CHAPTER 1

SET THEORY

[Part 2: Operation on Set]

Union

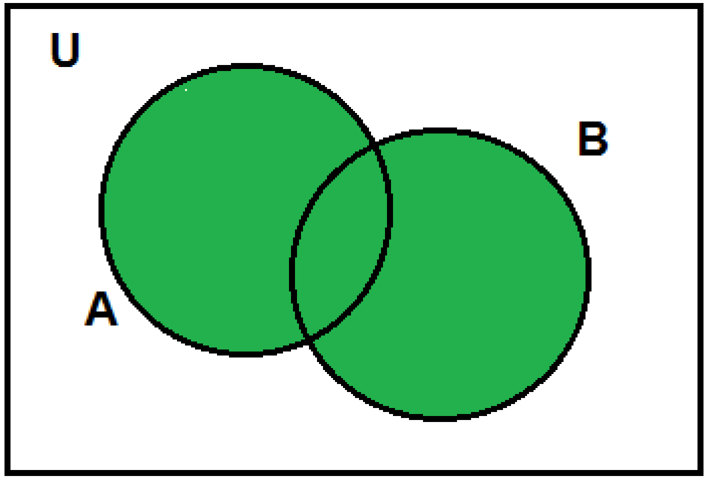
- The union of two sets A and B, denoted by $A \cup B$, is defined to be the set

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

- The union consists of all elements belonging to either A or B (or both)

Union

- Venn diagram of $A \cup B$



Example

A={1, 2, 3, 4, 5}, B={2, 4, 6} and C={8, 9}

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$

$B \cup C = \{2, 4, 6, 8, 9\}$

$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$

Union

- If *A* and *B* are finite sets, the cardinality of $A \cup B$,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Intersection

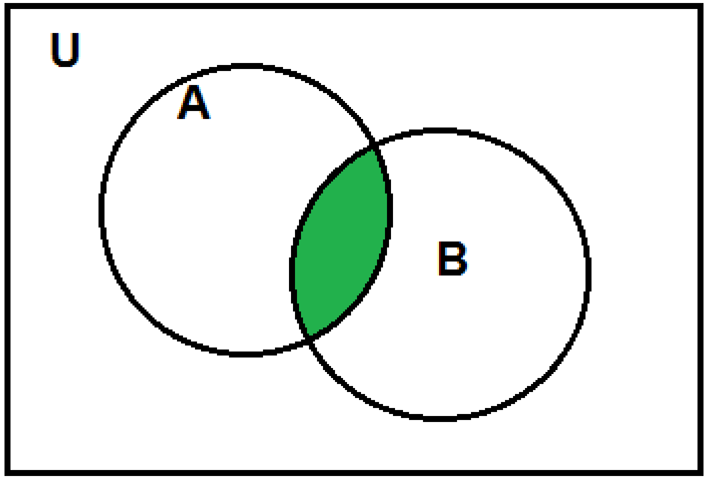
- The intersection of two sets A and B, denoted by $A \cap B$, is defined to be the set

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

- The intersection consists of all elements belonging to both A and B.

Intersection

- Venn diagram of $A \cap B$



Example

A={1, 2, 3, 4, 5, 6}, B={2, 4, 6, 8, 10} and
C={ 1, 2, 8, 10 }

$$A \cap B = \{2, 4, 6\}$$
$$A \cap C = \{1, 2\}$$
$$C \cap B = \{2, 8, 10\}$$
$$A \cap B \cap C = \{2\}$$

