SCSI 1013: Discrete Structure

CHAPTER 6

FINITE AUTOMATA

2018/2019 – SEM. 1

Deterministic Finite Automata (DFA)

- In computer science, we study different types of computer languages, such as Basic, Pascal, and C++.
- We will discuss a type of a language that can be recognized by special types of machines.
- A deterministic finite automaton (pl. automata) is a mathematical model of a machine that accepts languages of some alphabet.

Deterministic Finite Automata (DFA)

- Deterministic Finite Automaton is a quintuple M= { S, I, q_0, f_s, F}
  where,
  S is a finite nonempty set of states
  I is the input alphabet (a finite nonempty set of symbols)
  q_0 is the initial state
  f_s is the state transition function
  F is the set of final states, subset of S.

Note:
Tuple: is an ordered list of elements.
Quintuple: five times as much in size; e.g., $M = \{S, I, O, q_0, f_s\}$;
sextuple: six time as much in size, e.g., $M = \{S, I, O, q_0, f_s, f_0\}$.

Example 1

- Let $M=\{ \{q_0,q_1,q_2\}, \{0,1\}, q_0, f_s, \{q_2\}\}$ where f_s is defined as follows:

  $f_s(q_0,0)= q_1$, $f_s(q_1,1)= q_2$
  $f_s(q_0,1)= q_0$, $f_s(q_2,0)= q_0$
  $f_s(q_1,0)= q_2$, $f_s(q_2,1)= q_1$

- Note that for M:
  $S=\{q_0,q_1,q_2\}$, $I=\{0,1\}$, $F=\{q_2\}$
  q_0 is the initial state

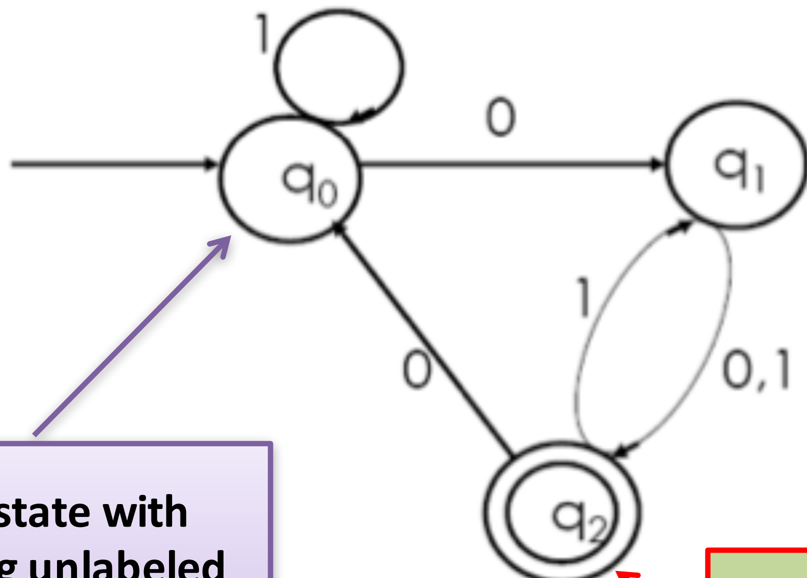
Example 1 (cont.)

- The state transition function of a DFA is often described by means of a table, called a **transition table**.

f_s	0	1
q_0	q_1	q_0
q_1	q_2	q_2
q_2	q_0	q_1

Example 1 (cont.)

- The **transition diagram** of this DFA is,



Each state represented by a small circle labeled with the state

Initial state with incoming unlabeled arrow not originating from any vertex

Final state with a double circle

Example 2

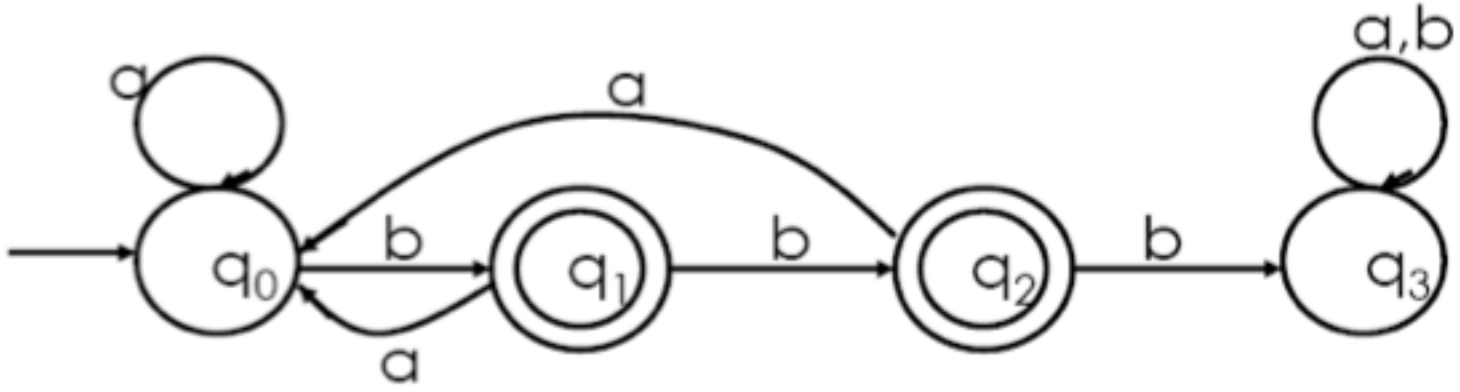
Let $M=(\{q_0,q_1,q_2,q_3\}, \{a,b\}, q_0, f_s, \{q_1,q_2\})$

where f_s is given by the table

f_s	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

Example 2 (cont.)

- The transition diagram of this DFA is,



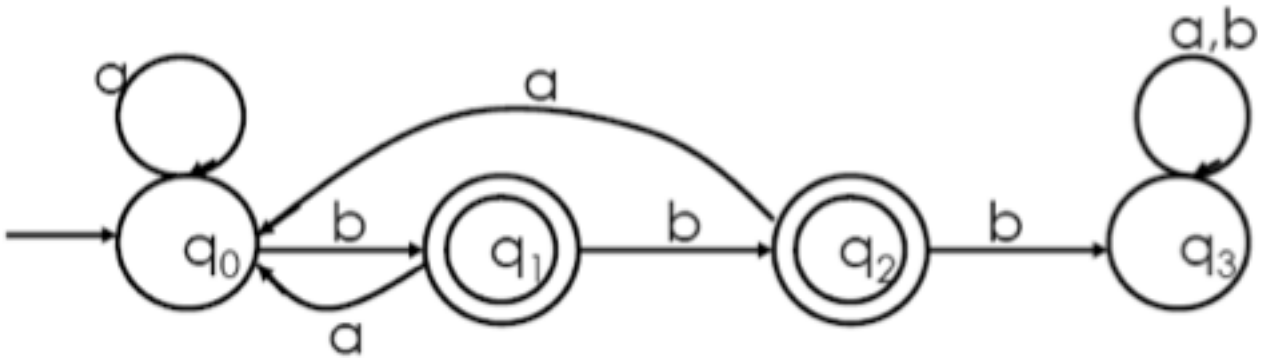
DFA – with extended transition function for M

