CHAPTER 6

FINITE AUTOMATA

Deterministic Finite Automata (DFA)

- In computer science, we study different types of computer languages, such as Basic, Pascal, and C++.

- We will discuss a type of a language that can be recognized by special types of machines.

Deterministic Finite Automata (DFA)

- A deterministic finite automaton (pl. automata) is a mathematical model of a machine that accepts languages of some alphabet.

Deterministic Finite Automata (DFA)

- Deterministic Finite Automaton is a quintuple M= { S, I, q_0, f_s, F}

  where,

  S is a finite nonempty set of states

  I is the input alphabet (a finite nonempty set of symbols)

  q_0 is the initial state

  f_s is the state transition function

  F is the set of final states, subset of S.

example

- Let M={ $\{q_0,q_1,q_2\}$, {0,1}, q_0, f_s, $\{q_2\}$}
  where f_s is defined as follows:

  $f_s(q_0,0)= q_1$, $f_s(q_1,1)= q_2$
  $f_s(q_0,1)= q_0$, $f_s(q_2,0)= q_0$
  $f_s(q_1,0)= q_2$, $f_s(q_2,1)= q_1$

- Note that for M:
  $S=\{q_0,q_1,q_2\}$, I= {0,1}, $F=\{q_2\}$
  q_0 is the initial state

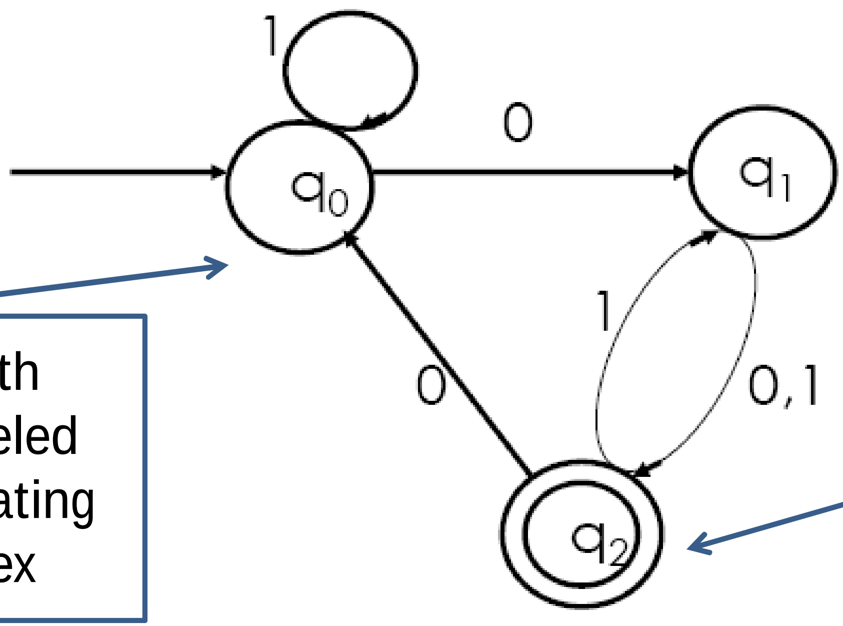
example

- The state transition function of a DFA is often described by means of a table, called a **transition table**.

f_s	0	1
q_0	q_1	q_0
q_1	q_2	q_2
q_2	q_0	q_1

example

- The transition diagram of this DFA is,



Each state represented by a small circle labeled with the state

Initial state with incoming unlabeled arrow not originating from any vertex

Final state with a double circle

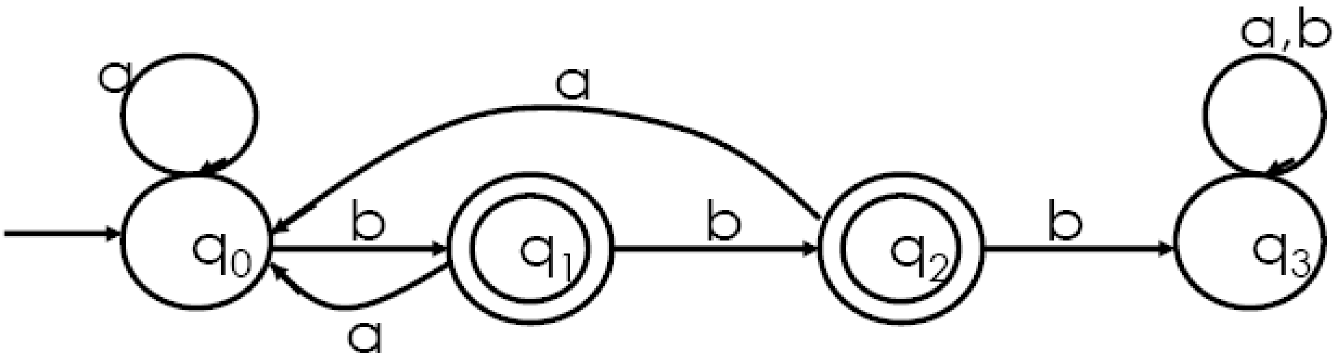
example

Let $M=(\{q_0,q_1,q_2,q_3\}, \{a,b\}, q_0, f_s, \{q_1,q_2\})$
where f_s is given by the table

f_s	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

example

- The transition diagram of this DFA is,



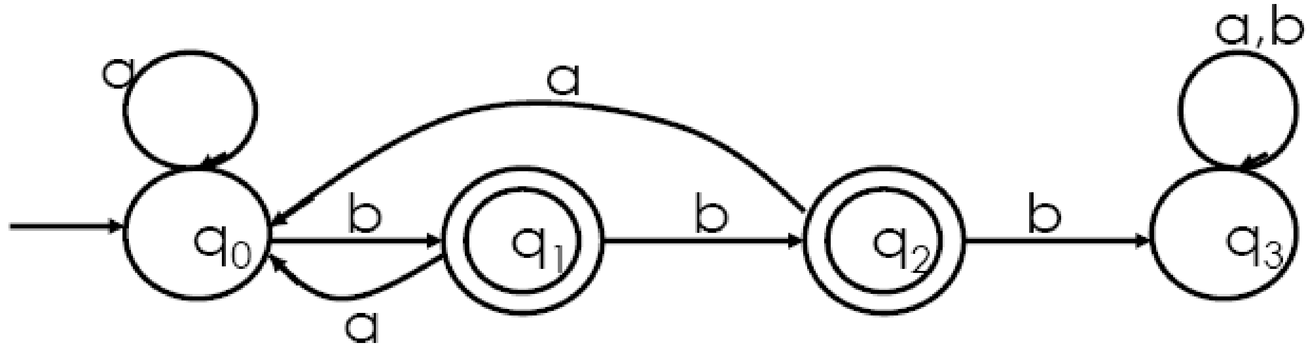
Deterministic Finite Automata (DFA)

- Let $M = \{ S, I, q_0, f_s, F\}$ be a DFA and *w* is an input string,
- *w* is said to be accepted by M if

