Notes on

Basic Circuit Theory

Voltage

$$B \xrightarrow{v_{AB}} A$$

$$B \xleftarrow{v_{BA}} A$$

$$v_{AB}=-v_{BA}$$

It is the difference in electrical potential between two points

Current

$$B \xrightarrow{i_{BA}} A$$

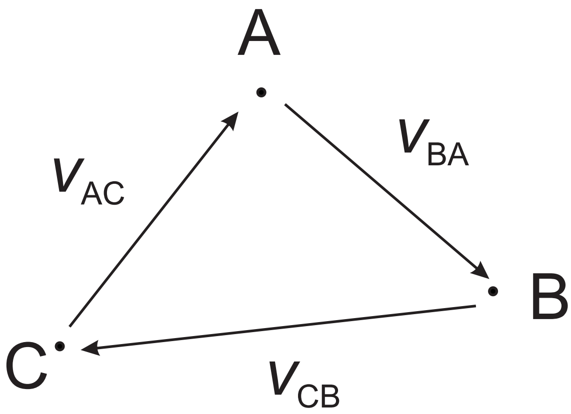
$$B \xleftarrow{i_{AB}} A$$

$$i_{AB}=-i_{BA}$$

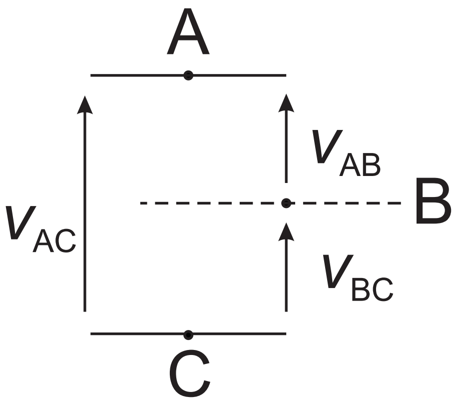
It is the charge flowing in a section of a conductor per unit time:

$$i=\frac{\mathrm{d}Q}{\mathrm{d}t}$$

Kirchoff's Voltage Law (KLV)

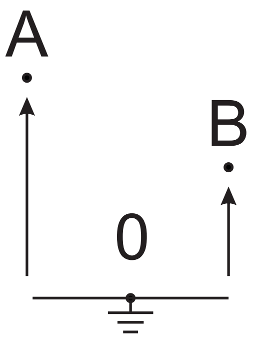


$$v_{BA} + v_{CB} + v_{AC} = 0$$



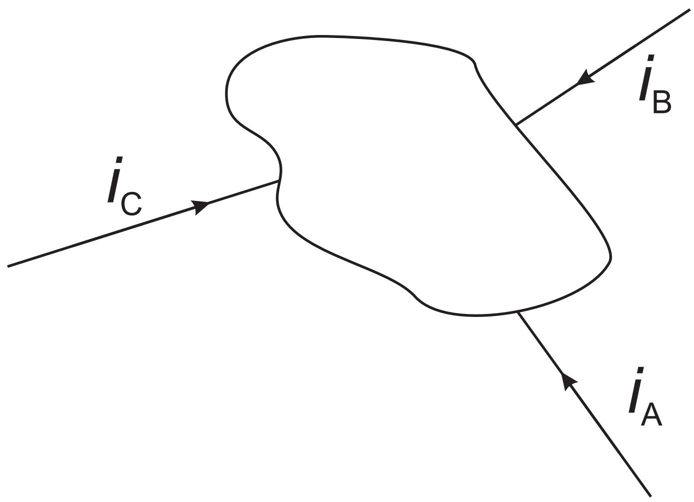
$$v_{AC} = v_{BC} + v_{AB}$$

like "quotes" on a drawing



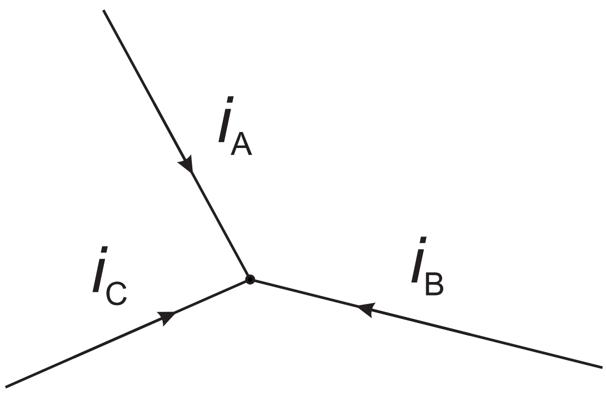
$$v_A = v_{A0}$$
$$v_B = v_{B0}$$

Kirchoff's Current Law (KLI)