- Let $M = \{ S, I, q_0, f_s, F\}$ be a DFA and *w* is an input string,
- *w* is said to be accepted by M if

$$f_s^*(q_0, w) \in F$$

- f_s^* - extended transition function for M

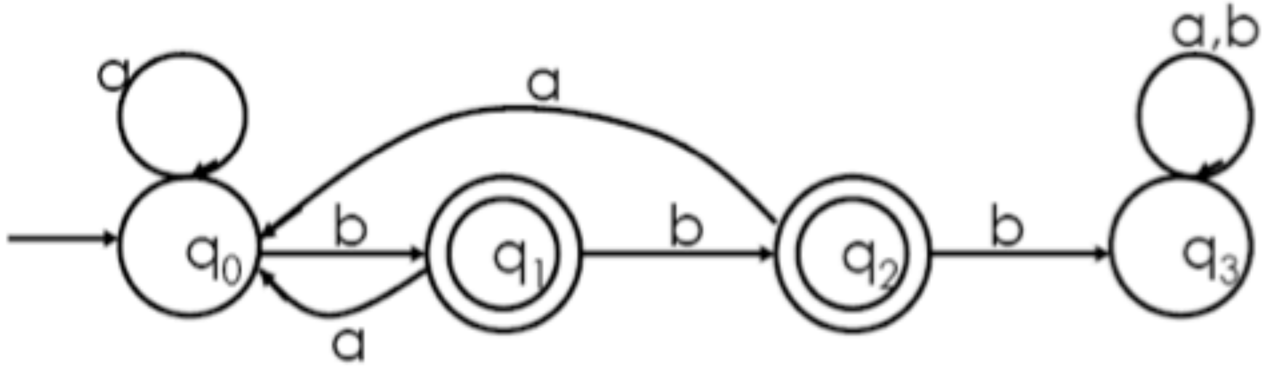
Example 3



w= abb

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2$ accepted by M

Example 4

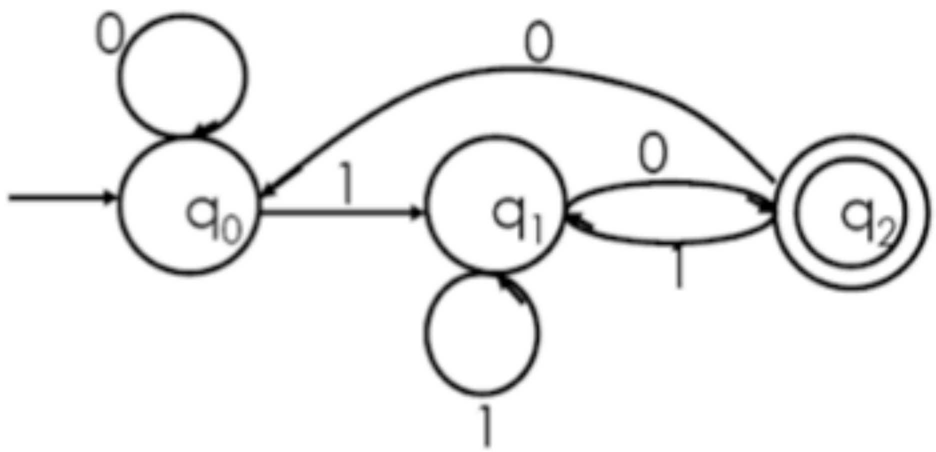


w= abba

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0$$

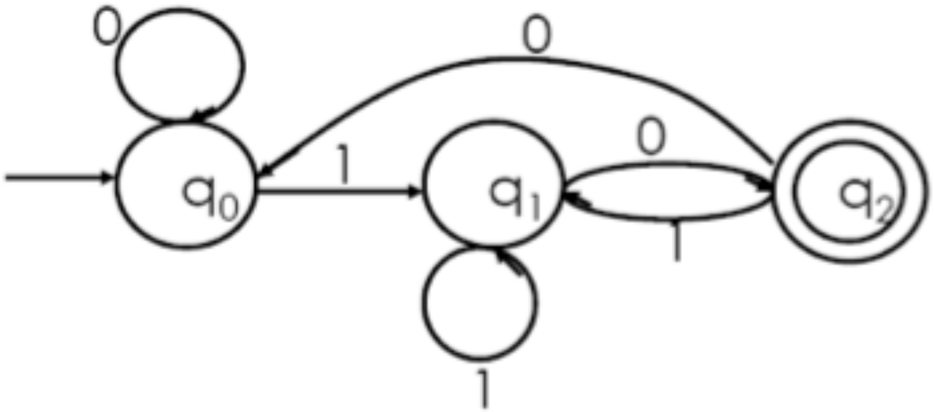
not accepted by M

Example 5



- What are the states of M?
- Write the set of input symbols.
- Which is the initial state?

Example 5 (cont.)



- Write the set of final states.
- Write the transition table for this DFA

Example 5 (cont.)

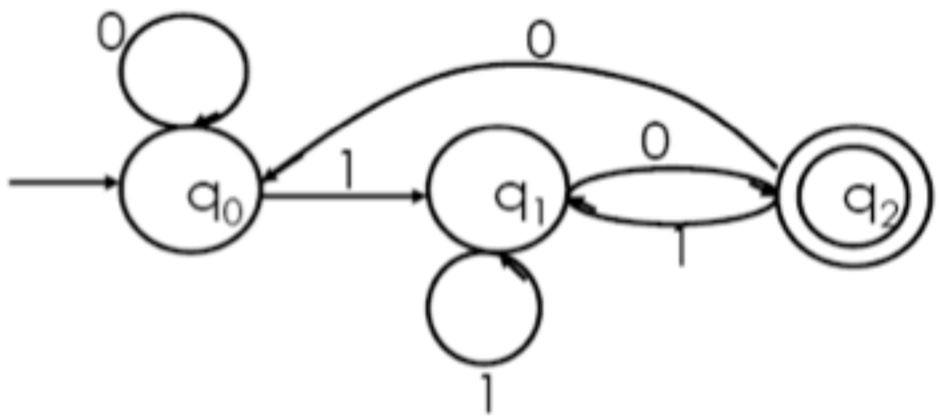
Which of the strings are accepted by M?

0111010, 00111, 111010,

0100, 1110

Example 5 - Solution

0111010

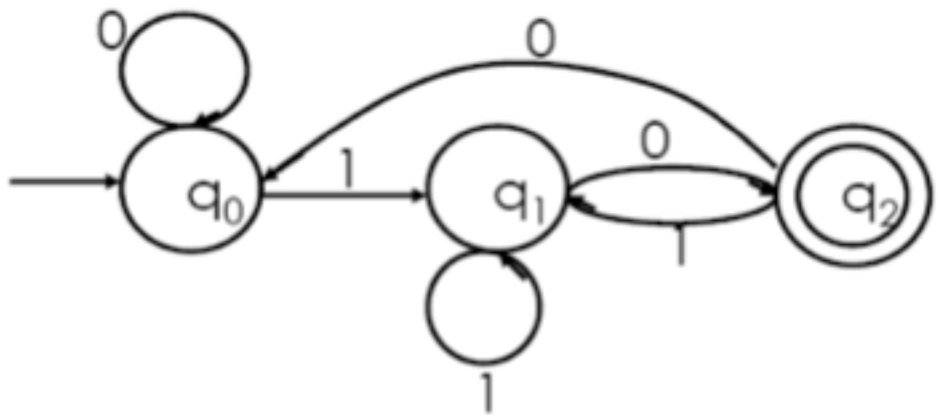


$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

Example 5 – Solution (cont.)