$$f_s^*(q_0, w) \in F$$

- f_s^* - extended transition function for M

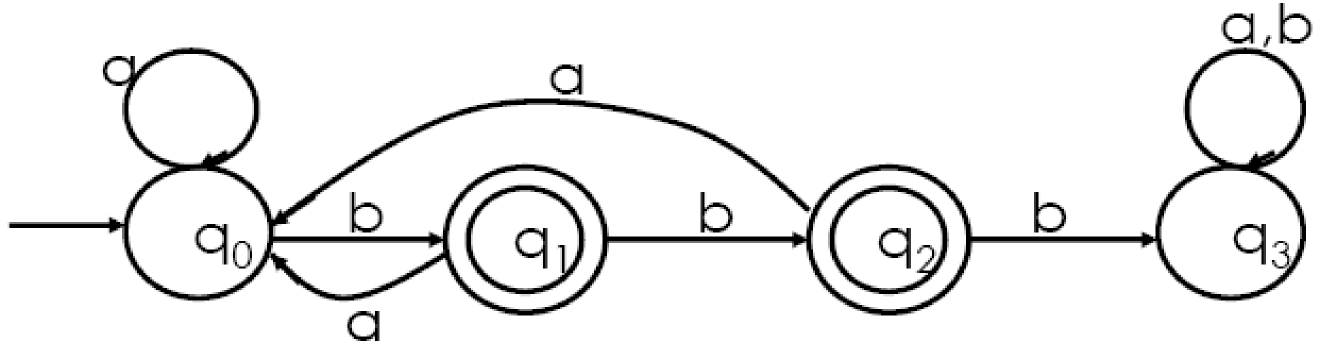
example



w= abb

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2$ accepted by M

example

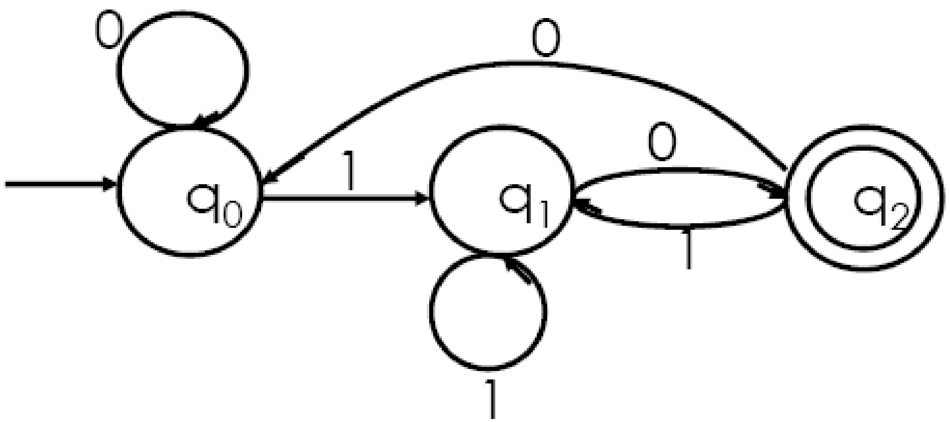


w= abba

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0$$

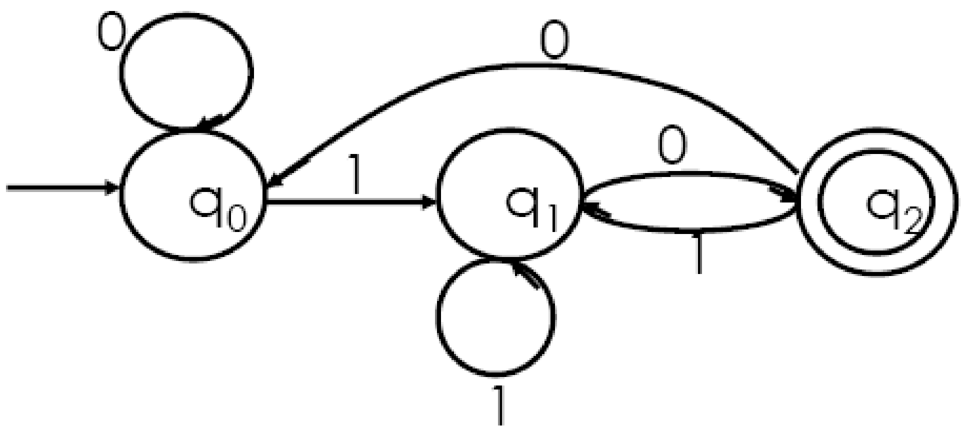
not accepted by M

example



- What are the states of M? q_0, q_1, q_2
- Write the set of input symbols. $I = \{0,1\}$
- Which is the initial state? q_0

example



- Write the set of final states. $F = \{q_2\}$
- Write the transition table for this DFA

example

The transition table, fs

f_s	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_1

example

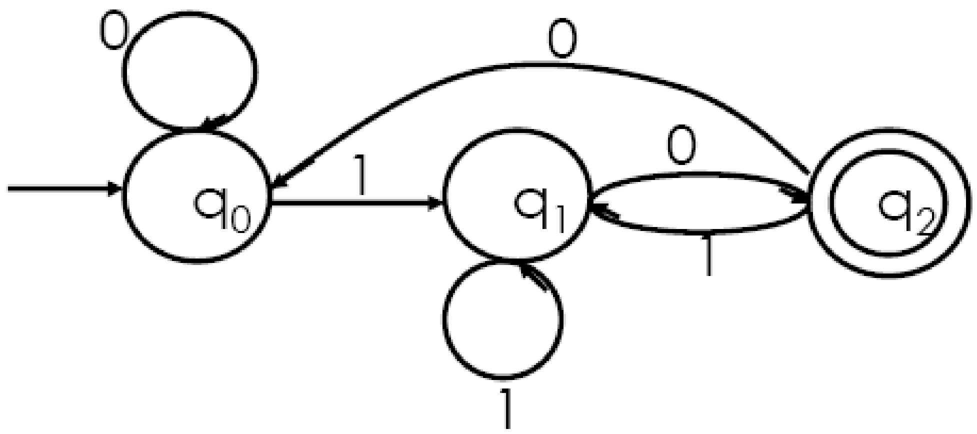
Which of the strings are accepted by M?

0111010, 00111, 111010,

0100, 1110

example

0111010

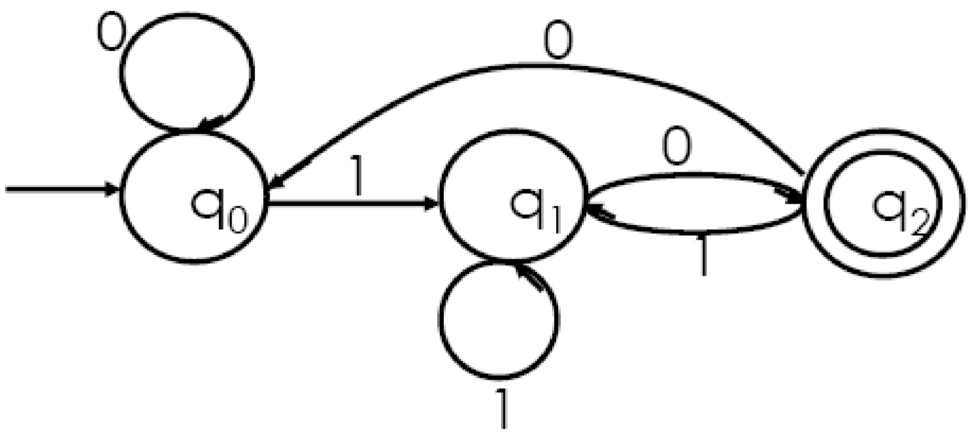


$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

example

00111

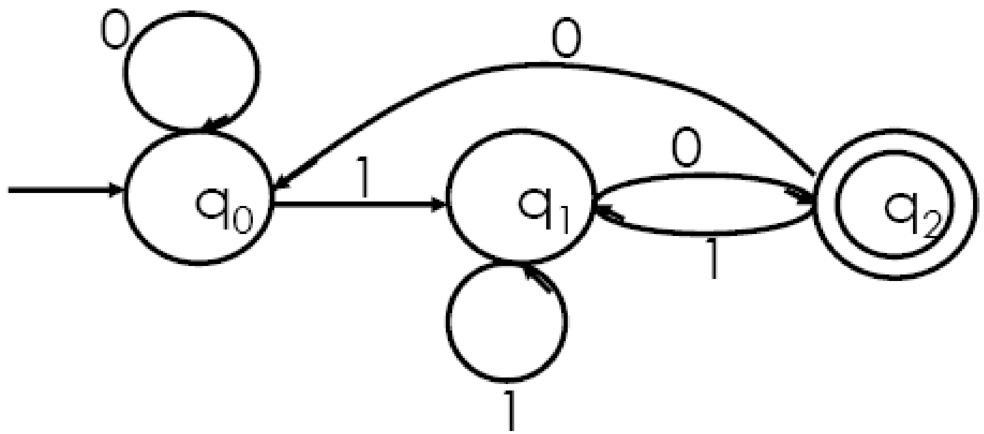


$$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1$$

not accepted by M

example

111010

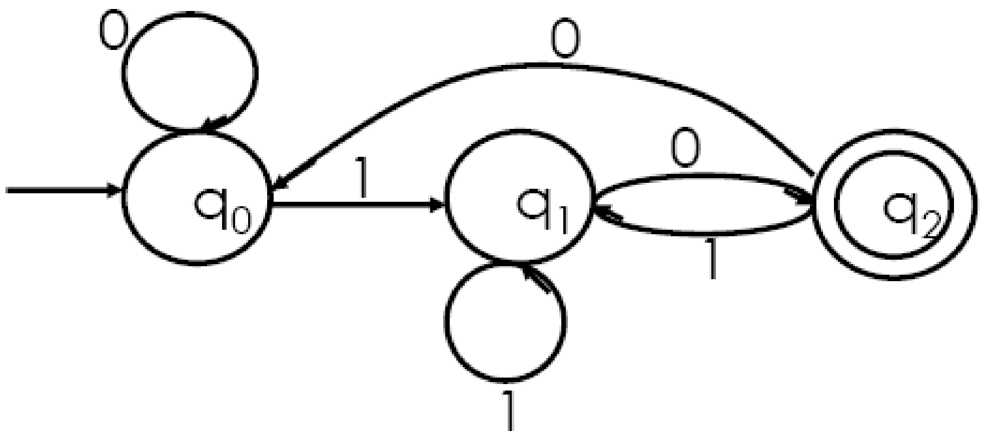


$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

example

0100

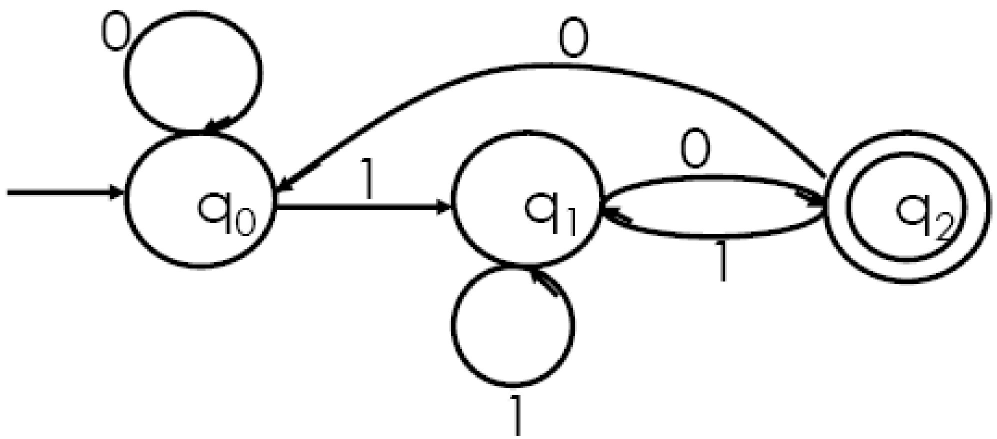


$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} q_0$

not accepted by M

example

1110



$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2$

accepted by M

example

Construct a state transition diagram of a DFA that accepts on {a,b} that contain an even number of a's and an odd number of b's.

