

UNIVERSITI TEKNOLOGI MALAYSIA

TEST 1

SEMESTER I - 2015/2016

SUBJECT CODE : SCS11013

SUBJECT NAME : DISCRETE STRUCTURE

YEAR/COURSE :

TIME : 2 HOURS

DATE :

VENUE :

INSTRUCTIONS TO THE STUDENTS:
Answer all questions in the given answer booklet.

Name	
I/C No.	
Year/Course	
Section	
Lecturer Name	

This questions paper consists of **6** printed pages including this page.

Question 1 [5 Marks]

- a) Calculate the size (cardinality) of these sets, given $A=\{1, 3, 5\}$ and $B=\{2, 4, 6, 8\}$. This can be done using only the facts that $|A|=3$ and $|B|=4$. You do not need to list all elements of the new sets.
- $|A \times B|$
 - $|P(A) \times P(B)|$
- (2 marks)
- b) Let A and B be sets. Use properties of sets to show that $(A \cap B) \cup (A' \cup B)' = A$.
- (3 marks)

Question 2 [15 marks]

- a) Let $D=\{1, 3, 5, 7, 8, 9\}$. Decide whether each of the following statements is true for all elements of D . For each that is not, give a counterexample. That is, provide a number in D for which the statement is not true.
- x is even and $x > 7$
 - x is odd or $x > 7$
 - x is not odd and $x \leq 7$
 - x is odd
- (4 marks)
- b) Use truth table to check if the symbolic logic statements are equivalent.

$$p \wedge (p \rightarrow q) \quad \text{and} \quad p \wedge q$$

(3 marks)

- c) Let $P(x)$ be the statement " x can speak Arabic" and let $Q(x)$ be the statement " x knows computer language C++". Express each of these sentences in term of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your faculty.
- There is a student at your faculty who can speak Arabic and who knows C++.
 - Every student at your faculty either can speak Arabic or knows C++.
- (4 marks)

d) For each of the compound propositions, state either TRUE or FALSE if p and r are TRUE and q is FALSE.

i. $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

ii. $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$

(4 marks)

Question 3 [20 Marks]

a) Let $A = \{3,4,5\}$ and $B = \{2,4,6,8,10\}$ and R be the relation from A to B defined by aRb if and only if $2a \leq b + 1$.

i. List the ordered pair of the relation R .

ii. Find the domain and range.

(4 marks)

b) Let $A = \{1,2,3,4,5\}$ and the relation R on A represented by the digraph in Figure 1.

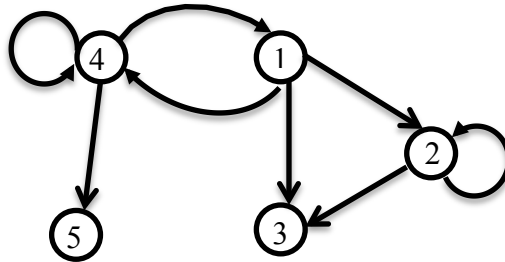


Figure 1

Based on the digraph in Figure 1, determine:

i. The in-degree and the out-degree of each vertex.

(3 marks)

ii. Matrix relation of R , M_R

(2 marks)

c) Determine whether the relation $R = \{(1,2),(1,4),(4,5),(5,3),(1,3),(2,2)\}$ on set $D = \{1,2,3,4,5\}$ is asymmetric? Explain.

(4 marks)

- d) Determine whether the relation R on $A = \{a, b, c, d\}$ whose matrix M_R is given below is an equivalence relation? Explain.

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

(7 marks)

Question 4 [20 marks]

- a) Let $J = \{0, 1, 2, 3, 4\}$ and define a function $f: J \rightarrow J$ as follows: For each $x \in J$, $f(x) = (x^3 + 2x + 4) \bmod 5$. Find the following:

i. $f(0)$

ii. $f(1)$

(2 marks)

- b) Let $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$ and $W = \{1, 2\}$.

- i. Draw the arrow diagram to define function $f: X \rightarrow Y$ that is one-to-one but not onto.

(2 marks)

- ii. List three ordered pairs to define function $g: X \rightarrow W$ that is onto but not one-to-one.

(2 marks)

- c) Let $A = \{p, q, r, s\}$ and $B = \{1, 2, 3, 4\}$. Which of the following can represent as bijection function from A to B ? Justify your answer.

i. $g = \{(p, 2), (q, 1), (r, 4), (s, 3)\}$

ii. $f = \{(p, 1), (q, 1), (r, 2), (s, 2)\}$

iii. $h = \{(p, 1), (p, 2), (p, 3), (p, 4)\}$

(2 marks)

d) Function f and g are defined by formulas:

$$f(x) = x^3 \quad \text{and} \quad g(x) = x - 1, \text{ for all real number } x.$$

Find $g \circ f$ and $f \circ g$. Determine whether $g \circ f$ equals $f \circ g$.

(6 marks)

e) Write a recursive algorithm to find the n term of the sequence defined by,

$$k_0 = -1, \quad k_1 = 2, \quad \text{and} \quad k_n = k_{n-1} + 2k_{n-2}, \quad \text{for } n \geq 2.$$

Then, trace the algorithm for $n = 4$.

(6 marks)

**** End of Questions ****

Properties On Sets

Commutative laws

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

Associative laws

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Absorption laws

$$A \cap (A \cup B) = A$$

$$A \cup (A \cap B) = A$$

Idempotent laws

$$A \cup A = A$$

$$A \cap A = A$$

Complement laws

$$(A')' = A$$

$$A \cup A' = U$$

$$A \cap A' = \emptyset$$

$$\emptyset' = U$$

$$U' = \emptyset$$

De Morgan's laws

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

Properties of universal set

$$A \cup U = U$$

$$A \cap U = A$$

Properties of empty set

$$A \cup \emptyset = A$$

$$A \cap \emptyset = \emptyset$$