00111

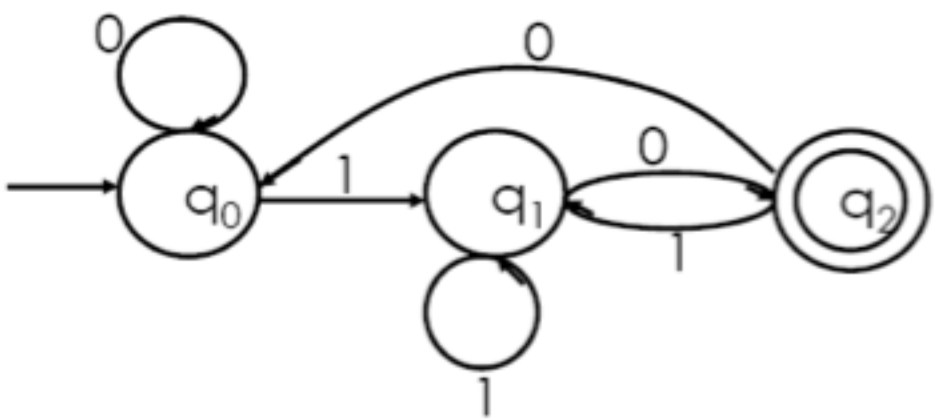


$$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1$$

not accepted by M

Example 5 – Solution (cont.)

111010

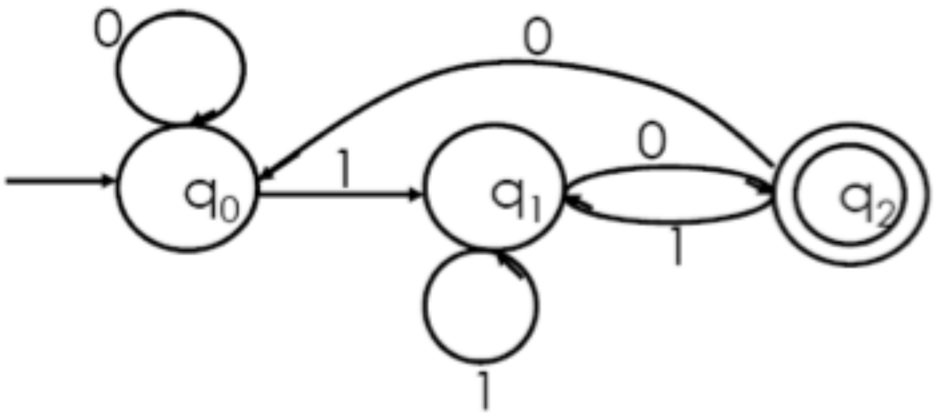


$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

Example 5 – Solution (cont.)

0100

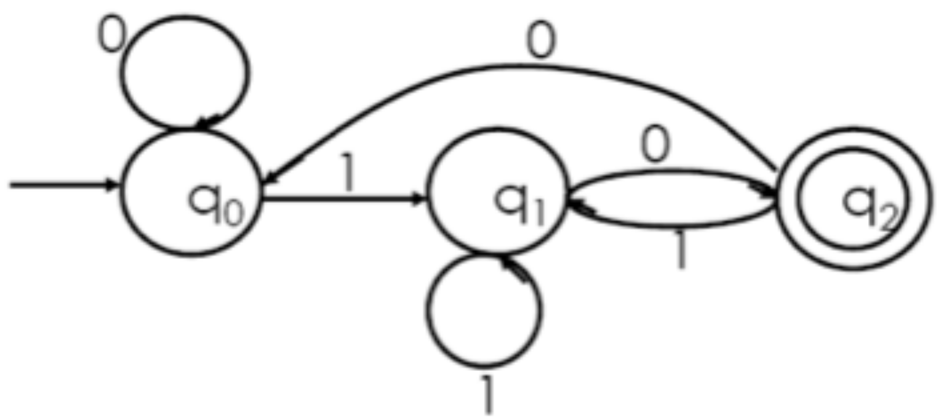


$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} q_0$

not accepted by M

Example 5 – Solution (cont.)

1110



$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2$

accepted by M

Example 6

Construct a state transition diagram of a DFA that accepts on {a,b} that contain an even number of a's and an odd number of b's.

Example of accepted strings:

aab, baa, baaabba

Example 6 - Solution

4 states,

q_0 even num. of a's & even num. of b's.
q_1 even num. of a's & odd num. of b's.
q_2 odd num. of a's & odd num. of b's.
q_3 odd num. of a's & even num. of b's.

$S = \{q_0, q_1, q_2, q_3\}$

Example 6 – Solution (cont.)

set of states, $S = \{q_0, q_1, q_2, q_3\}$

set of input symbols, $I = \{a, b\}$

initial state, q_0

final state, q_1

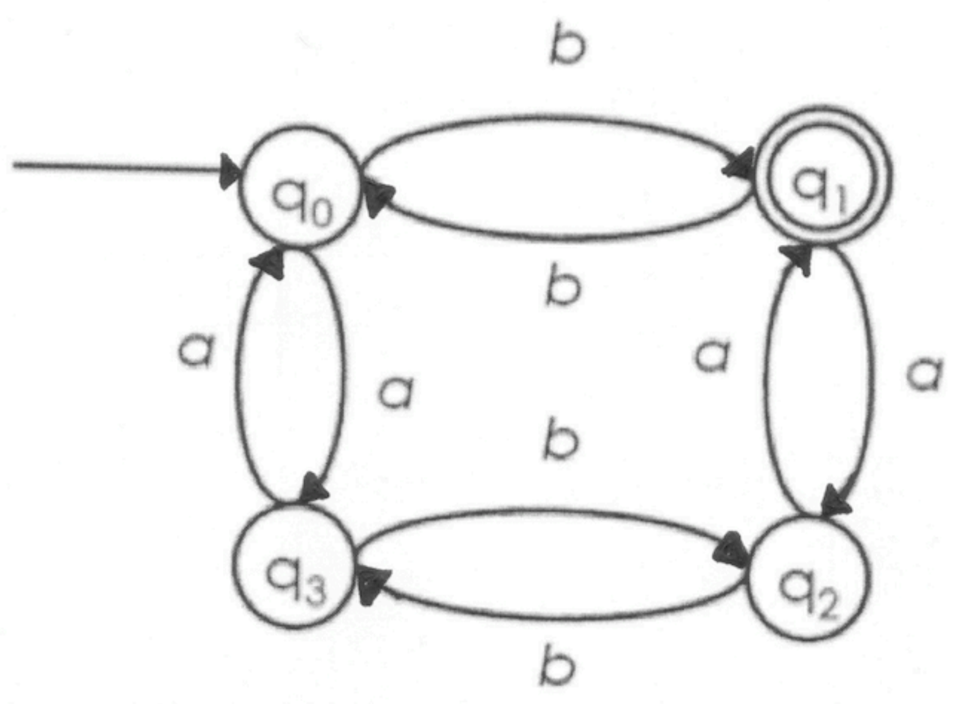
Example 6 – Solution (cont.)

State transition function

f_s	a	b
q_0	q_3	q_1
q_1	q_2	q_0
q_2	q_1	q_3
q_3	q_0	q_2

Example 6 – Solution (cont.)

State transition diagram



Exercise 1

Let $M=(S, I, q_0, f_s, F)$ be the DFA such that $S=\{q_0,q_1,q_2\}$, $I=\{a,b\}$, $F=\{q_2\}$, q_0 =initial state, and f_s is given by,

f_s	a	b
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

Draw the state diagram of M.

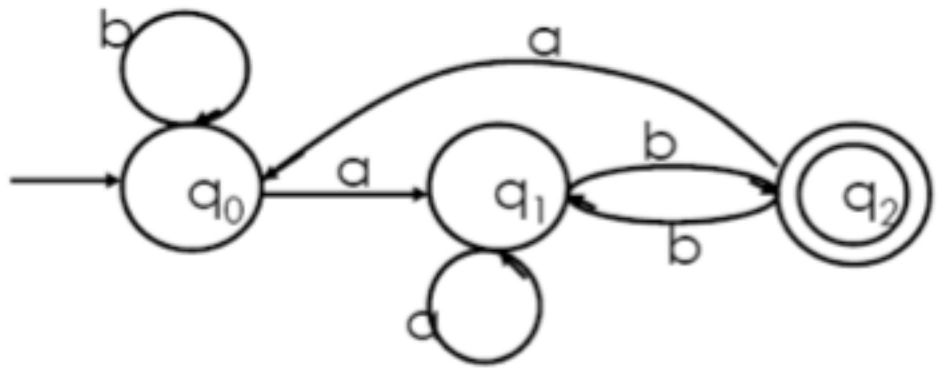
Which of the strings

abaa, bbbabb, bbbaa dan bababa

are accepted by M?