Example of accepted strings:

aab, baa, baaabba

example

4 states,

q_0	even num. of a's & even num. of b's.
q_1	even num. of a's & odd num. of b's.
q_2	odd num. of a's & odd num. of b's.
q_3	odd num. of a's & even num. of b's.

$S = \{q_0, q_1, q_2, q_3\}$

example

set of states, $S = \{q_0, q_1, q_2, q_3\}$

set of input symbols, $I = \{a, b\}$

initial state, q_0

final state, q_1

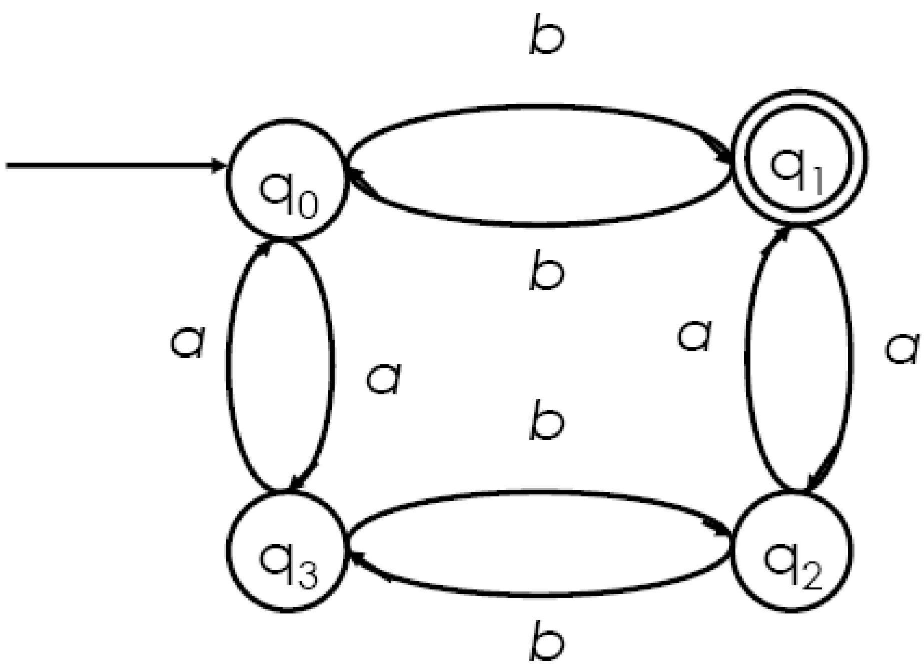
example

State transition function

f_s	a	b
q_0	q_3	q_1
q_1	q_2	q_0
q_2	q_1	q_3
q_3	q_0	q_2

example

State transition diagram



exercise

Let $M=(S, I, q_0, f_s, F)$ be the DFA such that $S=\{q_0,q_1,q_2\}$, $I=\{a,b\}$, $F=\{q_2\}$, q_0 =initial state, and f_s is given by,

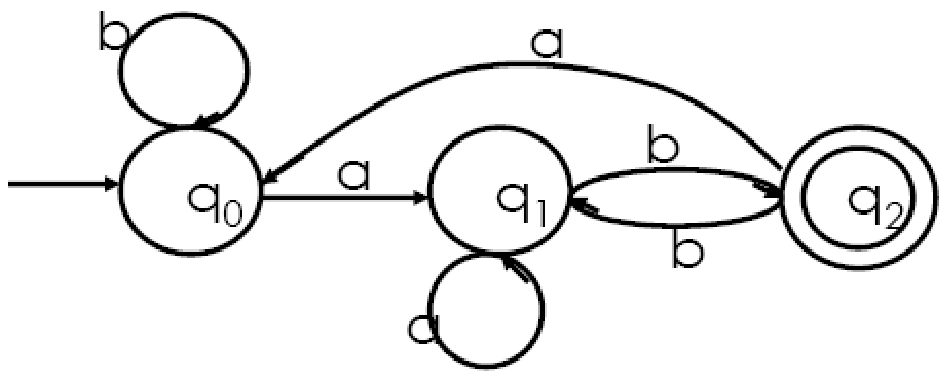
f_s	a	b
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

Draw the state diagram of M.

Which of the strings
abaa, bbbabb, bbbaa dan bababa
are accepted by M?

exercise

The transition diagram of M is,



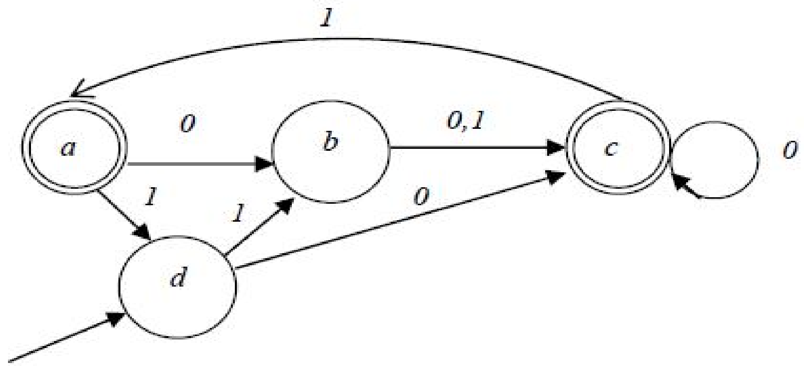
Construct the transition table of M.
Which of the strings
baba, baab, abab dan abaab
are accepted by M?

Exercise

- Construct DFA M with the input set{0,1} such that M accepts all and only those string that contain **001**
- Construct DFA M with the input set{0,1} such that M accepts an even number of 1 and any number of 0
- Construct DFA, M, with the input set{a,b,c} such that M accepts any string with **aab** as substring

Exercise

a) Let $M = (Q, I, q_0, \delta, \sigma)$ be the Deterministic Finite Automaton (DFA) with state transition diagram shown in Figure 3.



i) Find the initial state, set of input symbols and set of final states.

ii) Write the transition table for this DFA.

iii) Determine the state that the machine ends for the input string 0101011.

iv) Is the input string 1101 accepted by the DFA?

Exercise

a) Let M=(S, I, q_0, f_s, F) be the deterministic finite automaton (DFA) such that S={q_0, q_1, q_2, q_3}, I={a, b, c}, F={q_2, q_3}, q_0 is the initial state, and f_s is defined as follows:

$f_s(q_0, a) = q_0,$	$f_s(q_0, b) = q_1,$	$f_s(q_0, c) = q_1,$
$f_s(q_1, a) = q_0,$	$f_s(q_1, b) = q_2,$	$f_s(q_1, c) = q_3,$
$f_s(q_2, a) = q_2,$	$f_s(q_2, b) = q_1,$	$f_s(q_2, c) = q_3,$
$f_s(q_3, a) = q_0,$	$f_s(q_3, b) = q_0,$	$f_s(q_3, c) = q_0,$

i. Draw the state diagram of M.

ii. Which of the strings *abc*, *bac*, and *acb* are accepted by M?

Finite State Machines (FSM)

- Automata with input as well as output.
- Every state has an input and corresponding to the input the state also has an output.
- These types of automata are commonly called **finite state machines**.