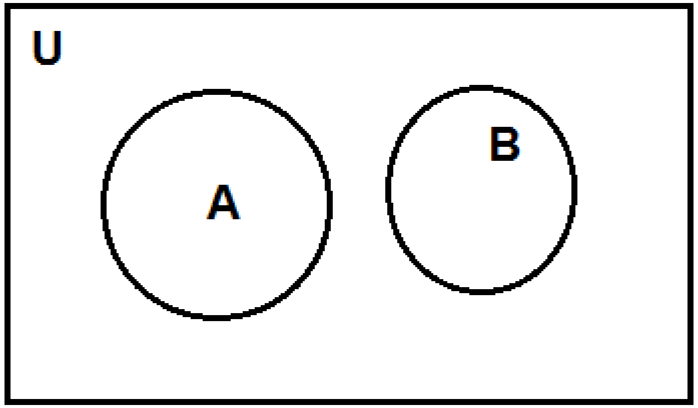
Disjoint

- Two sets *A* and *B* are said to be disjoint if,

$$A \cap B = \varnothing$$

Disjoint

- Venn diagram, $A \cap B = \varnothing$



Example

$A = \{1, 3, 5, 7, 9, 11\}$

$B = \{2, 4, 6, 8, 10\}$

$A \cap B = \varnothing$

Difference

- The set

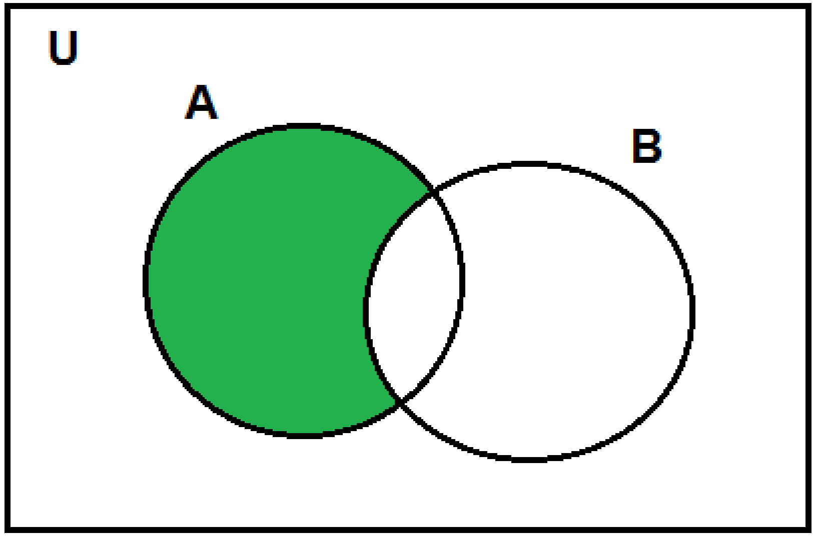
  $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

  is called the difference.

- The difference $A - B$ consists of all elements in A that are not in B.

Difference

- Venn diagram of $A - B$



Example

$A = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$

$B = \{ 2, 4, 6, 8 \}$

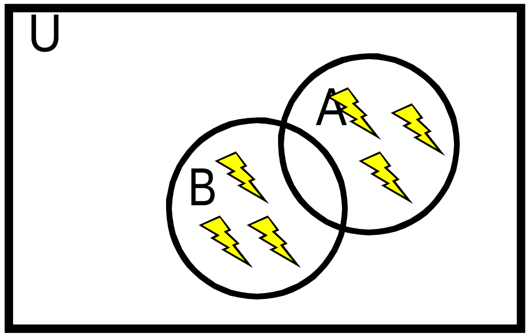
$A - B = \{ 1, 3, 5, 7 \}$

Symmetric Difference

- The *symmetric difference*,

$$A \oplus B = \{ x : (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$$

$$= (A - B) \cup (B - A)$$



Complement

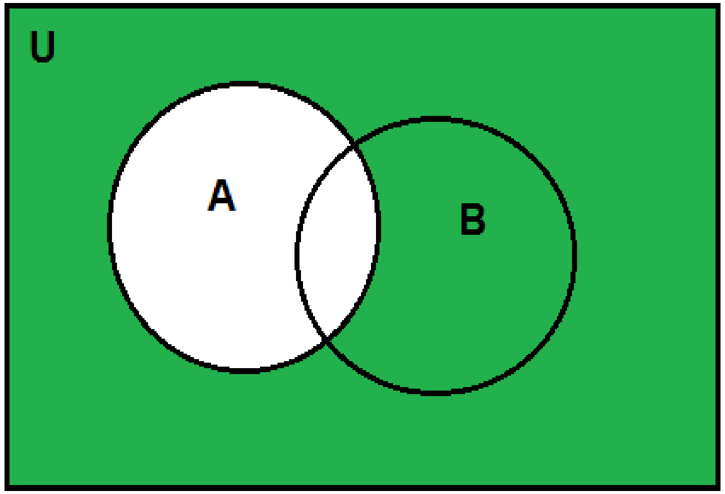
- The complement of a set A with respect to a universal set U, denoted by A' is defined to be

