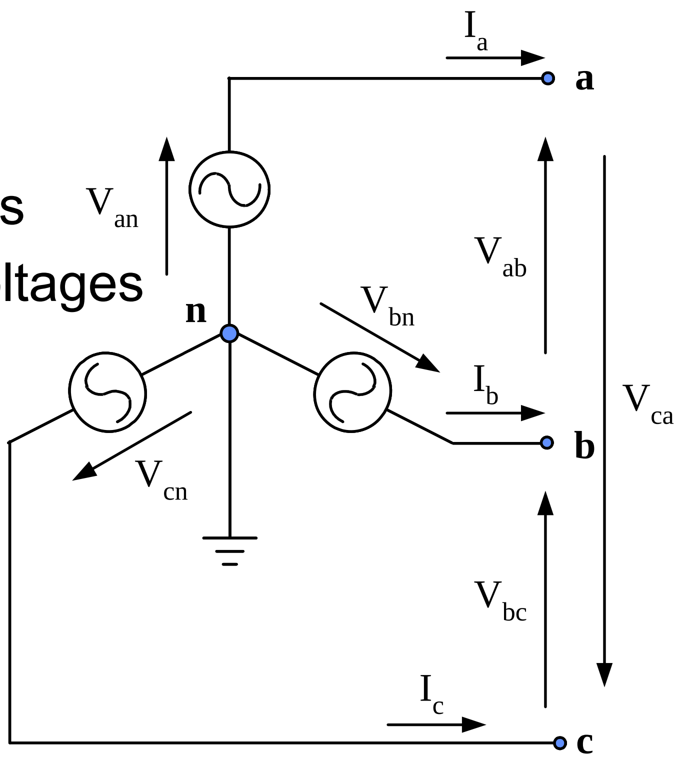
Star connection

Reference voltage: V_{an}

Phase current = Line current = I_L

The voltage from one live conductor to another is called the **line-to-line** voltages or simply **line** voltages

- e.g. V_{ab}, V_{bc}, V_{ca}
- To calculate V_{ab}, (KVL)



$$\mathbf{V_{ab}} = \mathbf{V_{an}} - \mathbf{V_{bn}}$$
$$= \mathbf{V_{an}} - \mathbf{V_{an}}\angle -120^\circ$$
$$= \mathbf{V_{an}}\left(1-(-0.5-j0.866)\right)$$
$$= \sqrt{3}\,\mathbf{V_{an}}\angle 30^\circ V$$

In a perfectly balanced system $\mathbf{V_{an}} + \mathbf{V_{bn}} + \mathbf{V_{cn}} = 0$

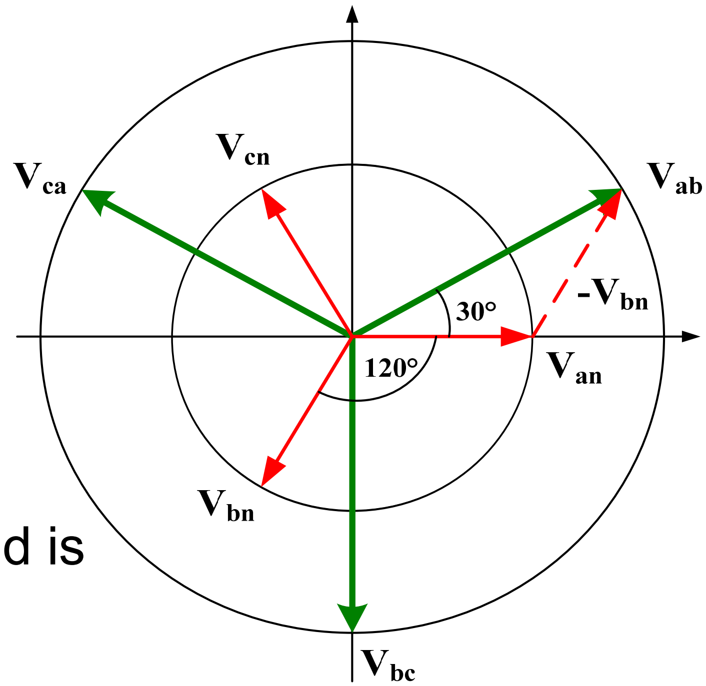
Relationship between voltages and currents

- The following vector diagram shows the relationship between phase angles of the phase and line voltages
- The 120º phase shift also exists between each line voltage