Exercise 2

The transition diagram of M is,



Construct the transition table of M.
Which of the strings
baba, baab, abab dan abaab
are accepted by M?

Exercise 3

Construct a state transition diagram of a DFA M with the input set {0,1} such that M accepts only the string 101.

Solution:

Finite State Machines (FSM)

- Automata with input as well as output.
- Every state has an input and corresponding to the input the state also has an output.
- These types of automata are commonly called **finite state machines**.

Finite State Machines (FSM)

- A finite state machine is a sextuple,
  $M = \{ S, I, O, q_0, f_s, f_o \}$
  where,
  S   is a finite nonempty set of states
  I   is the input alphabet
  O   is the output alphabet
  q_0 is the initial state
  f_s is the state transition function
  f_0 is the output function.

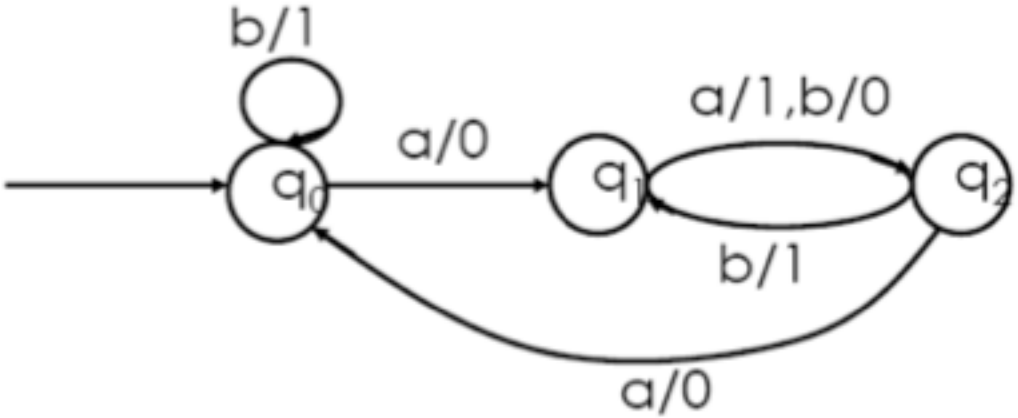
Example 1

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM
- where,
  - S = {q_0,q_1,q_2},
  - I = {a,b},
  - O={0,1},
  - q_0= initial state,
- The f_s and f_0 are =>

	f_s		f_o	
	a	b	a	b
q_0	q_1	q_0	0	1
q_1	q_2	q_2	1	0
q_2	q_0	q_1	0	1

Example 1 (cont.)

- The transition diagram:

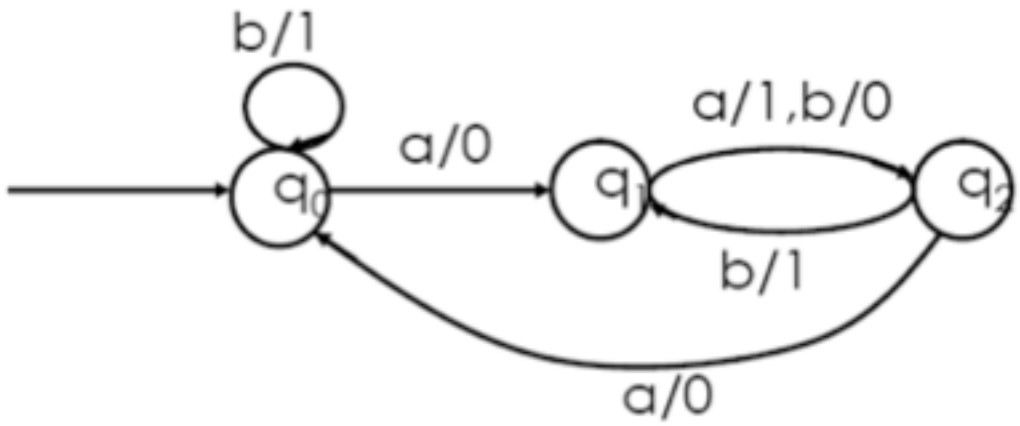


Input string: bbab

$$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2$$

Output string: 1100 Output: 0

Example 1 (cont.)



Input string: bababaa

$$q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2$$

1 0 0 0 1 0 1

Output string: 1000101

Output: 1

Example 2

- Let $M = \{ S, I, O, q_0, f_s, f_o \}$ be the FSM

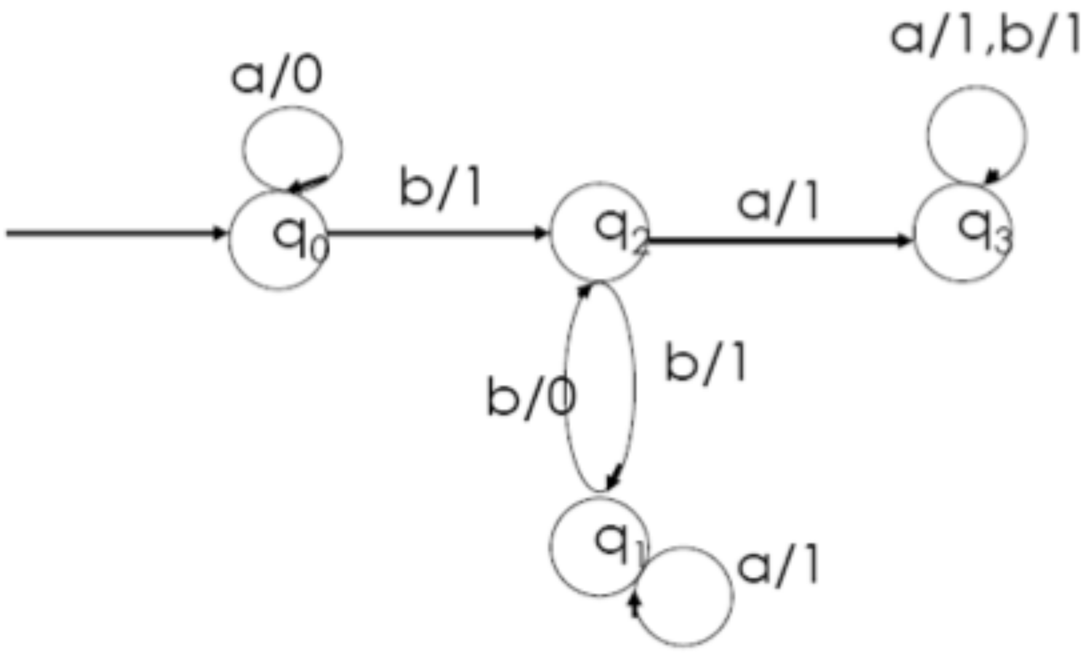
- where,
  $S = \{q_0, q_1, q_2, q_3\}$,
  $I = \{a, b\}$,
  $O = \{0, 1\}$,
  q_0 = initial state,

- f_s and f_0

	f_s		f_o	
	a	b	a	b
q_0	q_0	q_2	0	1
q_1	q_1	q_2	1	0
q_2	q_3	q_1	1	1
q_3	q_3	q_3	1	1

Example 2 (cont.)