Finite State Machines (FSM)

- A finite state machine is a sextuple,
  $M = \{ S, I, O, q_0, f_s, f_o \}$
  where,
  S is a finite nonempty set of states
  I is the input alphabet
  O is the output alphabet
  q_0 is the initial state
  f_s is the state transition function
  f_0 is the output function.

example

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM

- where,

  S = {q_0,q_1,q_2},

  I = {a,b},

  O={0,1},

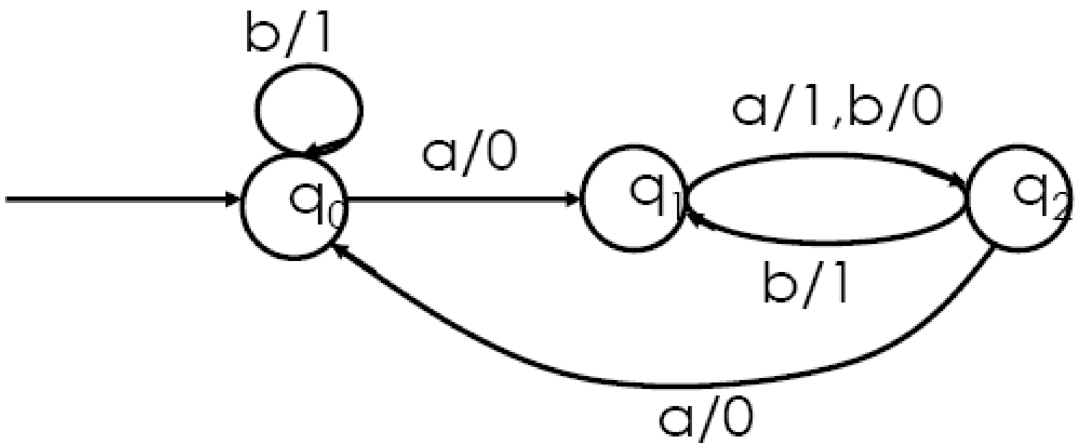
  q_0= initial state,

example

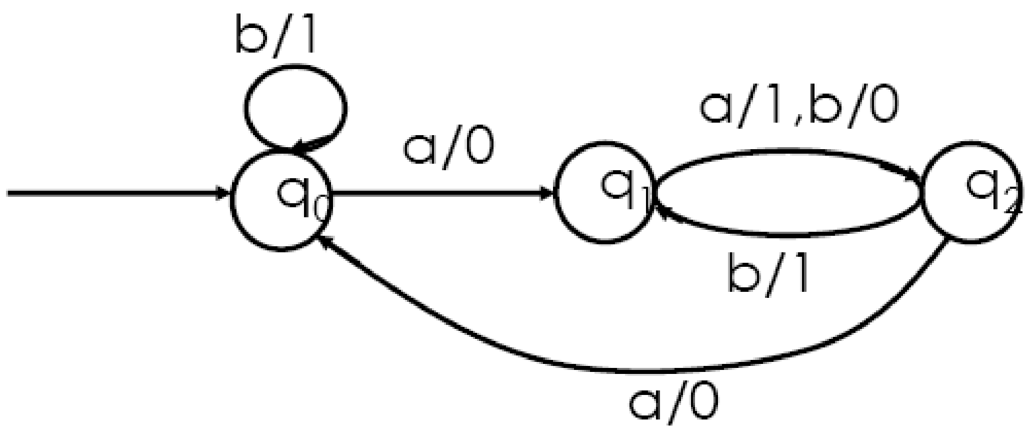
f_s and f_0

	f_s a	b	f_o a	b
q_0	q_1	q_0	0	1
q_1	q_2	q_2	1	0
q_2	q_0	q_1	0	1

example



example

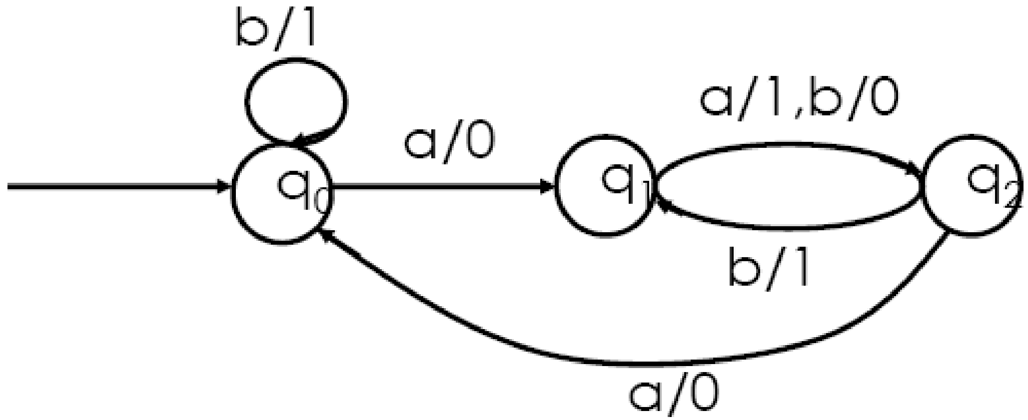


Input string: bbab

$$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2$$

Output string: 1100 Output: 0

example



Input string: babababaa

$$q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2$$

1 0 0 0 1 0 1

Output string: 1000101

Output: 1

example

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM

- where,

  S ={q_0,q_1 ,q_2 ,q_3},

  I ={a,b},

  O ={0,1},

  q_0 = initial state,

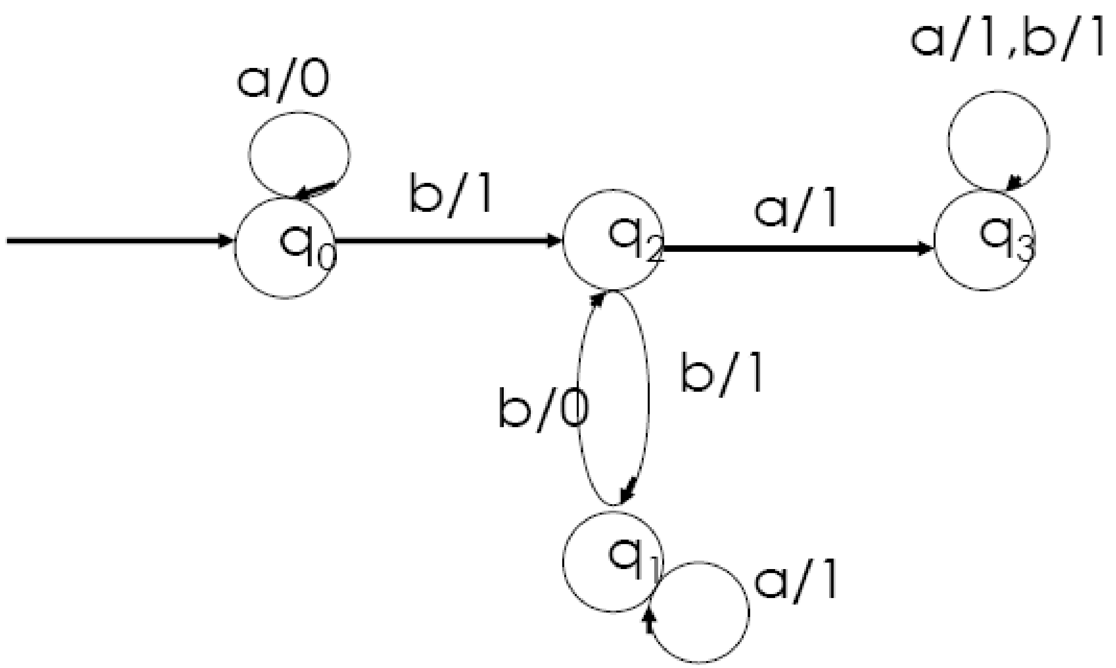
example

- f_s and f_0

	f_s a	b	f_o a	b
q_0	q_0	q_2	0	1
q_1	q_1	q_2	1	0
q_2	q_3	q_1	1	1
q_3	q_3	q_3	1	1

example

- Draw the transition diagram of M.



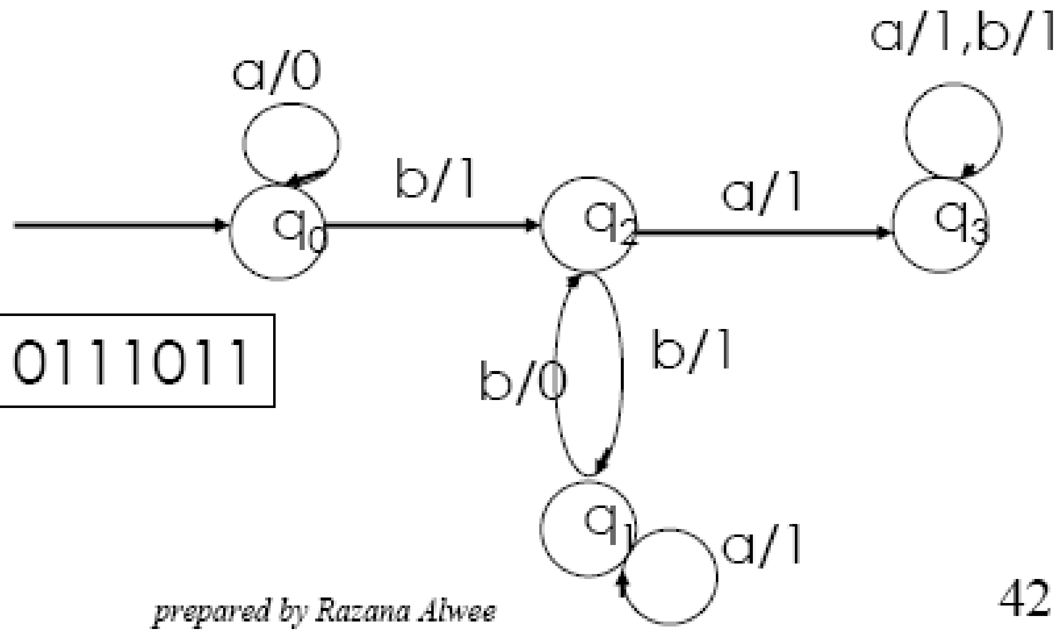
example

- What is the output string if the input string is *abbabab*?