$$\mathbf{V_{ab}} = \sqrt{3}\mathbf{V_{an}} \angle 30^\circ$$

$$\mathbf{V_{bc}} = \sqrt{3}\mathbf{V_{bn}} \angle -90^\circ$$

$$\mathbf{V_{ca}} = \sqrt{3}\ \mathbf{V_{cn}} \angle -210^\circ$$



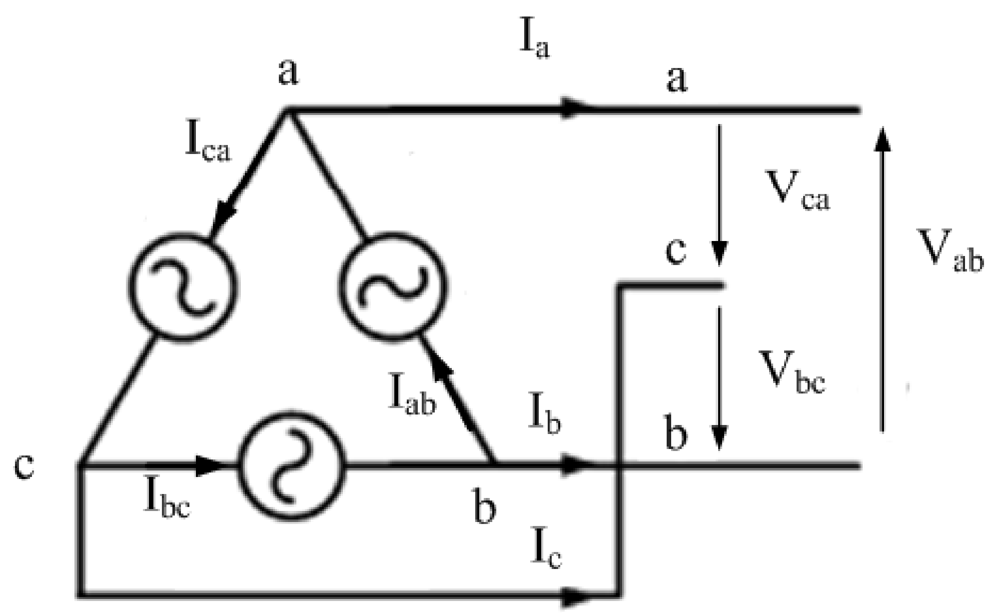
Line voltage (V_L) has magnitude √3 V_P and is leading phase voltage (V_{PH}) with 30°.

Relationship between voltages and currents

For balanced systems the relationships are:

Delta connection:
Phase voltage = Line-to-Line Voltage = V_{LL}

$$I_{ab} = I_{ab}\angle 0^\circ = I_P\angle 0^\circ$$
$$I_{bc} = I_{bc}\angle -120^\circ = I_P\angle -120^\circ$$
$$I_{ca} = I_{ca}\angle 120 = I_P\angle 120^\circ$$
$$KCL:$$
$$\Rightarrow I_a = I_{ab} - I_{ca}$$
$$= I_P\angle 0^\circ - I_P\angle 120^\circ$$
$$= I_p\left(1 - (-0.5 + j0.866)\right)$$
$$= \sqrt{3}\, I_P\angle -30^\circ$$



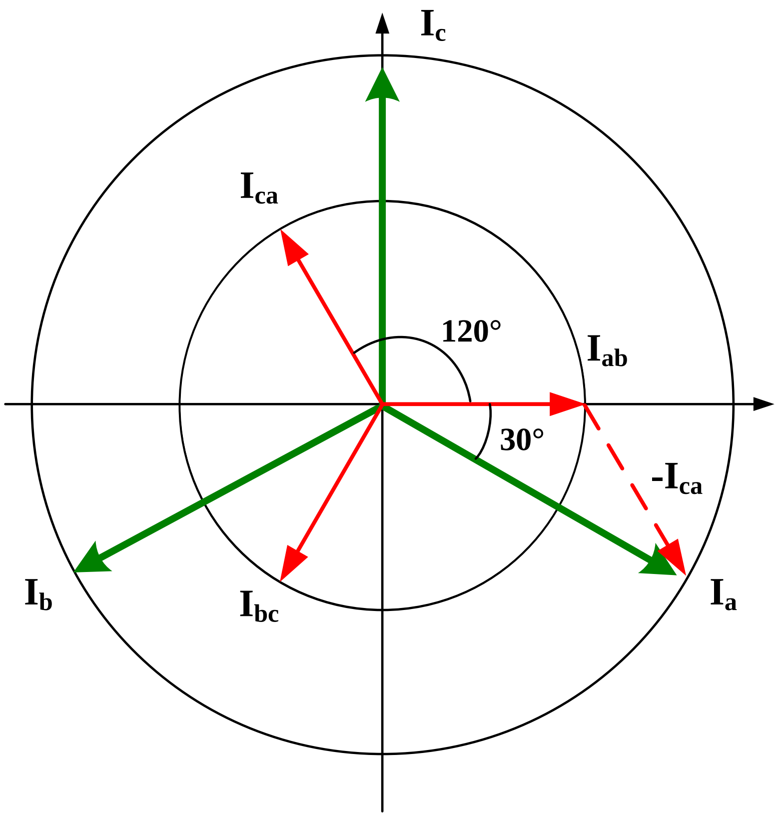
Relationship between voltages and currents