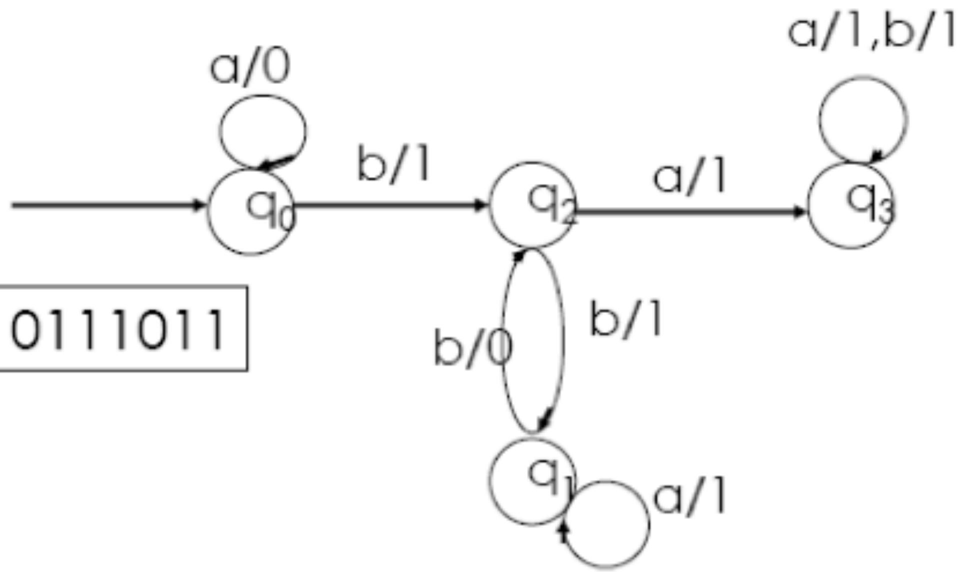
- Draw the transition diagram of M.



Example 2 (cont.)

- What is the output string if the input string is *abbabab*?

$$q_0 \xrightarrow[0]{a} q_0 \xrightarrow[1]{b} q_2 \xrightarrow[1]{b} q_1 \xrightarrow[1]{a} q_1 \xrightarrow[0]{b} q_2 \xrightarrow[1]{a} q_3 \xrightarrow[1]{b} q_3$$



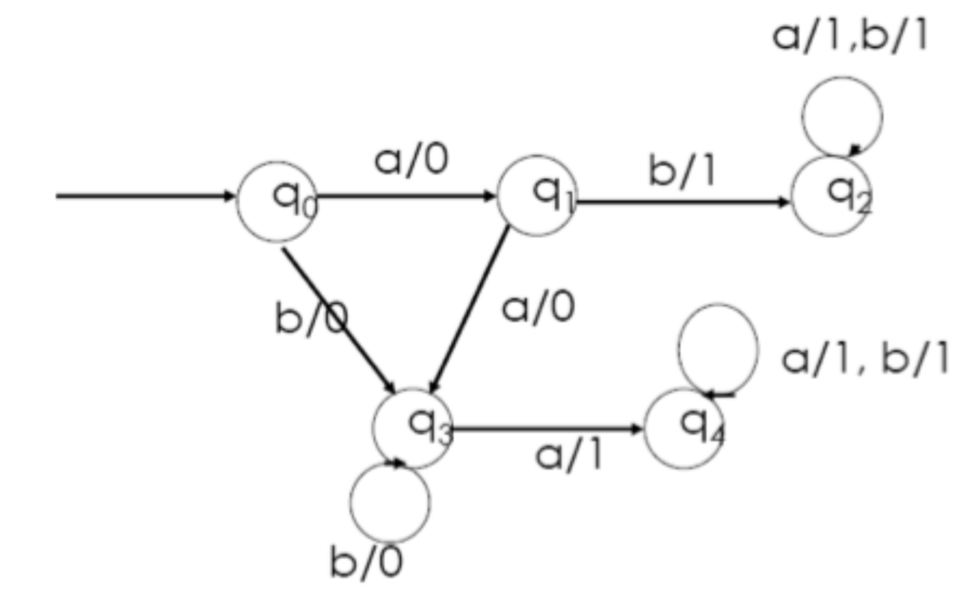
Output string: 0111011

Output : 1

Example 3

- Let M be a FSM.
- Let x be a nonempty string in M.
- We say that x is accepted by M if and only if the output of x is 1.

State transition diagram



Example 3 (cont.)

Based on the given information, answer the following:

- Write the transition table of M.
- What is the output string if the input string is *aaabbbb*?
- What is the output if the input string is *bbbaaaa*?
- Is the string *aaa* accepted by M?
- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

Example 3 - Solution

- The transition table of M.

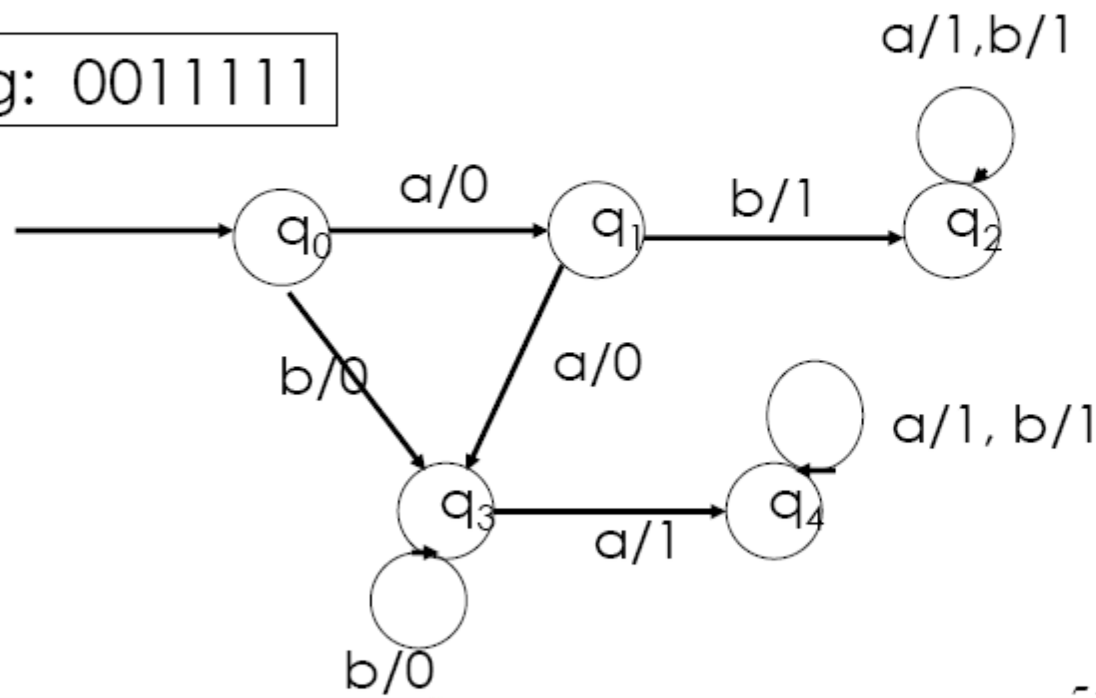
	f_s a	b	f_o a	b
q_0	q_1	q_3	0	0
q_1	q_3	q_2	0	1
q_2	q_2	q_2	1	1
q_3	q_4	q_3	1	0
q_4	q_4	q_4	1	1

Solution (cont.)