abbabab

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_2 \xrightarrow{b} q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_3 \xrightarrow{b} q_3$$

0 1 1 1 0 1 1



Output string: 0111011

example

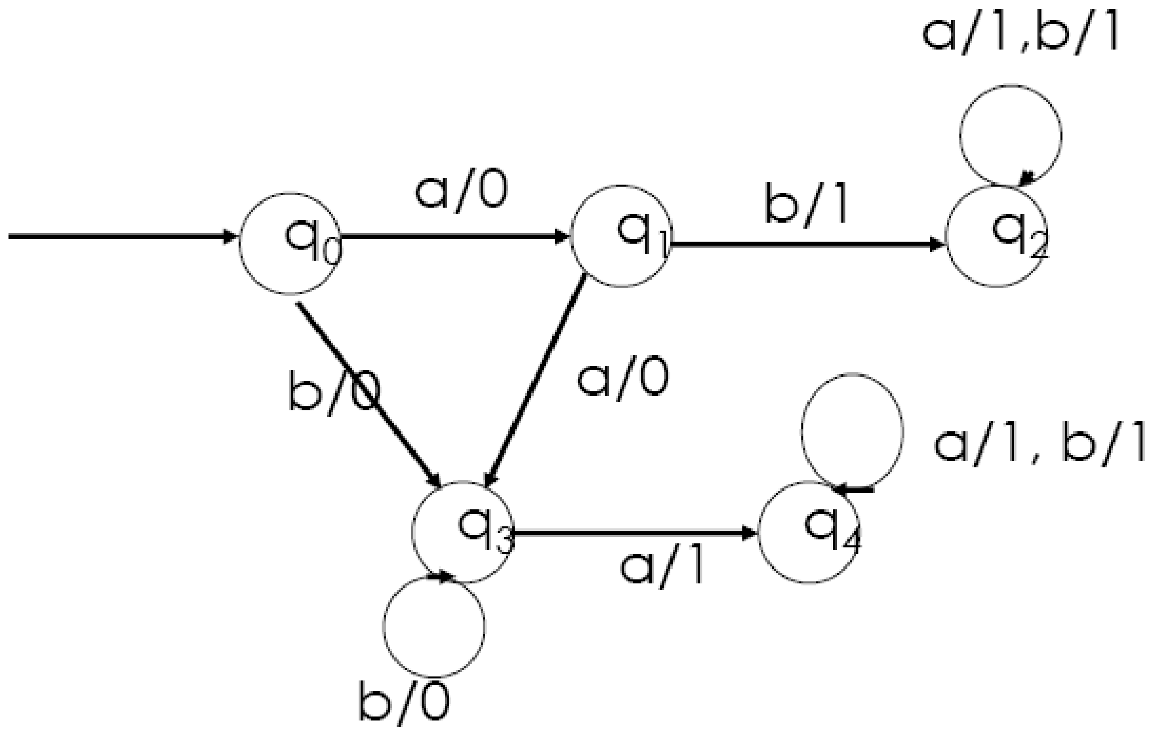
- What is the output of *abbabab*?

  Output: 1

Finite State Machines (FSM)

- Let M be a FSM.
- Let x be a nonempty string in M.
- We say that x is accepted by M if and only if the output of x is 1.

example



example

- Write the transition table of M.
- What is the output string if the input string is *aaabbbb*?
- What is the output if the input string is *bbbaaaa*?

example

- Is the string *aaa* accepted by M?
- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

example

- The transition table of M.

	f_s a	b	f_o a	b
q_0	q_1	q_3	0	0
q_1	q_3	q_2	0	1
q_2	q_2	q_2	1	1
q_3	q_4	q_3	1	0
q_4	q_4	q_4	1	1

example

- What is the output string if the input string is *aaabbbb*?

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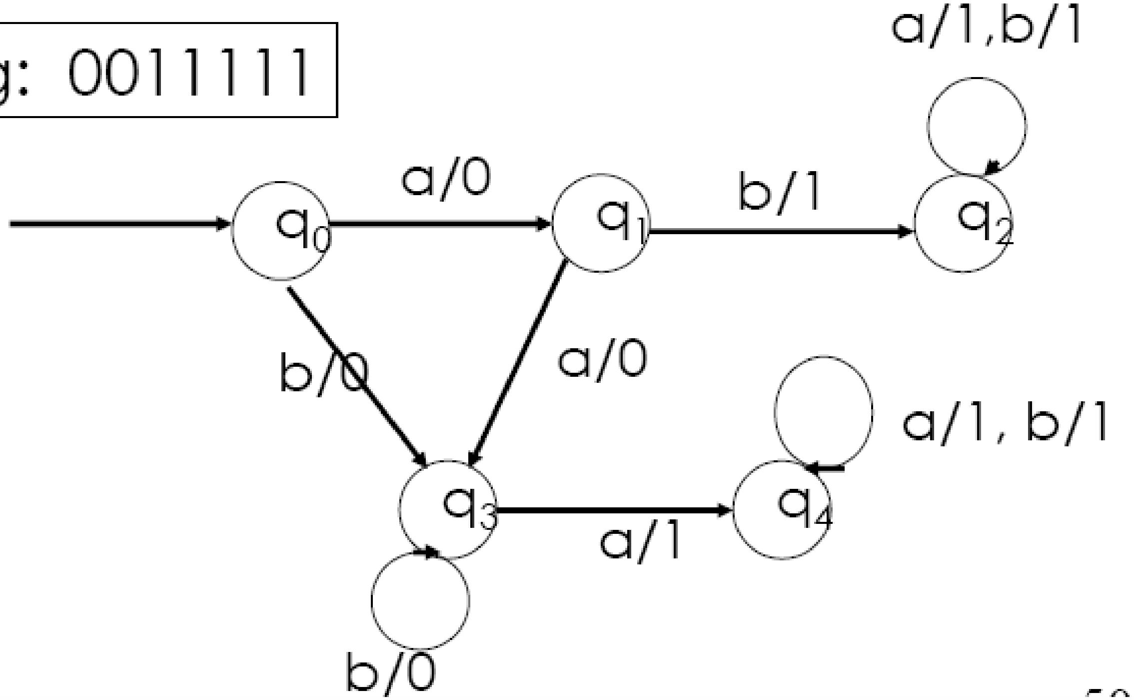
aaabbbb

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4$

0	0	1	1	1	1	1

Output string: 0011111



example

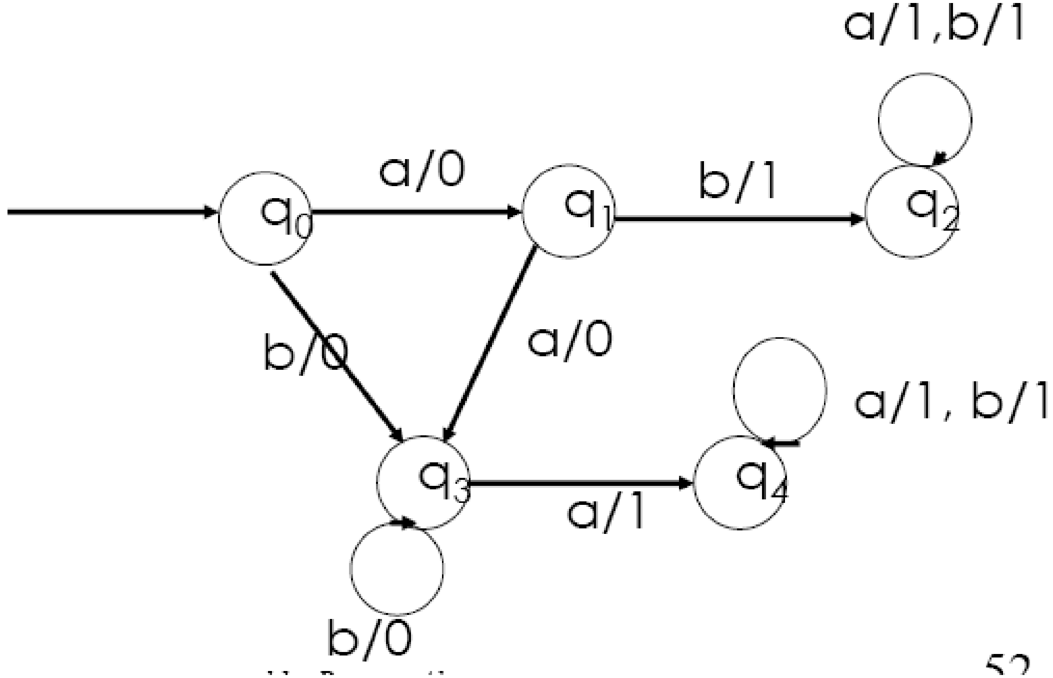
- What is the output if the input string is *bbbaaaa*?