$$I_a = I_{ab} - I_{ca} = \sqrt{3}\, I_P \angle -30^\circ$$

$$I_b = I_{bc} - I_{ab} = \sqrt{3}\, I_P \angle -150^\circ$$

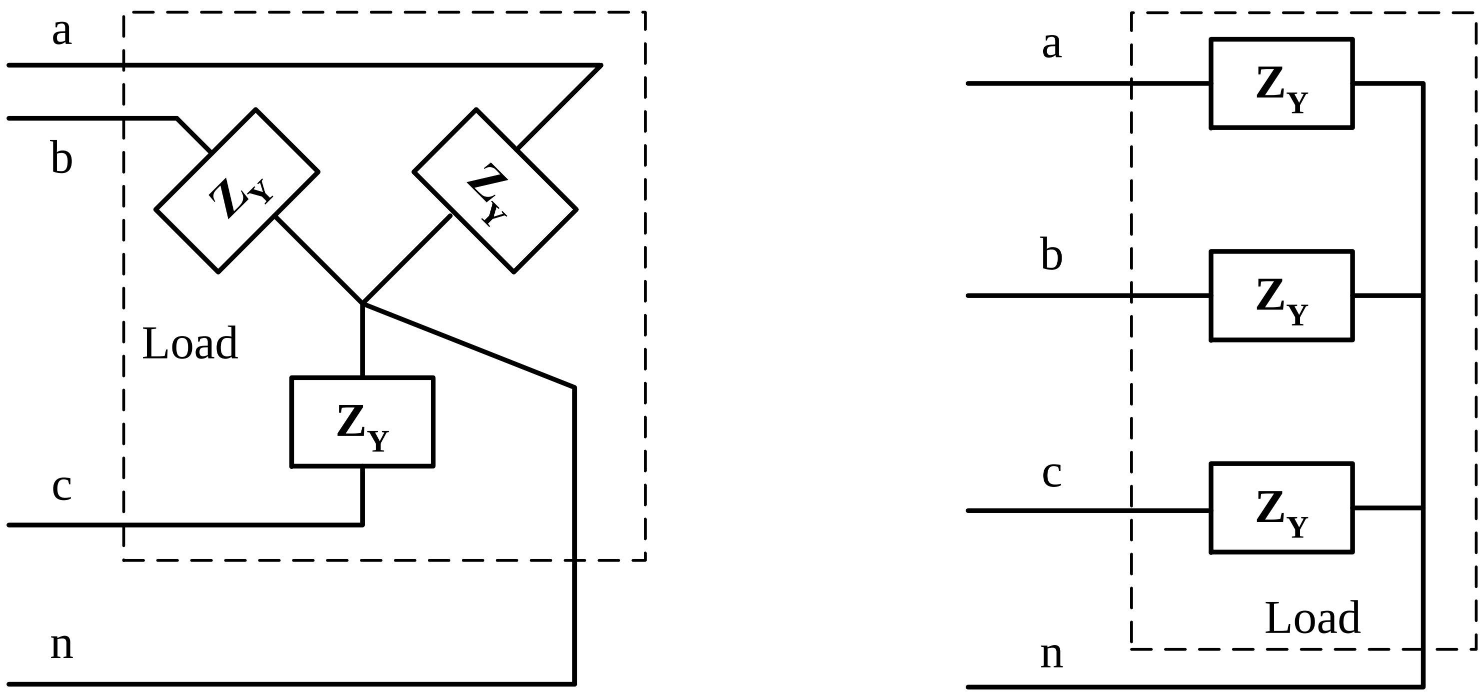
$$I_c = I_{ca} - I_{bc} = \sqrt{3}\, I_P \angle 90^\circ$$

Line Current (I_L) lagging current phase (I_{PH}) with 30°.



Loads

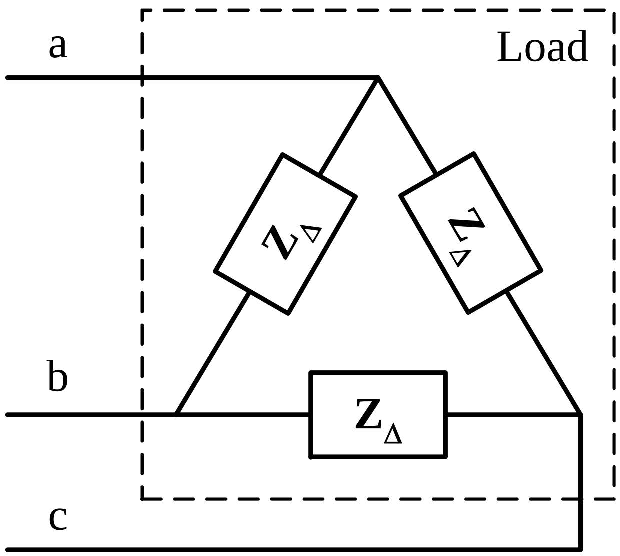
Loads can also be connected in a 3-phase fashion