- What is the output string if the input string is **aaabbbb**?

$$q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{a} q_3 \xrightarrow[1]{a} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4$$

Output string: 0011111

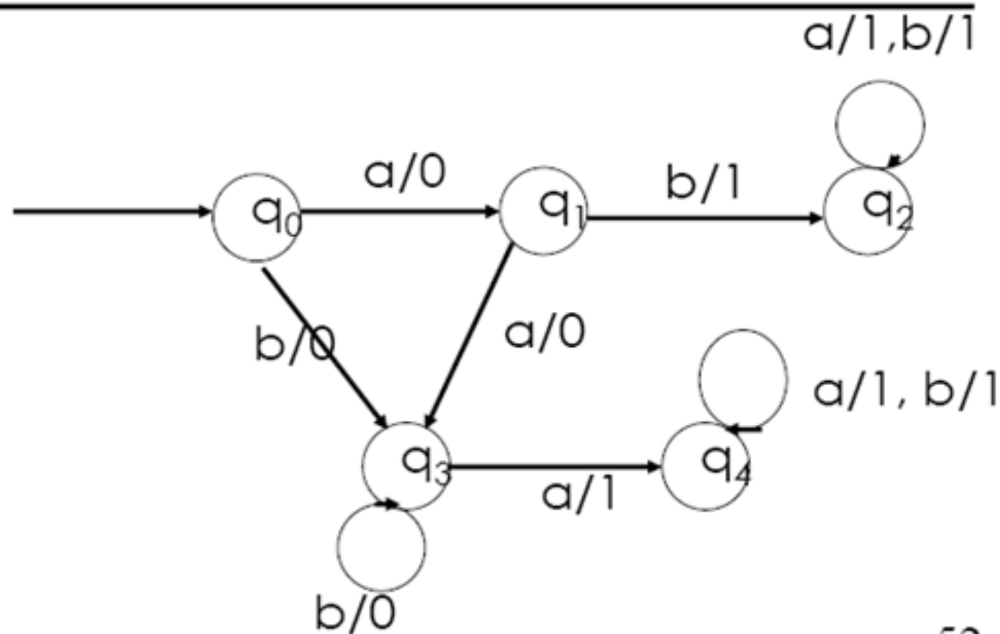


Solution (cont.)

- What is the output string if the input string is **bbbaaaa**?

$$q_0 \xrightarrow[0]{b} q_3 \xrightarrow[0]{b} q_3 \xrightarrow[0]{b} q_3 \xrightarrow[1]{a} q_4 \xrightarrow[1]{a} q_4 \xrightarrow[1]{a} q_4 \xrightarrow[1]{a} q_4$$

Output: 1



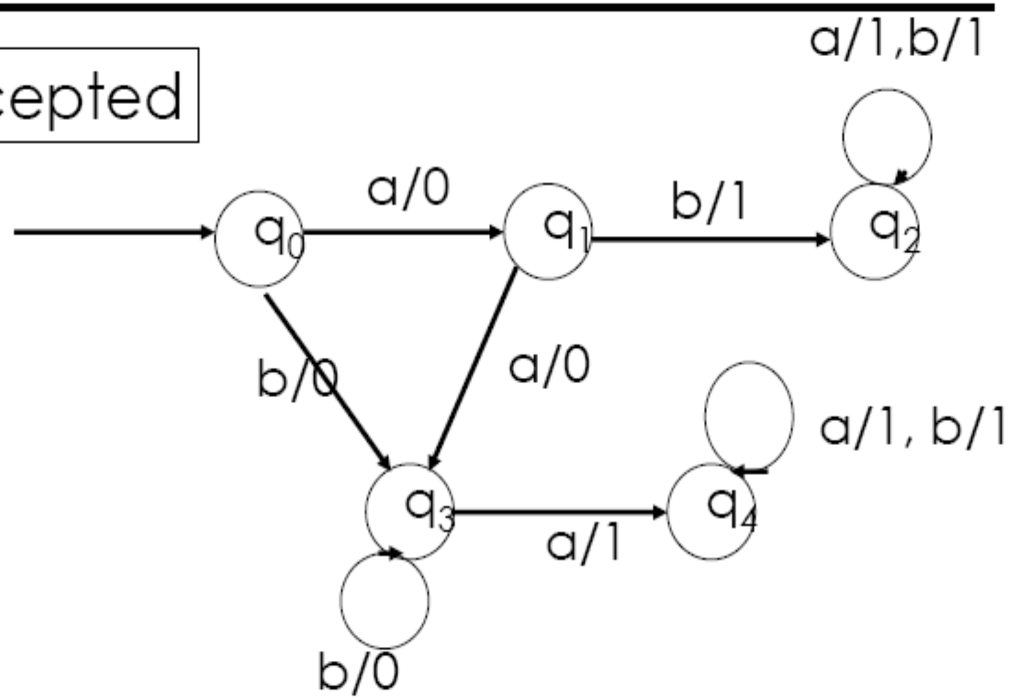
Solution (cont.)

- Is the string **aaa** accepted by M?

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4$$

0 0 1

Output: 1,accepted



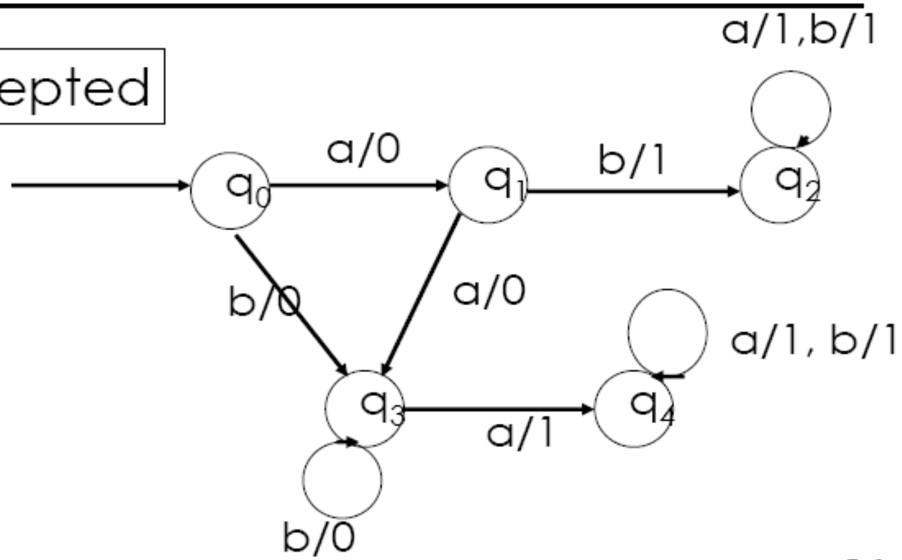
Solution (cont.)

- Which of the strings **ba, aabbba, bbbb, aaabbbb** are accepted by M?