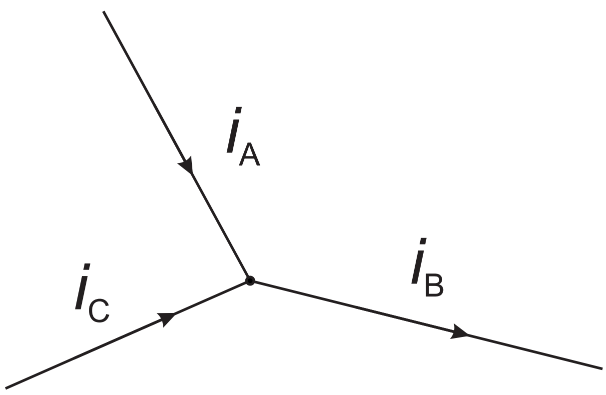


$$i_A + i_B + i_C = 0$$

special case: Node's Law



$$i_A + i_B + i_C = 0$$

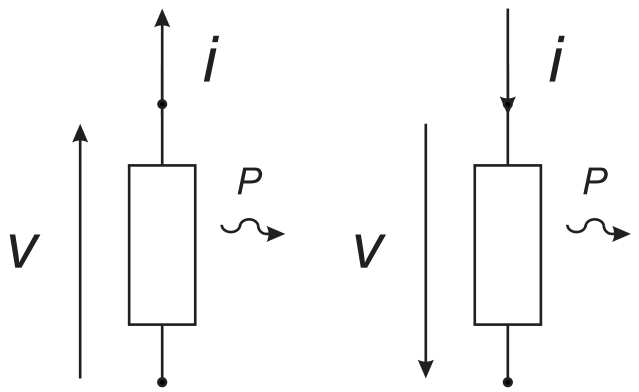


$$i_B = i_A + i_C$$

like a flow of a liquid

Sign Conventions

Active Sign Convention:

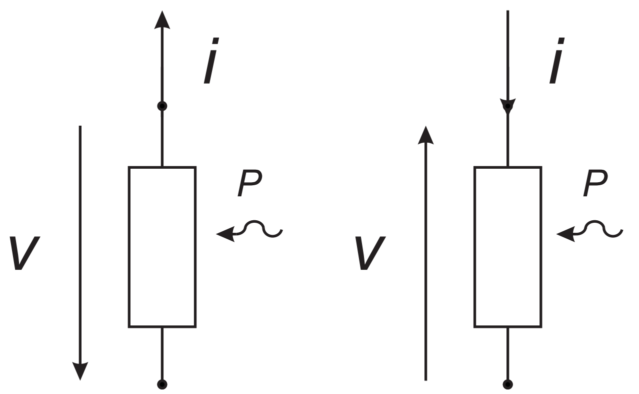


v and *i* arrows have *the same direction*

$$\boxed{P = v\,i}$$

P : electrical power *delivered by* the device

Passive Sign Convention:



v and *i* arrows have *opposite directions*

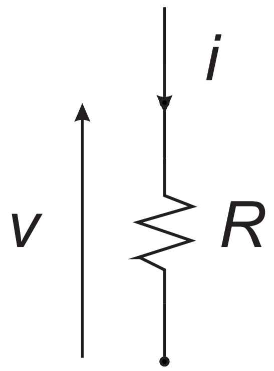
$$\boxed{P = v\,i}$$

P : electrical power *absorbed by* the device

Basic Circuit Elements

Ideal Resistor

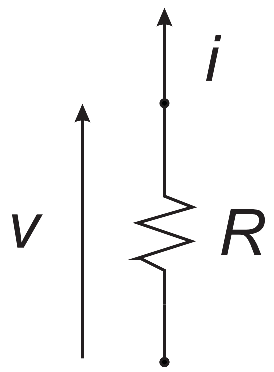
Passive Sign Convention:



$$v = Ri$$

Ohm's Law

Active Sign Convention:



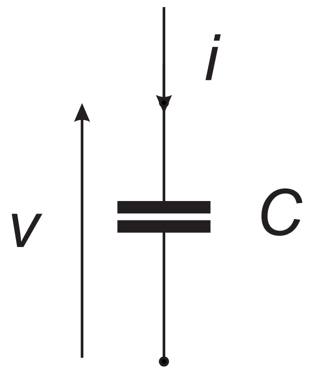
$$v = -Ri$$

Ohm's Law

Basic Circuit Elements

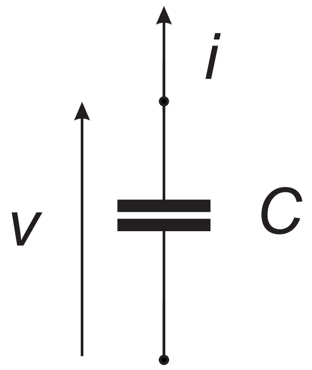
Ideal Capacitor

Passive Sign Convention:



$$i = C\,\frac{dv}{dt}$$

Active Sign Convention:

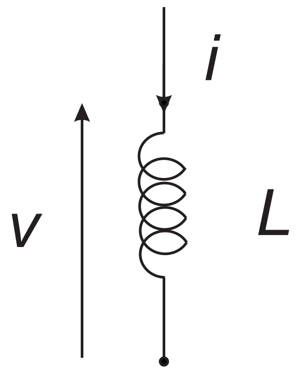


$$i = -C\,\frac{dv}{dt}$$

Basic Circuit Elements

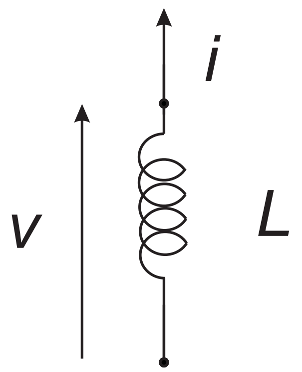
Ideal Inductor

Passive Sign Convention:



$$v = L\,\frac{\mathrm{d}i}{\mathrm{d}t}$$

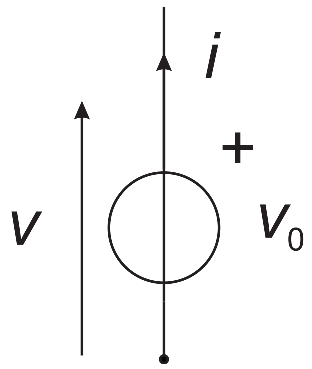
Active Sign Convention:



$$v = -L\,\frac{\mathrm{d}i}{\mathrm{d}t}$$

Basic Circuit Elements

Ideal Voltage Source



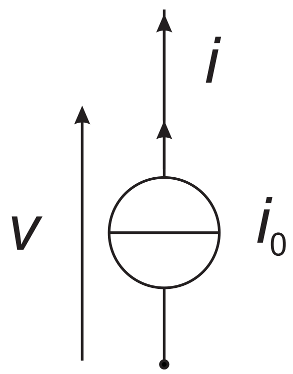
$$v = v_0$$

independently of *i*

special case: if v_0=0

short circuit (i.e. an ideal wire)

Ideal Current Source



$$i = i_0$$

independently of *v*

special case: if i_0=0

open circuit (i.e. no electrical connection)

Basic Circuit Elements

Switch

ON state

equivalent to a short circuit
(i.e. an ideal wire)