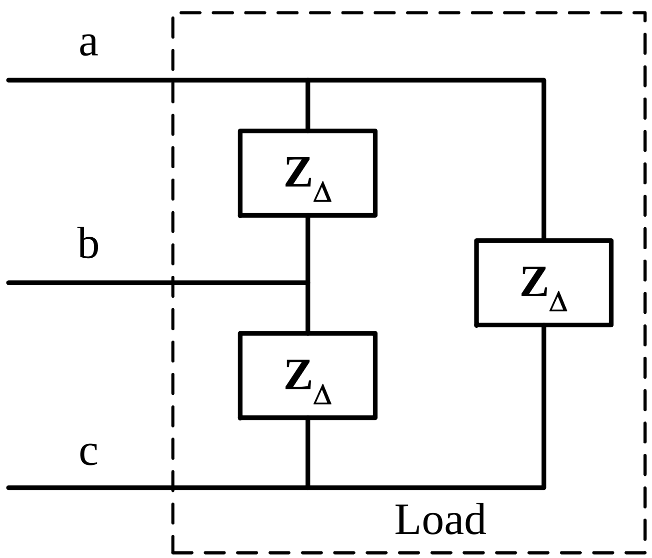


Star (Wye) connection of Load

Loads

Loads can also be connected in a 3-phase fashion





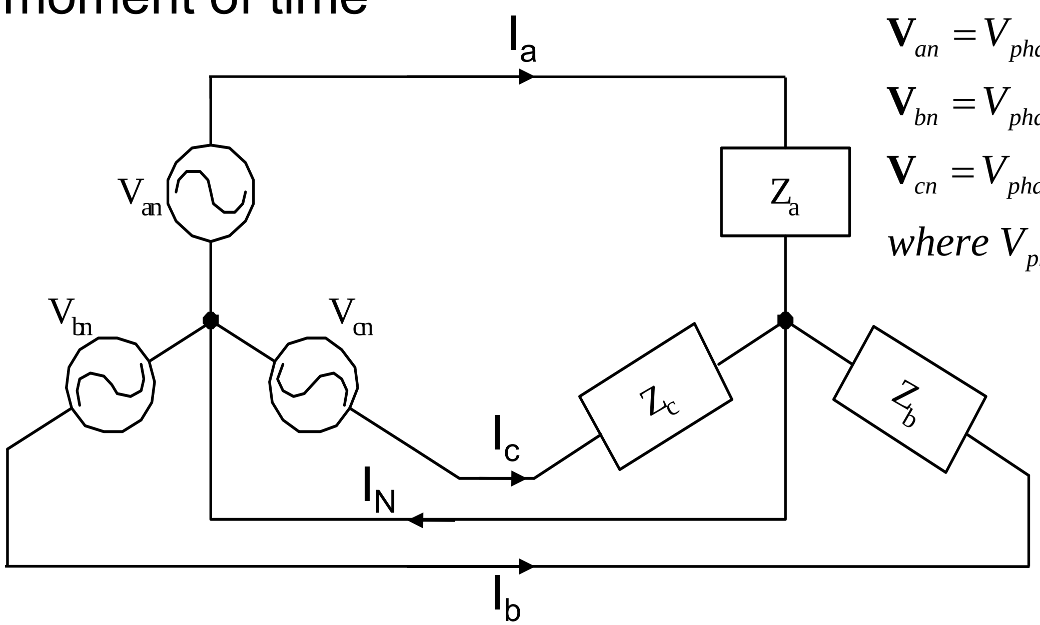
Delta connection at the Load

3-Phase balanced system

A 3-Phase 4 Wire Balanced System consists of a star connected source connected to a star connected load by 3 live wires and one neutral

Such a system will be balanced if all the three loads are equal

In a balanced system, $I_a + I_b + I_c = I_N$ → The neutral current is zero at each moment of time



$$\mathbf{V}_{an} = V_{phase} \angle 0°$$
$$\mathbf{V}_{bn} = V_{phase} \angle -120°$$
$$\mathbf{V}_{cn} = V_{phase} \angle -240°$$
$$where\ V_{phase} = |V_{an}| = |V_{bn}| = |V_{cn}|$$

$$I_a = \frac{V_{an}\angle 0^o}{Z_a}$$
$$I_b = \frac{V_{bn}\angle -120^o}{Z_b}$$
$$I_c = \frac{V_{cn}\angle 120^o}{Z_c}$$

3-Phase balanced system

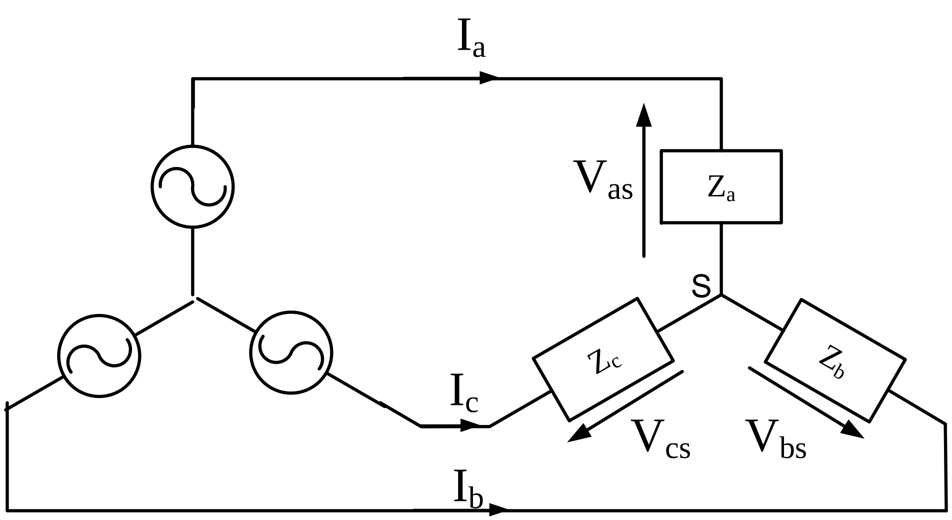
Wye-Connected Balanced Loads - Three wire system

$$\mathbf{V}_{as} = V_{phase}\angle 0^\circ$$

$$\mathbf{V}_{bs} = V_{phase}\angle -120^\circ$$

$$\mathbf{V}_{cs} = V_{phase}\angle -240^\circ$$

$$where\ V_{phase} = |V_{as}| = |V_{bs}| = |V_{cs}|$$



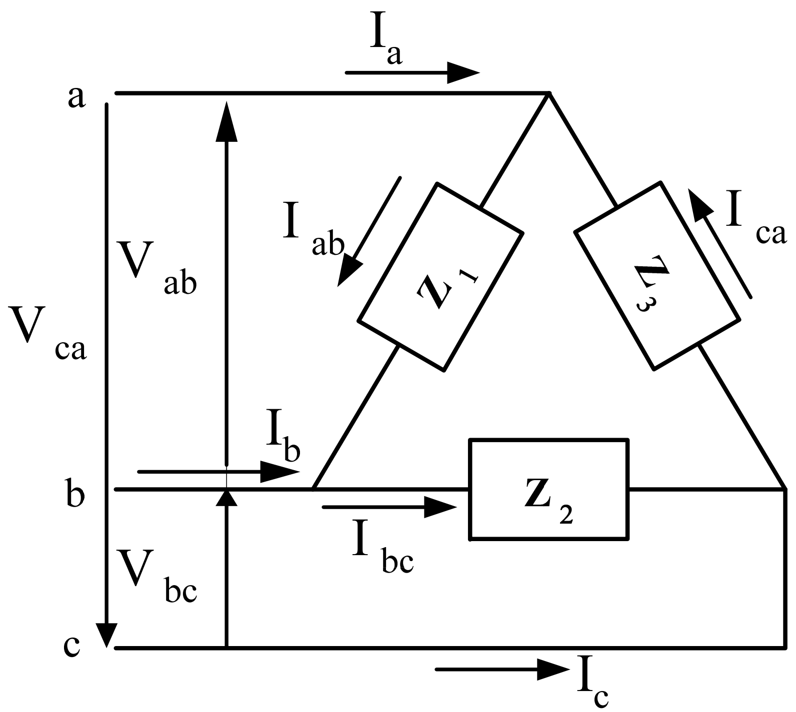
$$I_a = \frac{V_{as}\angle 0^o}{Z_a}$$

$$I_b = \frac{V_{bs}\angle -120^o}{Z_b}$$

$$I_c = \frac{V_{cs}\angle 120^o}{Z_c}$$

3-Phase balanced system

Delta-Connected Balanced Loads



where $|\mathbf{I}_{RY}| = |\mathbf{I}_{YB}| = |\mathbf{I}_{BR}| = \mathbf{I}_{phasa}$

and $|\mathbf{I}_{R}| = |\mathbf{I}_{Y}| = |\mathbf{I}_{B}| = \mathbf{I}_{line}$

Phase currents:

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}\angle 0^{o}}{\mathbf{Z}_{1}}$$

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}\angle -120^{o}}{\mathbf{Z}_{2}}$$

$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}\angle 120^{o}}{\mathbf{Z}_{3}}$$

Line currents:

$$\mathbf{I}_{a} = \mathbf{I}_{ab} - \mathbf{I}_{ca}$$

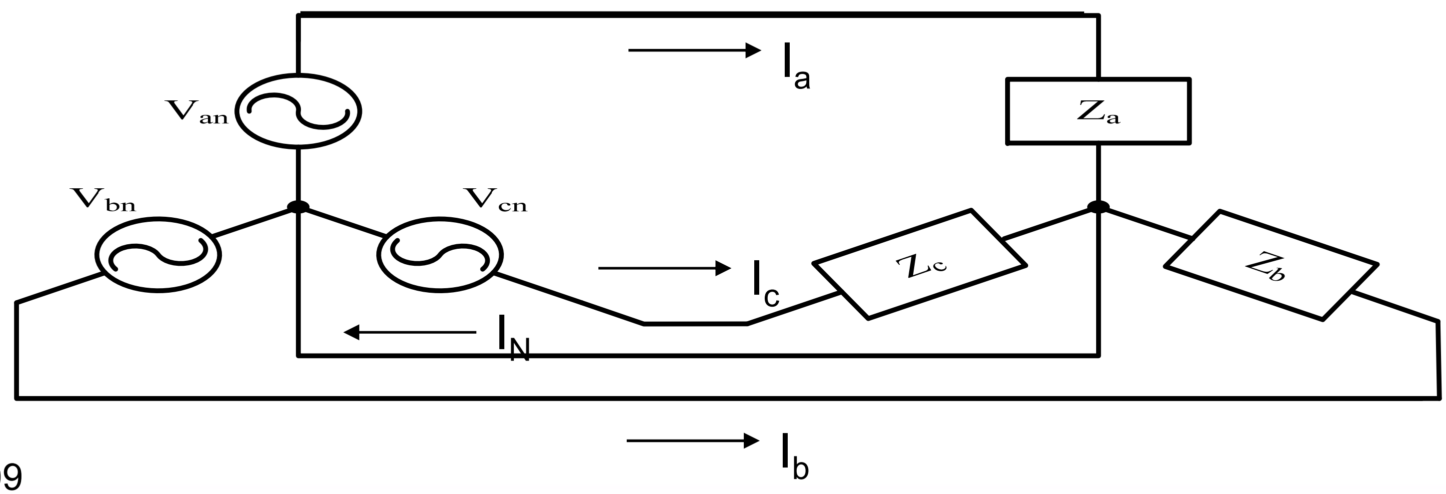
$$\mathbf{I}_{b} = \mathbf{I}_{bc} - \mathbf{I}_{ab}$$

$$\mathbf{I}_{c} = \mathbf{I}_{ca} - \mathbf{I}_{bc}$$

Example Problem

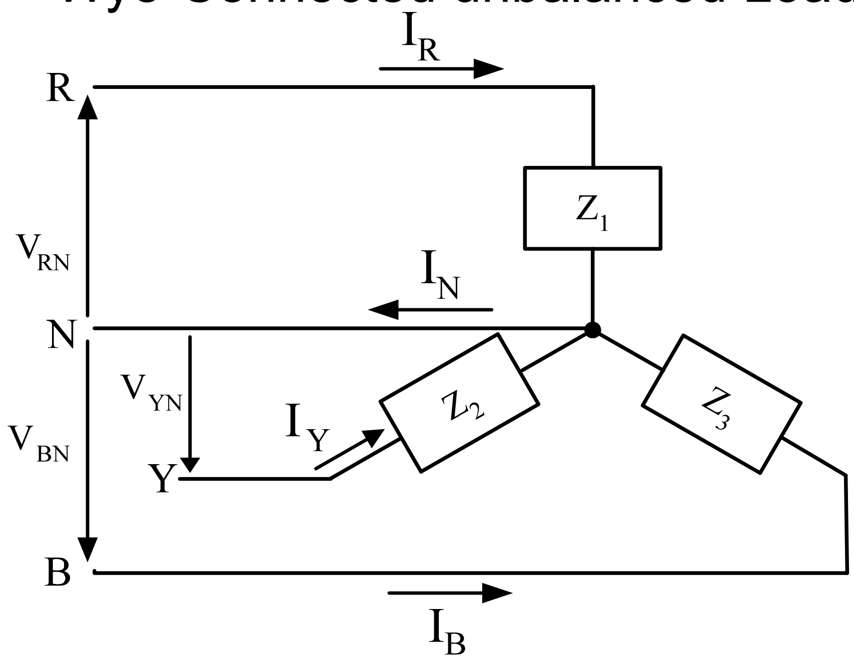
Given a balanced three phase four wire system with a wye-connected inductive load whose series impedance is: Z = 4 + j4 (per phase), and a 415 V supply,

i) Calculate the phase and line voltages
ii) Line currents
iii) Demonstrate that the neutral current is zero