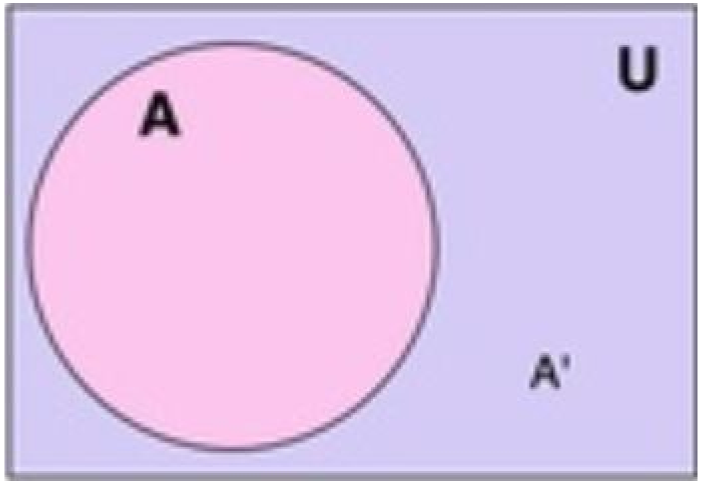
$$A' = \{x \in U \mid x \notin A\}$$

$$A' = U - A$$

Complement

Venn diagram of *A’*





Example

Let U be a universal set,

$U = \{ 1, 2, 3, 4, 5, 6, 7 \}$

$A = \{ 2, 4, 6 \}$

$A' = U - A = \{ 1, 3, 5, 7 \}$

Exercise

- Let,

  $U = \{ a, b, c, d, e, f , g, h, i, j, k, l, m \}$

  $A = \{ a, c, f, m\}$

  $B = \{ b, c, g, h, m \}$

- Find:

  $A \cup B$, $A \cap B$, $| A \cup B|$, $A - B$ dan A'.

Exercise

Let the universe be the set U={1, 2, 3, 4,.....,10}.

Let A={1, 4, 7, 10}, B={1, 2, 3, 4, 5} and C={2, 4, 6, 8}.

List the elements of each set:

a) U'

b) $B' \cap (C - A)$

c) $B - A$

d) $(A \cup B) \cap (C - B)$

Properties of Sets

- Commutative laws

$$A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

Properties of Sets

- Associative laws

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Properties of Sets

- Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Properties of Sets

- Absorption laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

Properties of Sets

- Idempotent laws

$$A \cap A = A$$

$$A \cup A = A$$

Properties of Sets

- Complement laws

$$(A')' = A$$

$$A \cap A' = \varnothing$$

$$A \cup A' = \mathrm{U}$$

$$\varnothing' = U$$

$$U' = \varnothing$$

Properties of Sets

- De Morgan's laws

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

Properties of Sets

- Properties of universal set

$$A \cup U = U$$

$$A \cap U = A$$

Properties of Sets

- Properties of empty set

$$A \cup \varnothing = A$$

$$A \cap \varnothing = \varnothing$$

Example

- Let A, B and C denote the subsets of a set S and let C' denote a complement of C in S.

- If $A \cap C = B \cap C$ and $A \cap C' = B \cap C'$, then prove that $A = B$

Example

$$
\begin{aligned}
A &= A \cap S \\
&= A \cap (C \cup C') \\
&= (A \cap C) \cup (A \cap C') && \text{by distributivity} \\
&= (B \cap C) \cup (B \cap C') && \text{by the given conditions} \\
&= B \cap (C \cup C') && \text{by distributivity} \\
&= B \cap S \\
&= B
\end{aligned}
$$

Simplification using the Set Law

Simplify the set

$(((A \cup B) \cap C)' \cup B')' =$

$= ((A \cup B) \cap C)'' \cap B''$	[DeMorgan]
$= ((A \cup B) \cap C) \cap B$	[Double Complement]
$= (A \cup B) \cap (C \cap B)$	[Associativity of $\cap$]
$= (A \cup B) \cap (B \cap C)$	[Commutativity of $\cap$]
$= ((A \cup B) \cap B) \cap C$	[Associativity of $\cap$]
$= B \cap C$	[Absorption]

