



UNIVERSITI TEKNOLOGI MALAYSIA

TEST 1

SEMESTER I - 2017/2018

SUBJECT CODE	:	SCSI1013
SUBJECT	:	DISCRETE STRUCTURE
TIME	:	2 HOURS (8.00 P.M - 10.00 P.M)
DATE	:	23rd OCTOBER 2017

INSTRUCTIONS TO THE STUDENTS:

Please answer all questions in the answer booklet.

QUESTION 1**[10 MARKS]**

- a) Sketch Venn diagrams that show the universal set, U , the sets A and B , and a single element x in each of the following cases:

(i) $x \in A; x \notin B; B \subset A$ (2 marks)

(ii) $x \in A; x \in B; A$ is not a subset of $B; B$ is not a subset of A (2 marks)

- b) Prove the following identity, stating carefully which of the set laws you are using at each stage of the proof.

$$B \cup (\emptyset \cap A) = B \quad (4 \text{ marks})$$

- c) In a certain programming language, all variable names have to be 3 characters long. The first character must be a letter from 'a' to 'z'; the others can be letters or digits from 0 to 9. If $L = \{a, b, c, \dots, z\}$, $D = \{0, 1, 2, \dots, 9\}$, and $V = \{\text{permissible variable names}\}$, use a Cartesian product to complete:

$$V = \{pqr \mid (p, q, r) \in \dots\} \quad (2 \text{ marks})$$

QUESTION 2**[20 MARKS]**

- a) Suppose that the variable x represents student, $F(x)$ means " x is freshman," and $M(x)$ means " x is a math major". Determine the symbolic statement equivalent to the following statement.

i) No math major is a freshman. (1 mark)

ii) Some freshmen are math major. (1 mark)

- b) Let $P(x, y): 2x + y = 1$, where the domain of discourse is the set of all integers. What are the truth values of the following:

i) $\forall x \exists y P(x, y)$ (2 marks)

ii) $\forall x \forall y P(x, y)$ (2 marks)

c) Evaluate the following logical equivalent using truth table.

$$(\neg p \rightarrow (q \vee r)) \equiv (q \rightarrow (p \vee r)) \quad (5 \text{ marks})$$

e) Prove the following compound proposition using logical law.

$$(P \vee \neg Q) \vee (R \wedge (Q \vee \neg Q)) \equiv R \vee (P \vee \neg Q) \quad (4 \text{ marks})$$

f) Proof directly that,

“If m is an even integer and n is an odd integer then $m+n$ is an odd”
(5 marks)

QUESTION 3

[25 MARKS]

a) $A = \{(1,3), (2,4), (-4,-8), (3,9), (1,5), (3,6)\}$. For all $(a,b), (c,d) \in A$. Determine R , a relation defined on A as

$$(a,b)R(c,d) \Leftrightarrow ad = bc \quad (5 \text{ marks})$$

b) Determine whether the following relation is reflexive, symmetric, antisymmetric and/or transitive. (Note: Use diagram to assist your justification and you can define an appropriate set if necessary).

i) R is relation on the set of people where $(a,b) \in R$, if a is taller than b .
(4 marks)

ii) R is a relation on the set of all web pages where $(a,b) \in R$, if everyone who has visited web page a also visited web page b .
(3 marks)

iii) R is a relation on the set of people in the Facebook friend list, where $(a,b) \in R$, if a is a friend of b .
(3 marks)

c) Determine whether the relation presented by the following matrices is equivalence relation.

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

(10 marks)

QUESTION 4**[14 MARKS]**

- a) Which of the following relations are functions from A to B . Write their domain and range. If it is not a function, give a reason.

i) $R = \{(1, -2), (3, 7), (4, -6), (8, 1)\}$, $A = \{1, 3, 4, 8\}$, $B = \{-2, 7, -6, 1, 2\}$

(2 marks)

ii) $R = \{(1, 0), (1, -1), (2, 3), (4, 1)\}$, $A = \{1, 2, 4\}$, $B = \{0, -1, 3, 1, 0\}$

(2 marks)

i) $R = \{(1, -1), (2, -2), (3, -3), (4, -4), (5, -5)\}$, $A = \{0, 1, 2, 3, 4, 5\}$, $B = \{-1, -2, -3, -4, -5\}$

(2 marks)

- b) Prove that

$f : X \rightarrow Y$ defined by $f(x) = 3x - 7$ is a one-to-one function.

(4 marks)

- c) If $f(x) = \sqrt{x+1}$ and $g(x) = x^2 + 2$

i) Calculate $f \circ g$

(2 marks)

ii) Calculate $g \circ f$

(2 marks)

QUESTION 5**[16 MARKS]**

- a) Determine s_5 if $s_0, s_1, s_2, \dots$ is a sequence satisfying the given recurrence relation and initial conditions.

$$s_n = 2s_{n-1} + s_{n-2} - s_{n-3} \quad \text{for } n \geq 3, \quad s_0 = 2, \quad s_1 = -1, \quad s_2 = 4$$

(3 marks)

- b) A consumer purchased items costing RM280 with a department store credit card that charges 1.5% interest per month compounded monthly. Write a recurrence relation and initial condition for b_n , the balance of the consumer's account after n months if no further charges occur and the minimum monthly payment of RM25 is made.

(3 marks)

- c) Write a recursive algorithm to find the n term of sequence defined by

$$a_1 = 2, \quad a_2 = 3, \quad \text{and} \quad a_n = a_{n-1}a_{n-2} \quad \text{for } n \geq 3. \quad \text{Then, use the algorithm to trace } a_7.$$

(10 marks)

Theorem for Logic

Indempotent Laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Double negation laws	$\neg \neg p \equiv p$
Commutative law	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Absorption Laws	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$
De Morgan's Laws	$\neg (p \wedge q) \equiv (\neg p) \vee (\neg q)$ $\neg (p \vee q) \equiv (\neg p) \wedge (\neg q)$