Unbalanced Loads

Wye-Connected unbalanced Loads - Four wire system



$$V_{RN} = V_{phasa} \angle 0°$$

$$V_{YN} = V_{phasa} \angle -120°$$

$$V_{BN} = V_{phasa} \angle 120°$$

$$I_R = \frac{V_{RN}\angle 0^o}{Z_1}$$

$$I_Y = \frac{V_{YN}\angle -120^o}{Z_2}$$

$$I_B = \frac{V_{BN}\angle 120^o}{Z_3}$$

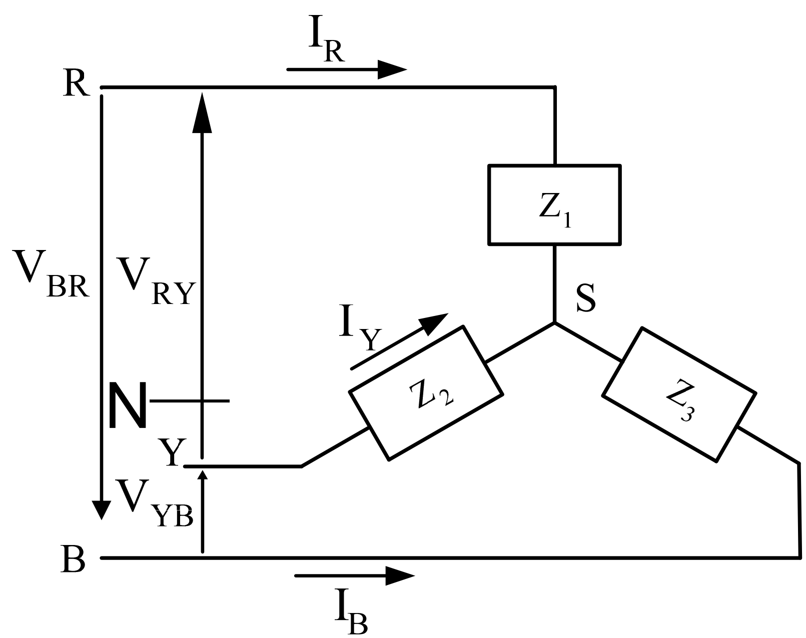
$$I_N = I_R + I_Y + I_B$$

For unbalanced load system,

$I_N \neq 0$ and $Z_1 \neq Z_2 \neq Z_3$

Unbalanced Loads

Wye-Connected unbalanced Loads - Three wire system



Note:

Point N # Point S

Potential between two point, displacement Neutral voltage, V_{SN}

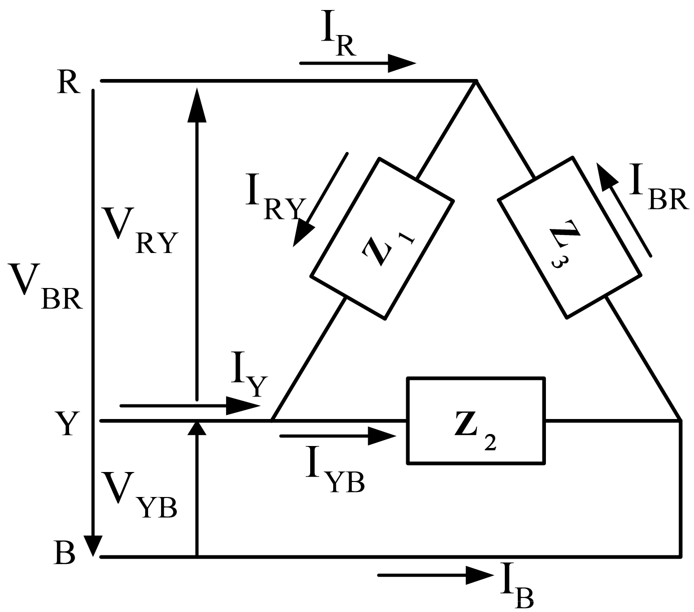
Unbalanced Loads

Two methods to solve Wye-Connected unbalanced Loads i.e. three wire/without neutral:

- **Change star connection to delta (Refer: Y- Star to delta transformation)**
  - Calculate I_{PH} from delta
  - Calculate I_L using KCL
  - Phase current (star) = line current (delta) because line I=phase I in Y connection
- **Using loop analysis**
  - Indicate direction of loop current
  - Write loop equations
  - Calculate loop current
  - Calculate line current = branch current

Unbalanced Loads

Delta-Connected unbalanced Loads - Three wire system



$$V_{RY} = V_{phase}\angle 0°$$
$$V_{YB} = V_{phase}\angle -120°$$
$$V_{BR} = V_{phase}\angle 120°$$

Phase currents:

$$I_{RY} = \frac{V_{RY}\angle 0^o}{Z_1}$$
$$I_{YB} = \frac{V_{YB}\angle -120^o}{Z_2}$$
$$I_{BR} = \frac{V_{BR}\angle 120^o}{Z_3}$$

Line currents:

$$I_R = I_{RY} - I_{BR}$$
$$I_Y = I_{YB} - I_{RY}$$
$$I_B = I_{BR} - I_{YB}$$

Example Problem

Given an unbalanced delta-connected load which is:
Z_1 = 6 + j8 Ω (per phase), Z_2 = 4 – j3 Ω , Z_3 = 8 + j6 Ω , Supply=110 V

i) Calculate the phase and line currents
ii) Construct the phasor diagram

Example Problem

A 3-Phase 4 Wire unbalanced load is supplied by a three phase 220V.