bbbaaaa

$$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{a} q_4 \xrightarrow{a} q_4 \xrightarrow{a} q_4$$

0 0 0 1 1 1 1

Output: 1



example

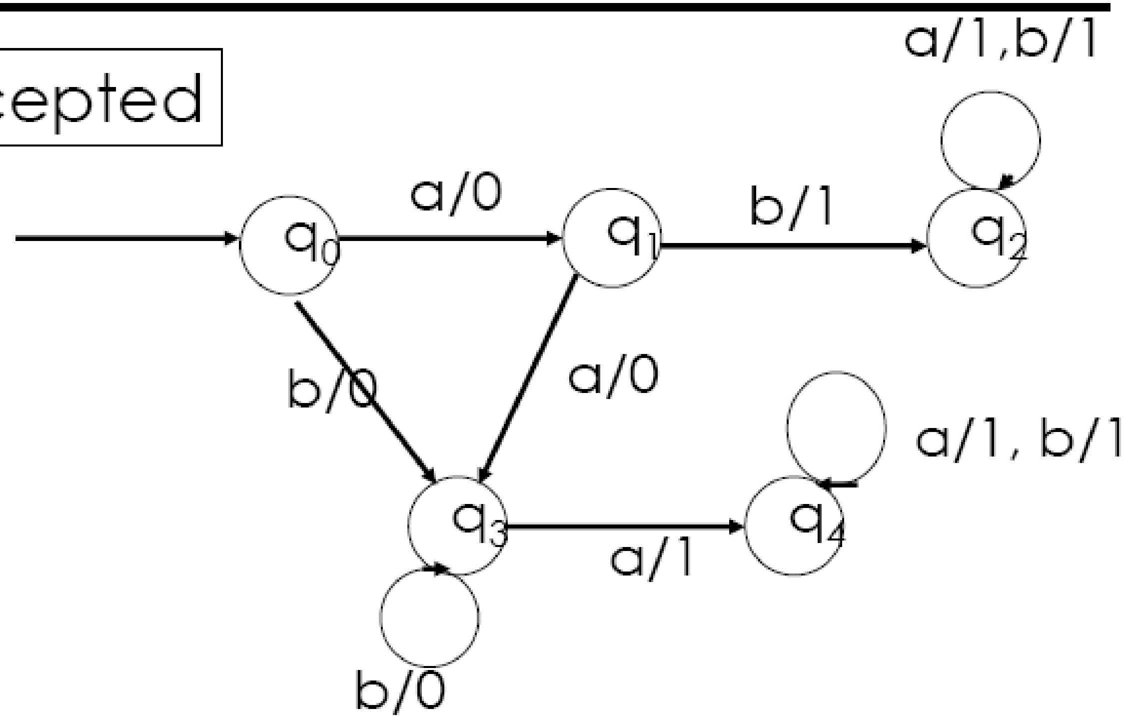
- Is the string *aaa* accepted by M?

aaa

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4$$

0 0 1

Output: 1,accepted



example

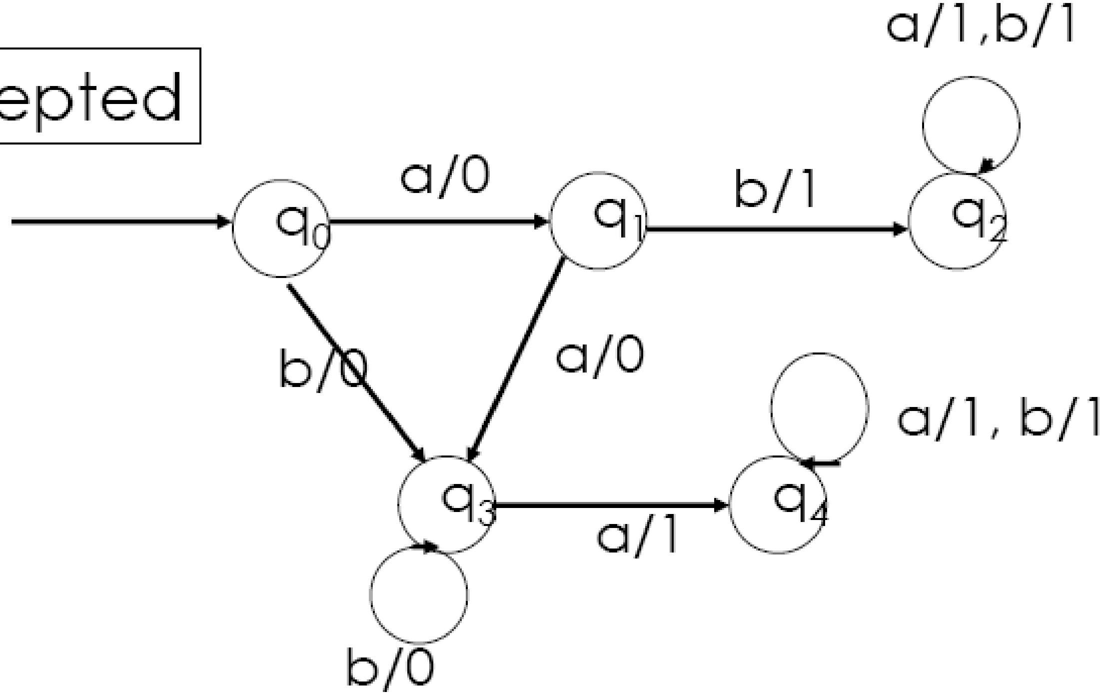
- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