Exercise

- Let A, B and C be sets.
- Show that

$$(A \cup (B \cap C))' = A' \cap (B' \cup C')$$

Exercise

- Let A, B and C be sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$

- Prove that $B = C$

solution

$B = B \cup (B \cap A)$	Absorption law
$= B \cup (A \cap B)$	commutative law
$= B \cup (A \cap C)$	Given condition
$= (B \cup A) \cap (B \cup C)$	Distributive law
$= (A \cup B) \cap (B \cup C)$	commutative law
$= (A \cup C) \cap (B \cup C)$	Given condition
$= (C \cup A) \cap (C \cup B)$	commutative law
$= C \cup (A \cap B)$	Distributive law
$= C \cup (A \cap C)$	Given condition
$= C$	Absorption law

Generalized Union/Intersection

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$

Generalized Union/Intersection

Ex. Suppose that:

$$A_i = \{1,2,3,\ldots,i\} \quad i = 1,2,3,\ldots$$

$$\bigcup_{i=1}^{\infty} A_i = ? \qquad \bigcup_{i=1}^{\infty} A_i = Z^+$$

$$\bigcap_{i=1}^{\infty} A_i = ? \qquad \bigcap_{i=1}^{\infty} A_i = \{1\}$$

Cartesian Product

- Let A and B be sets.
- An ordered pair of elements $a \in A$ dan $b \in B$ written (a, b) is a listing of the elements a and b in a specific order.
- The ordered pair (a, b) specifies that a is the first element and b is the second element.

Cartesian Product

- An ordered pair (a, b) is considered distinct from ordered pair (b, a), unless $a=b$.

- Example $(1, 2) \neq (2, 1)$

Cartesian Product

- The Cartesian product of two sets A and B, written $A \times B$ is the set,

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$$

- For any set A,

$$A \times \varnothing = \varnothing \times A = \varnothing$$

Example

$A = \{a, b\}$, $B = \{1, 2\}$.

$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$

Cartesian Product

- if $A \neq B$, then $A \times B \neq B \times A$.
- if $|A| = m$ and $|B| = n$, then $|A \times B| = mn$.

Example

- $A = \{1, 3\}$, $B = \{2, 4, 6\}$.

$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$
$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$

$A \neq B$, $A \times B \neq B \times A$

$|A| = 2$, $|B| = 3$, $|A \times B| = 2.3 = 6$.

Cartesian Product

- The Cartesian product of sets $A_1, A_2, ..., A_n$ is defined to be the set of all n-tuples $(a_1, a_2, ... a_n)$ where $a_i \in A_i$ for $i=1,...,n$;

- It is denoted $A_1 \times A_2 \times .... \times A_n$

  $|A_1 \times A_2 \times .... \times A_n| = |A_1|.|A_2| .... |A_n|$

Example

- $A = \{a, b\}$, $B = \{1, 2\}$, $C = \{x, y\}$

  $A \times B \times C = \{(a,1,x),(a,1,y), (a,2,x), (a,2,y),$
  $(b,1,x), (b,1,y), (b,2,x), (b,2,y)\}$

- $|A \times B \times C| = 2.\ 2.\ 2 = 8$

Exercise

- Let A= {w, x}, B={1, 2} and C={KB, SD, PS}.
- Find $|A\times B|$, $|B\times C|$, $|A\times C|$, $|A\times B\times C|$, $|B\times C\times A|$, $|A\times B\times A\times C|$
- Determine the following set,

  a) $A\times B$, $B\times C$, $A\times C$

  b) $A\times B\times C$

  c) $B\times C\times A$

  d) $A\times B\times A\times C$

Exercise

- Let $X = \{1,2\}$, $Y=\{a\}$ and $Z=\{b,d\}$.
- List the elements of each set.

  a) $X \times Y$

  b) $Y \times X$

  c) $X \times Y \times Z$

  d) $X \times Y \times Y$

  e) $X \times X \times X$

  f) $Y \times X \times Y \times Z$