ba

$$q_0 \xrightarrow[0]{b} q_3 \xrightarrow[1]{a} q_4$$

Output: 1,accepted

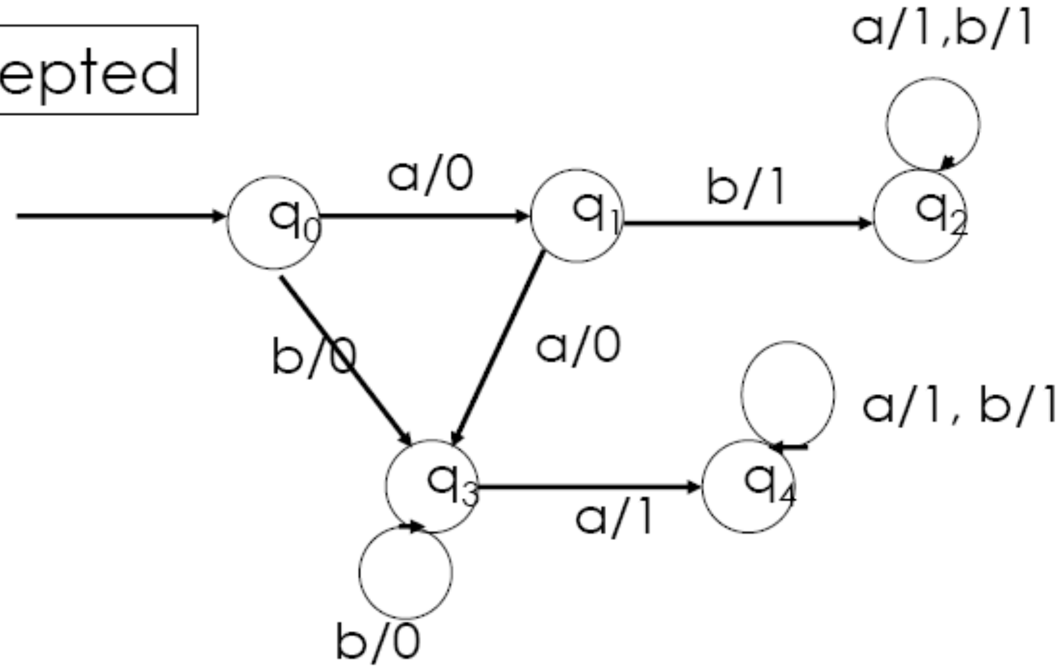


aabbba

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4$$

0	0	0	0	0	1

Output: 1,accepted

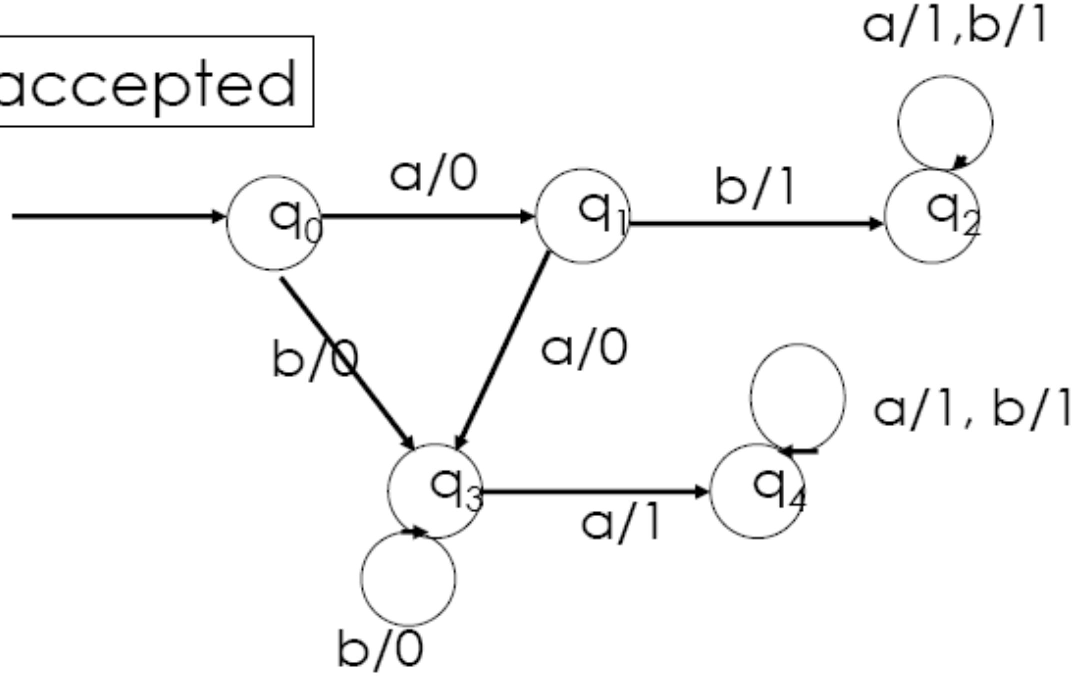


bbbb

$$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3$$

0 0 0 0

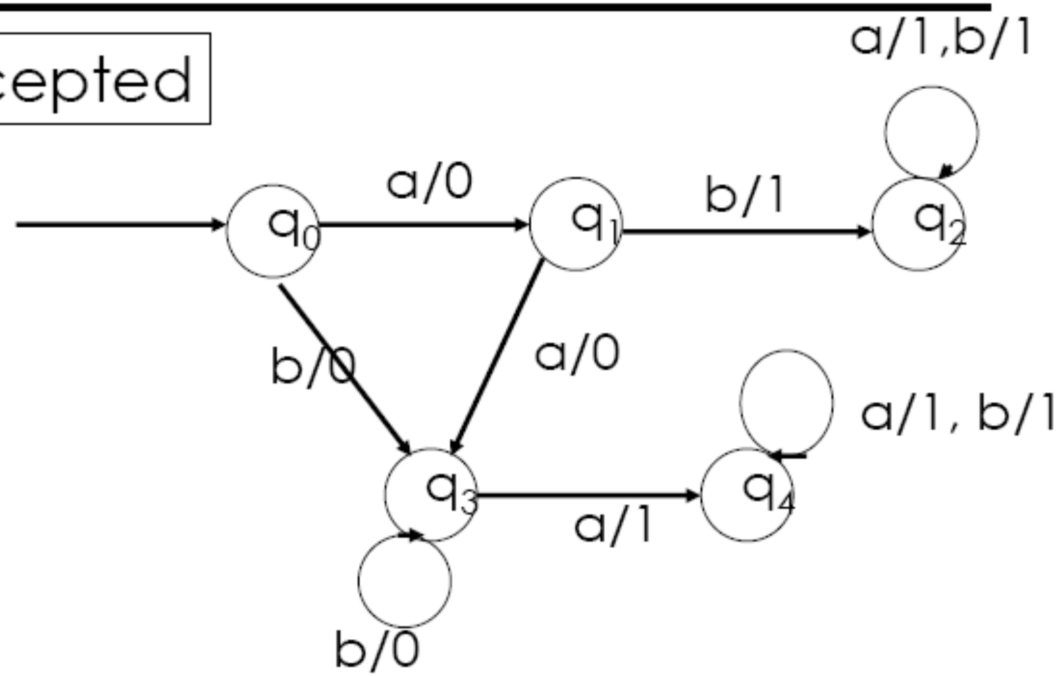
Output: 0,not accepted



aaabbbb

$$q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{a} q_3 \xrightarrow[1]{a} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4 \xrightarrow[1]{b} q_4$$

Output: 1,accepted



Example 4

- Consider a vending machine that sells candy and the cost of a candy is 50 cents.
- The machine accepts any sequence of 10-, 20-, or 50 cent coins.
- After inserting at least 50 cents, the customer can press the button to release the candy.

- If the customer inputs more than 50 cents, the machine does not return the change.

- After selling the candy, the machine returns to initial state.

- Construct a finite state machine that models this vending machine.