ba

$q_0 \xrightarrow{b} q_3 \xrightarrow{a} q_4$

0 1

Output: 1,accepted

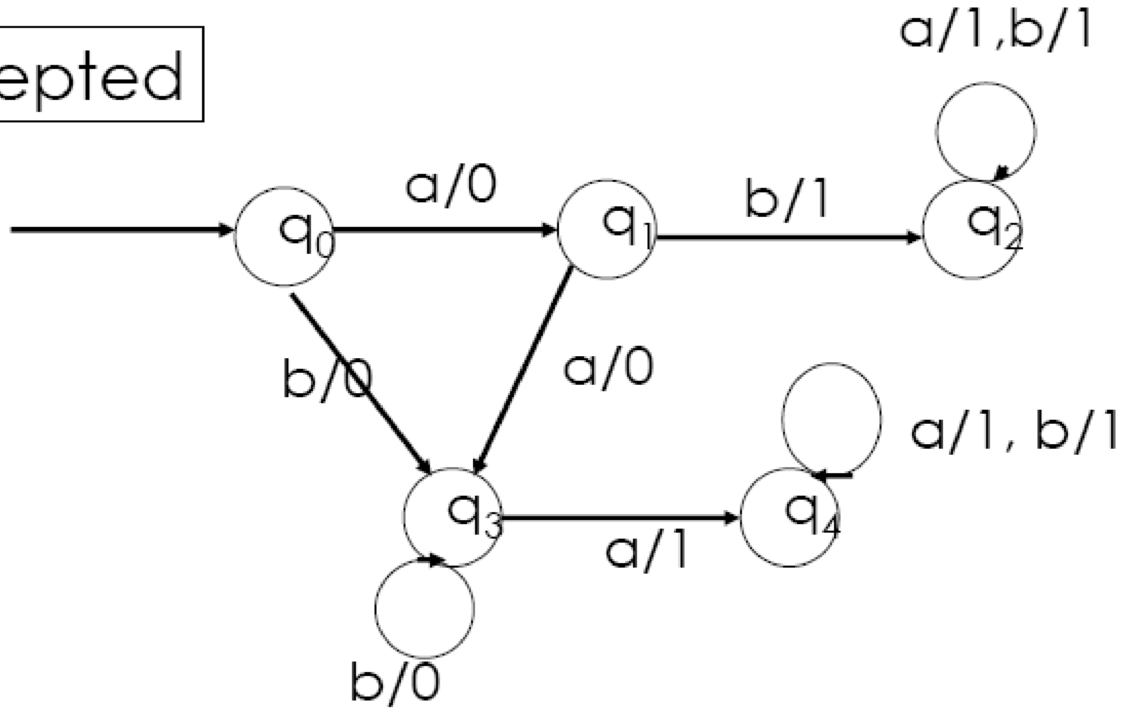


aabbba

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4$

0 0 0 0 0 1

Output: 1,accepted

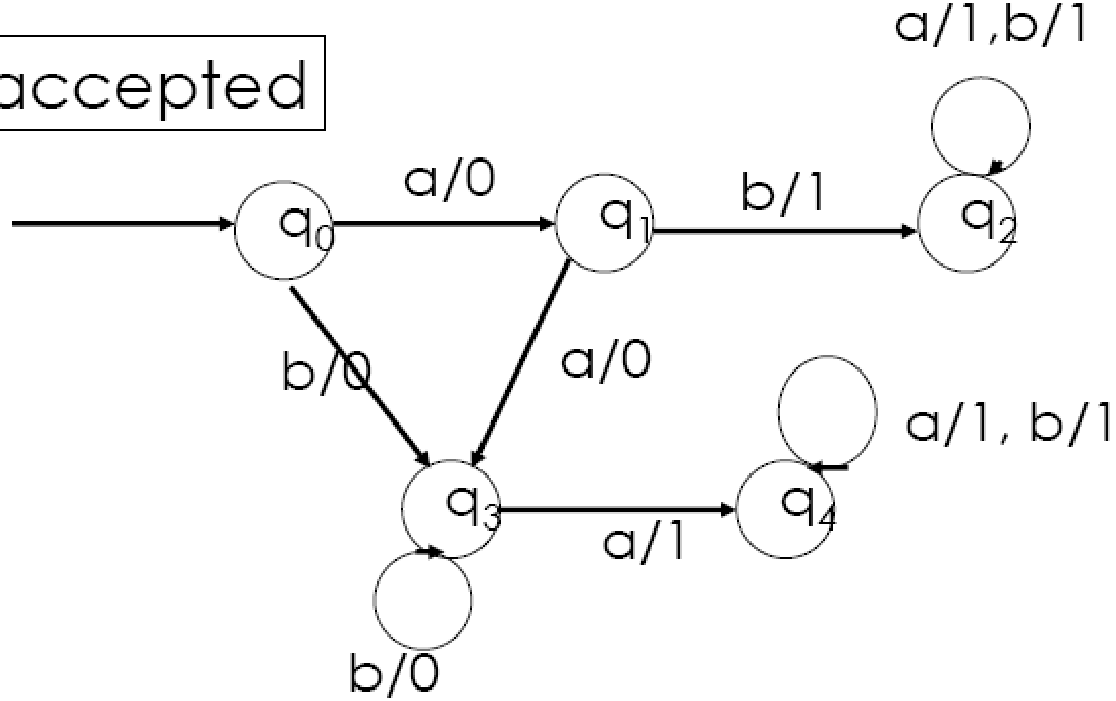


bbbb

$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3$

0 0 0 0

Output: 0,not accepted

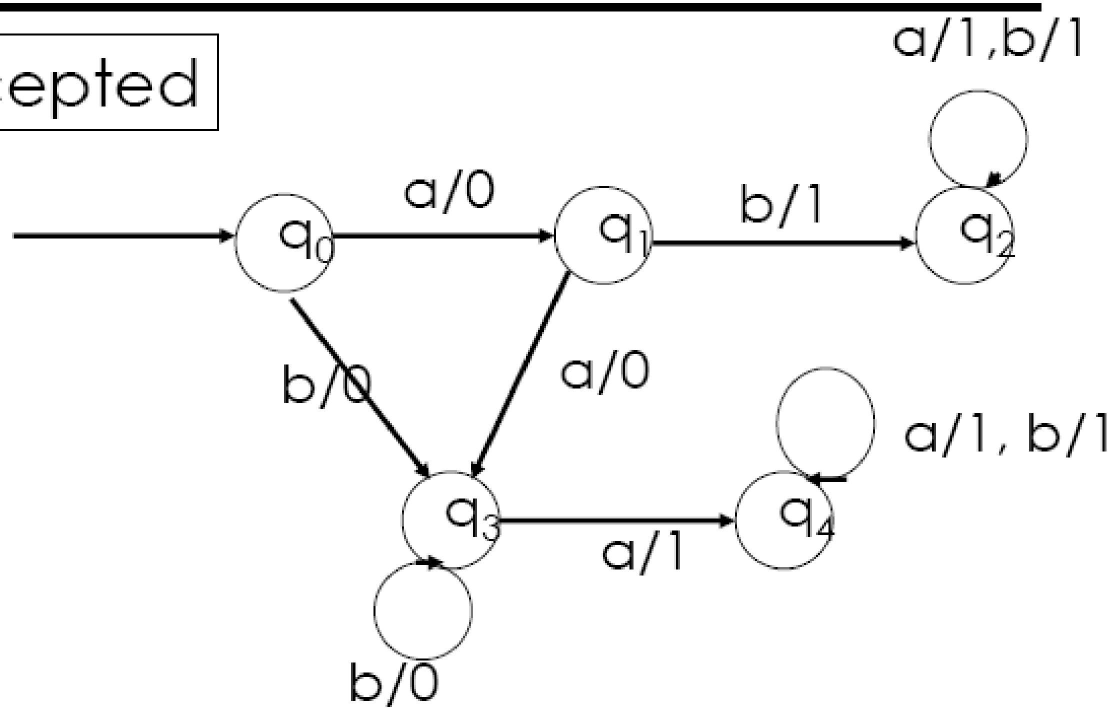


aaabbbb

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4$$

0 0 1 1 1 1 1

Output: 1,accepted



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example

- Consider a vending machine that sells candy and the cost of a candy is 50 cents.

- The machine accepts any sequence of 10-, 20-, or 50 cent coins.

- After inserting at least 50 cents, the customer can press the button to release the candy.

example

- If the customer inputs more than 50 cents, the machine does not return the change.
- After selling the candy, the machine returns to initial state.
- Construct a finite state machine that models this vending machine.

example

States,

q_0,	initial state (0)
q_1,	10 cents
q_2,	20 cents
q_3,	30 cents
q_4,	40 cents
q_5,	$\geq$ 50 cents