The impedance per phase are as follows:

$Z_1 = 10\angle -50^\circ\ \Omega$, $Z_2 = 15\angle 25^\circ\ \Omega$, and $Z_3 = 5\angle -90^\circ\ \Omega$

i) Calculate the line currents and neutral current
ii) Construct the phasor diagram

Example Problem

A balanced line voltage 415 V is supplied to the three impedance as follows:

$Z_1 = 10\angle 30° \ \Omega$, $Z_2 = 20\angle 60° \ \Omega$, and $Z_3 = 10\angle -45° \ \Omega$.

Note: All the three impedance form Y connected without neutral line.

i) Calculate the line currents
ii) Calculate the voltage across each impedance

Example Problem

A 3 phase supply, 4 wire, 415 V, 50Hz supplies power to the loads. The values of loads are given below:

Load 1 : P_1 =11 kW, p.f 0.85 lag

Load 2 : Q_2 =8 kVar, p.f 0.8 lag

Load 3 : S_3 =15 kVA, p.f 0.75 lag

$Z_4 = 10\angle 15^\circ\ \Omega$

Calculate the line currents (I_R , I_Y , and I_B)
Hint: Choose V_{RN} as a reference

Example Problem

A factory is supplied by a balanced three phase source. The factory has two plants, each a balanced three phase load as follows:

i) Load 1: 150 kVA at 0.8 pf leading – Y connected

ii) Load 2: 80 kW and 95 kVAR (inductive)- delta connected

The loads are connected in parallel. Assume a lossless line, find the current at each load and total current if the source voltage is 415 V.

Example Problem

A 3 phase Y connected load is supplied by a three phase source 346.4 V, 50Hz. The magnitudes of line currents are I_R =50 A, I_Y =40 A and I_B =33.33 A . By using a phasor diagram in figure 1, calculate the each load impedance value. Choose phase R voltage as a reference.

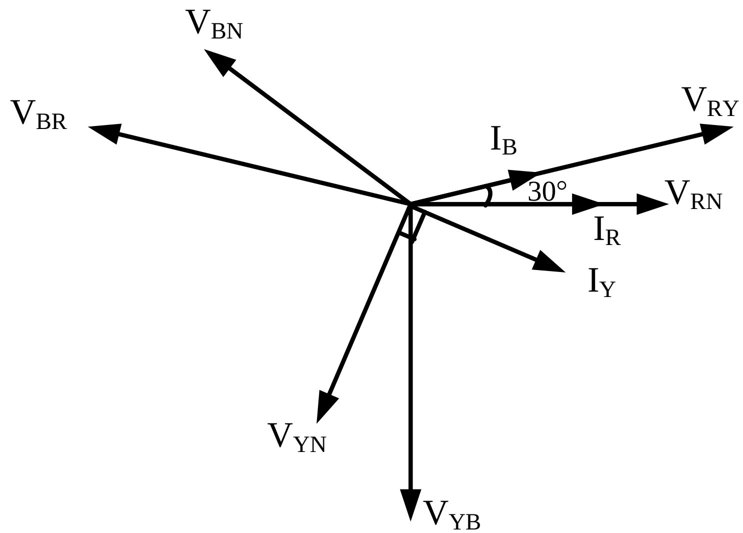


Figure 1

Example Problem

Three similar coils having 20 Ω resistance and 0.8 H inductance are connected in Y for a three phase supply 400 V, 50 Hz.

I) Calculate per phase impedance

ii) Total real power of the loads

3-Phase power

The three phase power is equal to the algebraic sum of individual phase powers

If the load is balanced: $P_{3ph} = 3\ P_{phase} = 3\ V_{phase}\ I_{phase} \cos\theta$

- **Wye connection system**

$I_{phase} = I_L$ **and** $V_{LL} = \sqrt{3}\ V_{phase}$

Real Power, $P_{3ph} = 3\ V_{phase}\ I_{phase} \cos\theta$

$$= \sqrt{3}\,|V_{LL}|\,|I_L|\cos\theta\ Watt$$

3-Phase power

- **Wye connection system**

I phase = I$_L$ and $|V_{LL}| = \sqrt{3}\,|V_{phase}|$

Reactive power, $Q_{3ph} = 3\,V_{phase}\,I_{phase}\,\sin\theta$

$$= \sqrt{3}\,|V_{LL}|\,|I_L|\,sin\theta \;\; VAr$$

Apparent power, $S_{3ph} = 3\,V_{phase}\,I_{phase}$

$$= \sqrt{3}\,|V_{LL}|\,|I_L| \;\; VA$$

3-Phase power

- **Delta connection system**

$V_{LL} = V_{phase}$ $\qquad$ $I_L = \sqrt{3}\ I_{phase}$

Real Power, $P = 3\ V_{phase}\ I_{phase} \cos\theta$

$$= \sqrt{3}|V_{LL}||I_L|\cos\theta\ Watt$$

Reactive power, $Q_{3ph} = 3\ V_{phase}\ I_{phase} \sin\theta$

$$= \sqrt{3}|V_{LL}||I_L|\sin\theta\ VAr$$

Apparent power, $S_{3ph} = 3\ V_{phase}\ I_{phase}$

$$= \sqrt{3}|V_{LL}||I_L|\ VA$$

3-Phase power

For connection of wye as well as delta, the formula for calculation of all the 3ϕ powers that are P(real), Q(reactive) and S(apparent) are the same

Example Problem

A balanced wye-connected load of $(8+j6)\Omega$ per phase is connected to a balanced 3-phase 400V supply. Find:

i)line currents

ii)power factor

iii) Real power, Reactive power and total volt-amperes.