Example 4 - Solution

1. List the states, S:

States,

q_0,	initial state (0)
q_1,	10 cents
q_2,	20 cents
q_3,	30 cents
q_4,	40 cents
q_5,	$\geq$ 50 cents

UTM UNIVERSITI TEKNOLOGI MALAYSIA

2. List the elements of M:

$S = \{q_0, q_1, q_2, q_3, q_4, q_5\}$,

$I = \{10, 20, 50, B\}$,

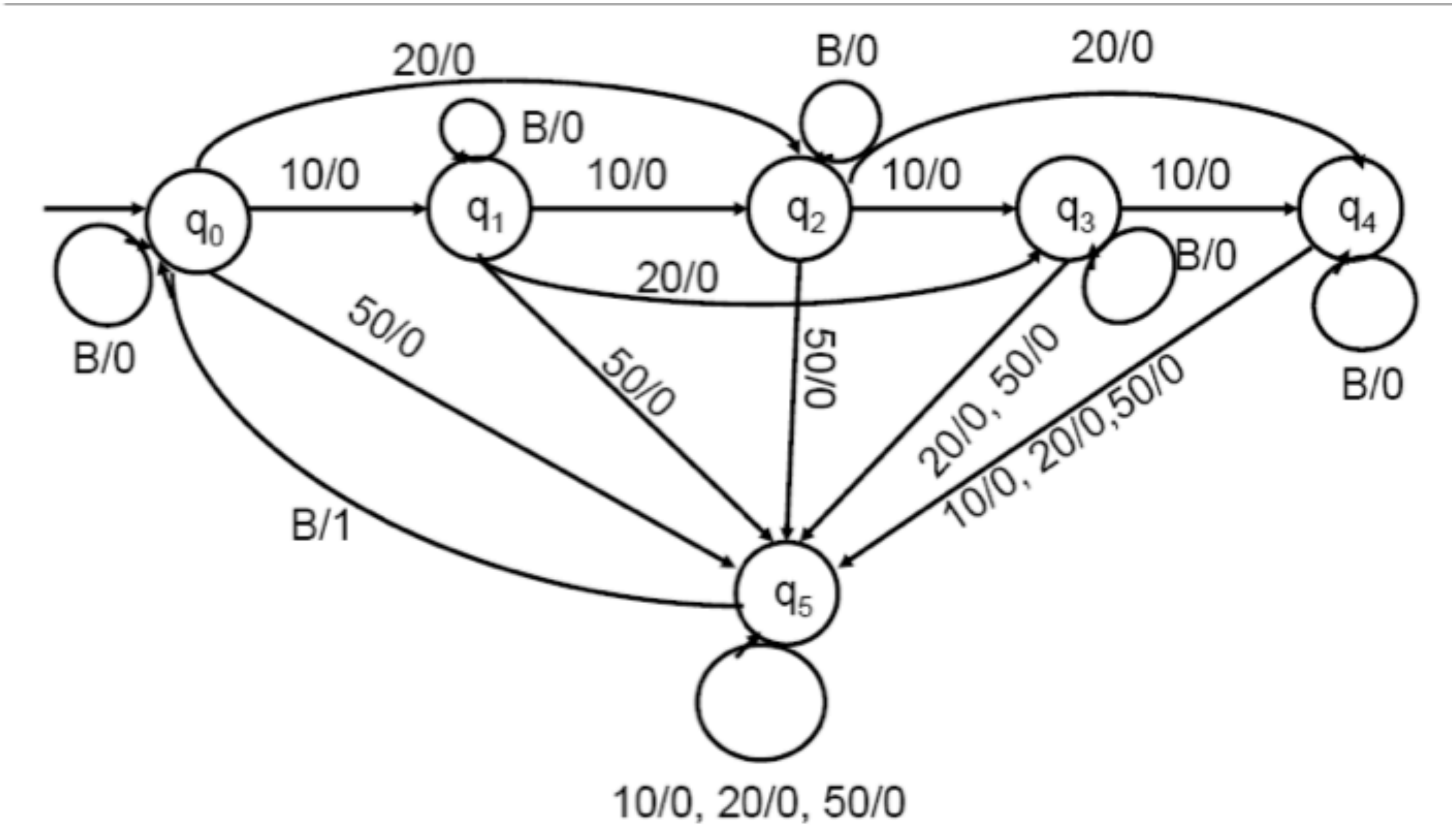
$O = \{0, 1\}$,

q_0 = initial state,

3. List the f_s and f_0:

	f_s				f_o			
	10	20	50	B	10	20	50	B
q_0	q_1	q_2	q_5	q_0	0	0	0	0
q_1	q_2	q_3	q_5	q_1	0	0	0	0
q_2	q_3	q_4	q_5	q_2	0	0	0	0
q_3	q_4	q_5	q_5	q_3	0	0	0	0
q_4	q_5	q_5	q_5	q_4	0	0	0	0
q_5	q_5	q_5	q_5	q_0	0	0	0	1

4. Draw the transition diagram of M:



UTM UNIVERSITI TEKNOLOGI MALAYSIA

Exercise #1

Let $M = \{ S, I, O, q_0, f_s, f_o \}$ be a FSM

where,

$S = \{q_0, q_1, q_2\}$,

$I = \{a, b\}$,

$O = \{0, 1\}$,

q_0 = initial state,

f_s and f_0

	f_s a	b	f_o a	b
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

- Draw the transition diagram of M.
- What is the output string if the input string is *aabbb*?
- What is the output string if the input string is *ababab*?
- What is the output if the input string is *abbbaba*?
- What is the output if the input string is *bbbababa*?

Exercise #2

Let $M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b, c\}, q_0, f_s, \{q_1, q_3, q_5\})$ be the Deterministic Finite Automaton (DFA) with state transition function, f_s defined as follows:

$f(q_0, a) = q_1$ $\quad f(q_0, b) = q_0$ $\quad f(q_0, c) = q_0$

$f(q_1, a) = q_1$ $\quad f(q_1, b) = q_2$ $\quad f(q_1, c) = q_1$

$f(q_2, a) = q_2$ $\quad f(q_2, b) = q_3$ $\quad f(q_2, c) = q_4$

$f(q_3, a) = q_3$ $\quad f(q_3, b) = q_3$ $\quad f(q_3, c) = q_3$

$f(q_4, a) = q_4$ $\quad f(q_4, b) = q_5$ $\quad f(q_4, c) = q_4$

$f(q_5, a) = q_5$ $\quad f(q_5, b) = q_5$ $\quad f(q_5, c) = q_5$

(a) Draw the transition table for the above machine.

(b) Determine the final state for the input string *abcc*.

(c) Is the input string *abcb* accepted by the DFA?

Exercise #3

Let $A = (S, I, O, Z, f, g)$ be a finite state machine (FSM) defined by the transition table shown in Table 1.

Table 1: Transition table of FSM *A*

	f			g		
Input / State	a	b	c	a	b	c
X	Z	X	Y	1	0	1
Y	X	X	Z	0	1	0
Z	Y	X	Z	1	0	1

(a) Draw the transition diagram of the finite state machine A.

(b) Find the output string for the input string *babccaab*.

(c) Find the output generated from the input string *cabcccba*.

(d) Determine whether the input string *abcbcbcabcc* is accepted.

Exercise #4

A sliding barrier turnstile (shown in Figure 1), used to control access to subways, is an automated gate at waist height with a barrier across the entryway. Initially the barrier is locked, barring the entry, preventing passengers from passing through. Depositing a token in a slot on the turnstile unlocks the barrier, allowing a single customer to push through. After the customer passes through, the barrier is locked again until another coin is inserted.

Figure 1

Considered as a state machine, the turnstile has two states: ***Locked*** and ***Unlocked***. There are two inputs that affect its state: putting a token in the slot (***token***) and retract the barrier (***retract***). In the locked state, retracting the arm has no effect; no matter how many times the input ***retract*** is given, it stays in the locked state. Putting a ***token*** as an input, shift the state from ***Locked*** to ***Unlocked***. In the unlocked state, putting additional tokens does not change the state. However, a customer passing through the retracted barrier, giving a ***retract*** input, shifts the state back to ***Locked***.

Given
- State
  - L: Locked
  - U: Unlocked
- Input
  - T: Token
  - R: Retract
- Output
  - 0: Nothing happened
  - 1: Retract the barrier so passenger can pass through
  - 2: Lock the barrier when passenger has passed through

(a) Complete the transition table below.

Table 1

State	Input, f_s		Output, f_o	
	T	R	T	R
L				
U				

(b) Draw the transition diagram for the turnstile system described above.