example

$S = \{q_0, q_1, q_2, q_3, q_4, q_5\}$,

$I = \{10, 20, 50, B\}$,

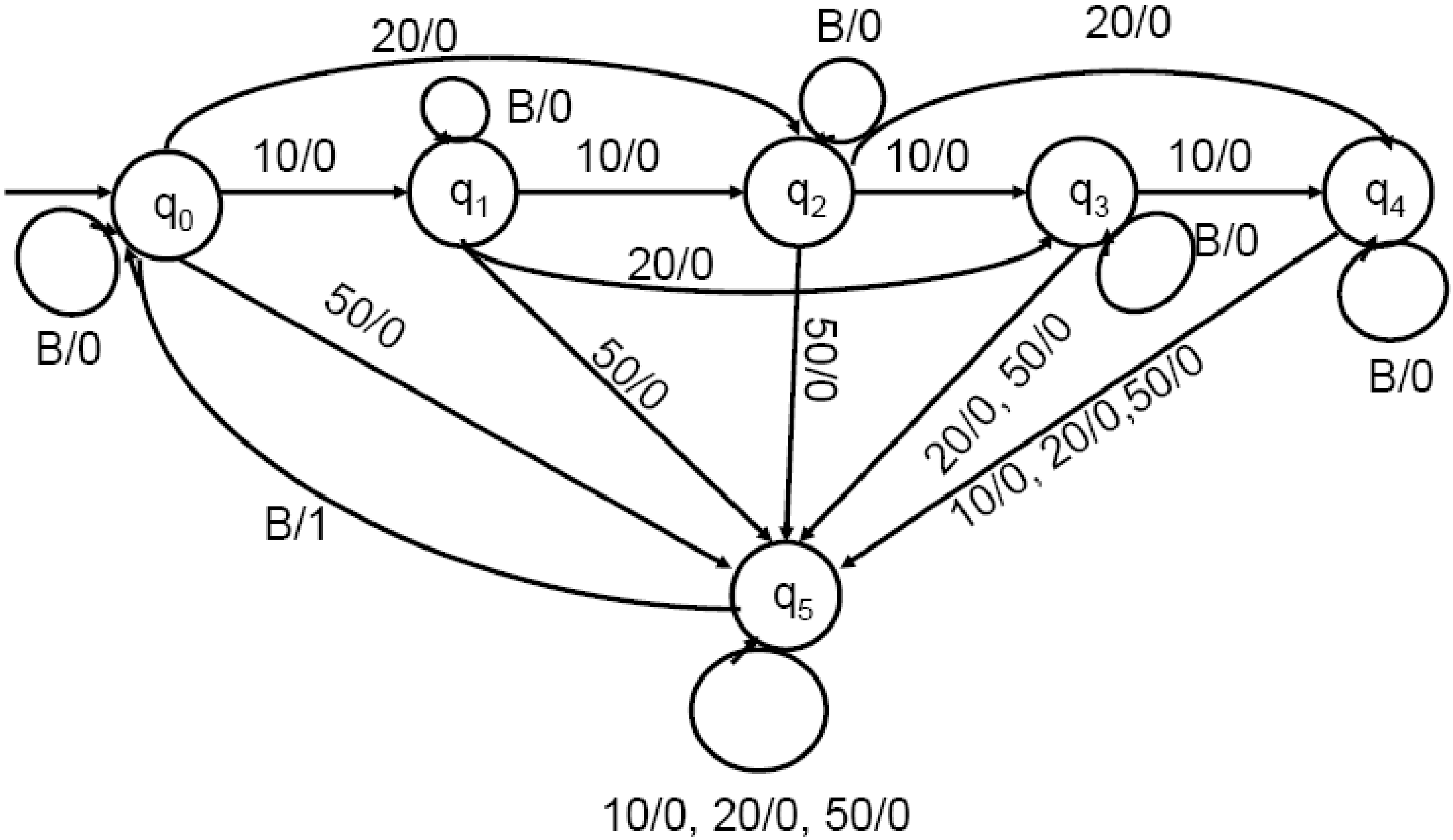
$O = \{0, 1\}$,

q_0 = initial state,

example

	f_s				f_o			
	10	20	50	B	10	20	50	B
q_0	q_1	q_2	q_5	q_0	0	0	0	0
q_1	q_2	q_3	q_5	q_1	0	0	0	0
q_2	q_3	q_4	q_5	q_2	0	0	0	0
q_3	q_4	q_5	q_5	q_3	0	0	0	0
q_4	q_5	q_5	q_5	q_4	0	0	0	0
q_5	q_5	q_5	q_5	q_0	0	0	0	1

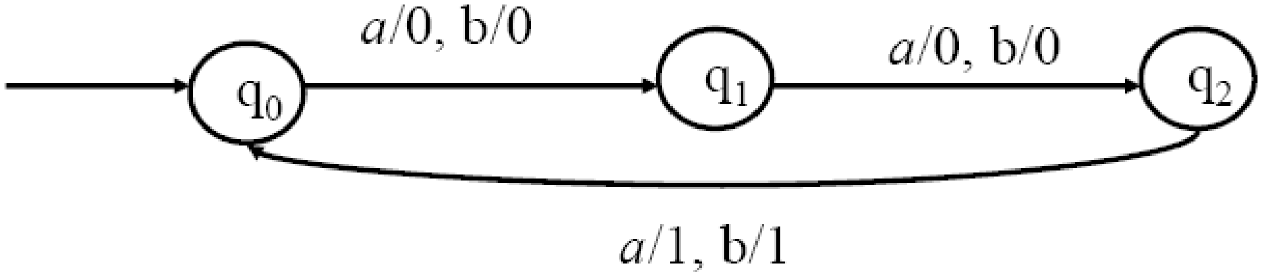
example


20/0
B/0
20/0
B/0
10/0
10/0
10/0
10/0
q0
q1
q2
q3
q4
B/0
20/0
B/0
50/0
50/0
50/0
20/0, 50/0
10/0, 20/0,50/0
B/0
B/1
q5
10/0, 20/0, 50/0

example

- Design a FSM, with input alphabet I={a, b}, that outputs a 1 if the number of input symbols read so far is divisible by 3.

example



exercise

Let M= { S, I, O, q_0, f_S, f_O} be a FSM

where,

S ={q_0,q_1,q_2},

I ={a,b},

O ={0,1},

q_0 = initial state,

exercise

f_s and f_0

	f_s		f_o	
	a	b	a	b
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

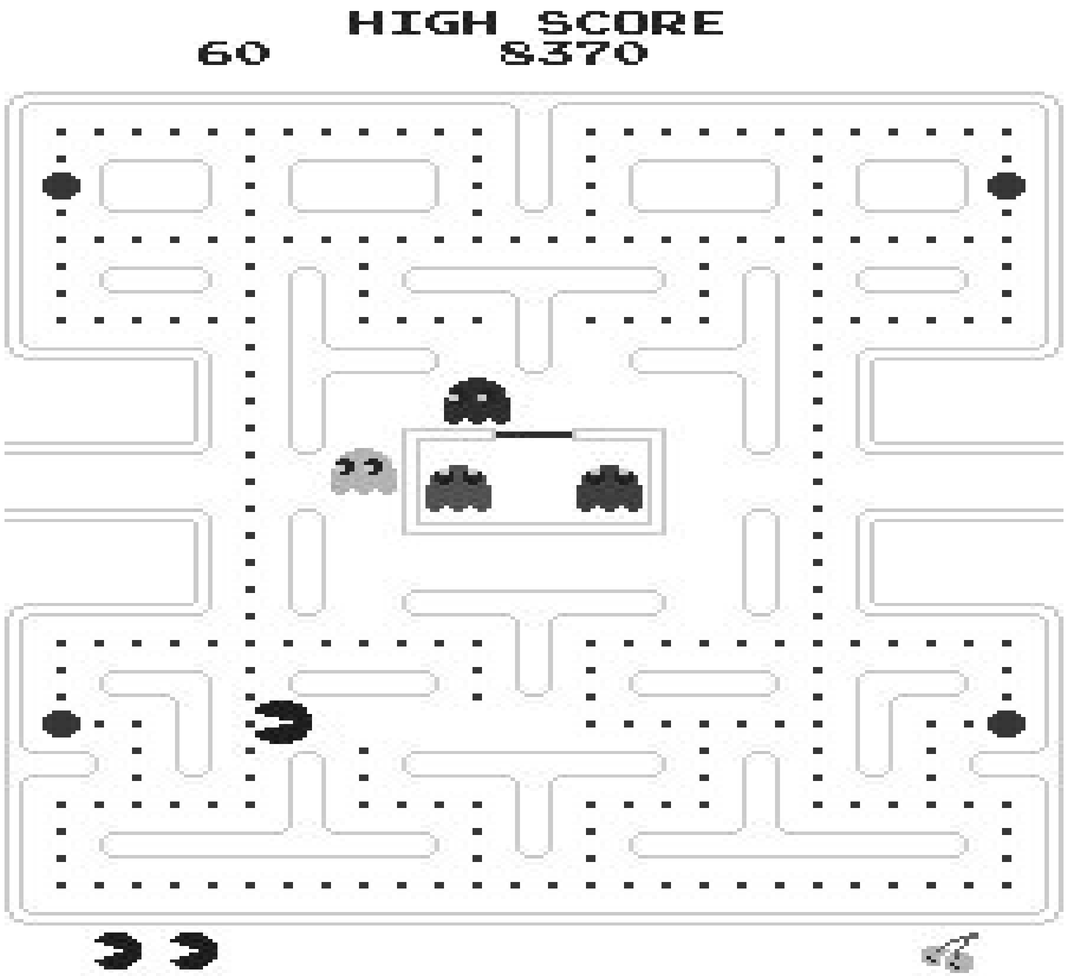
- Draw the transition diagram of M.
- What is the output string if the input string is *aabbb*?
- What is the output string if the input string is *ababab*?
- What is the output if the input string is *abbbaba*?
- What is the output if the input string is *bbbababa*?

exercise

- Design a FSM that accepts all string over {a,b} that begin with aa.
- For example: aaab, aabba, aababab

Exercise

- *Pac-Man* is an arcade game developed by a young Namco employee named Toru Iwatani and first released in Japan in 1980. It is considered one of the classics of the medium, virtually synonymous with video games, and an icon of 1980s. *Pac-Man* is one of the few games to have been consistently published for over three decades, having been remade on numerous platforms and spawned many sequels. Re-releases include ported and updated versions of the original arcade game.
- The typical version of Pac-Man is a one player game where he/she manoeuvres the Pac-Man around the maze, attempting to avoid four ‘ghosts’ characters while eating dots that distributed throughout the maze. Among the dots, there are four super dots that located at four corners of the maze. If the Pac-Man collides with the ghost, he loses one of his three lives and play resumes with the ghosts reassigned to their initial starting location. When Pac-Man eats a super dot, he is able to chase the ghosts for a few seconds of time before the super dot expires. The game ends when Pac-Man has lost all his three lives. Figure x shows a screenshot of the Pac-Man game.


HIGH SCORE
60
8370

Cont.,

- Noted that the ghosts in Pac-Man have four behaviours:
  - S_1 : randomly wander the maze
  - S_2 : chase Pac-Man when he is within the line of sight
  - S_3 : avoid Pac-Man when has consumed a super dot
  - S_4 : return to the initial position to restart the game
- The inputs are:
  - A : spot Pac-Man (Pac-Man is within the line of sight)
  - B : lose Pac-Man (Pac-Man is not within the line of sight)
  - C : Pac-Man eats super dot
  - D : super dot expires
  - E : collides with Pac-Man
  - F : reach the initial position
- The outputs are:

0 : nothing happened

1 : Pac-Man loses his life

2 : number of ghosts reduces by 1

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Cont.,

- Complete the transition table below.

State	Input, f_s						Output, f_o					
	A	B	C	D	E	F	A	B	C	D	E	F
S_1												
S_2												
S_3												
S_4												