Example Problem

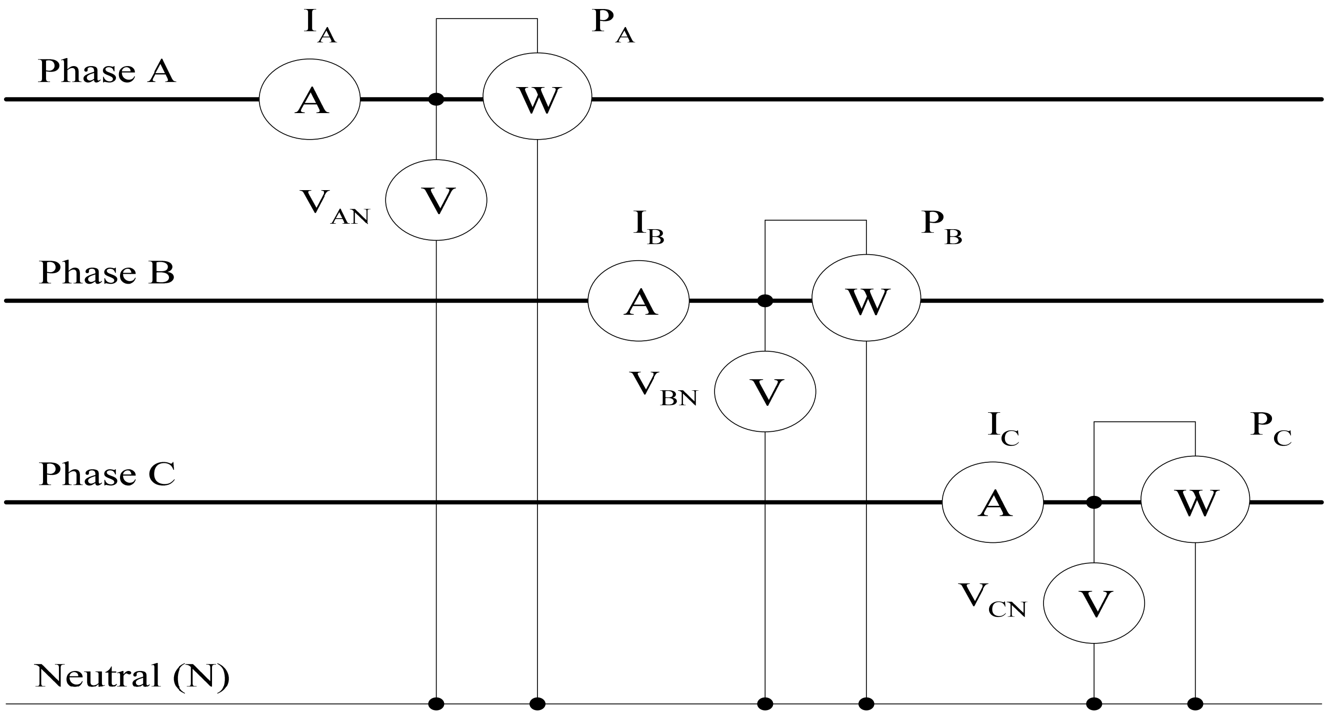
Three impedances each of magnitude (15-j20)Ω are connected in delta across a 3 phase, 400V supply. Determine the phase current, line current, real power and reactive power drawn from the supply.

Power measurement

- In a four-wire system (3 phases and a neutral) the real power is measured using three single-phase watt-meters.
- In a three-wire system (three phases without neutral) the power is measured using only two single phase watt-meters.
- The watt-meters are supplied by the line current and the line-to-line voltage.

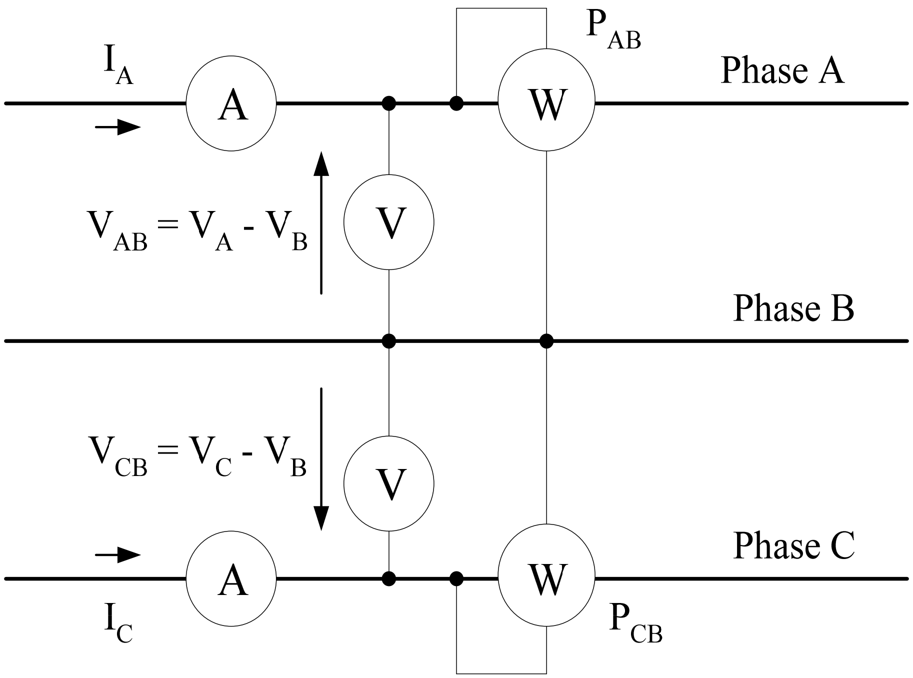
Power measurement

Four wire system - Each phase measured separately



Power measurement

Three wire system -The three phase power is the sum of the two watt-meters reading



$$P_{3\phi} = P_{AB} + P_{CB}$$

$$V_{ab} = 110\angle 0° V$$

$$I_a = 20\angle -70° A$$

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