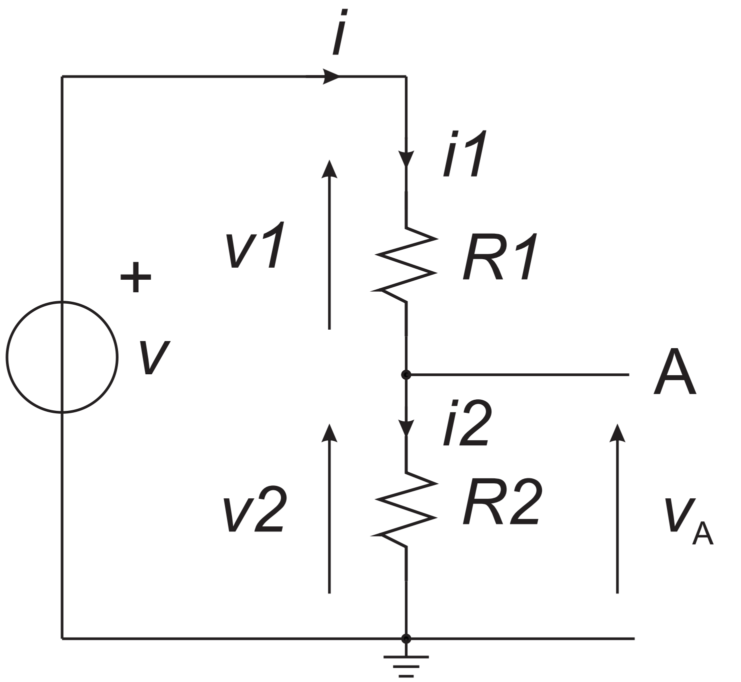
OFF state

equivalent to an open circuit
(i.e.two unconnected points)

Resistor Connections

Series Connection

Two resistors are connected in series if
- they share one terminal (A),
- such a terminal is not connected to nothing else



by Ohm's Law

$v1=R1\ i1$

$v2=R2\ i2$

by KLI

$i=i1=i2$

by KLV

$v=v1+v2$

as a consequence

$$\begin{aligned} v &= R1\ i + R2\ i \\ &= (R1+R2)\ i \quad (*) \\ &= R_{EQ}\ i \end{aligned}$$

Two resistors in series are equivalent (for external effects) to a single resistor with a resistance

$$\boxed{R_{EQ}=R1+R2}$$

Replacing two resistor in series with the equivalent resistor, node A is lost.
Node A voltage can be expressed in terms of the overall voltage *v* across the series-connected resistors

$v_A = v2 = R2\ i2 = R2\ i,$ from (*): $i = \dfrac{v}{R1+R2}$

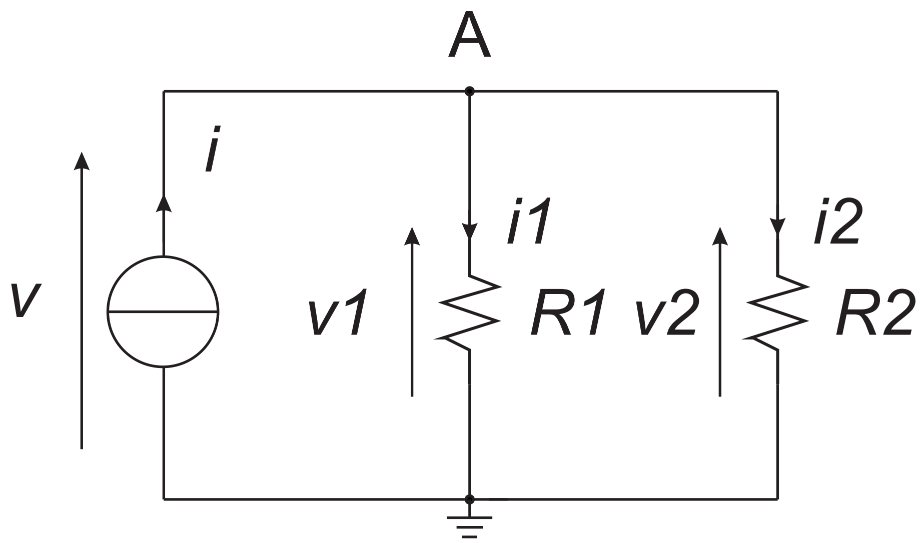
$$\boxed{v_A = \frac{R2}{R1+R2}\ v}$$

Voltage Divider Rule

Resistor Connections

Shunt Connection

Two resistors are connected in shunt if they share both terminals



by Ohm's Law

$v1=R1\ i1 \qquad v2=R2\ i2$

by KLI

$i=i1+i2$

by KLV

$v=v1=v2$

as a consequence

$$\begin{aligned} i &= v/R1 + v/R2 \\ &= v(1/R1+1/R2) \\ &= v/R_{EQ} \qquad (*) \end{aligned}$$

Two shunt-connected resistors are equivalent (for external effects) to a single resistor with a resistance

$$\boxed{R_{EQ} = \frac{1}{\dfrac{1}{R1} + \dfrac{1}{R2}} = \frac{R1R2}{R1+R2}}$$

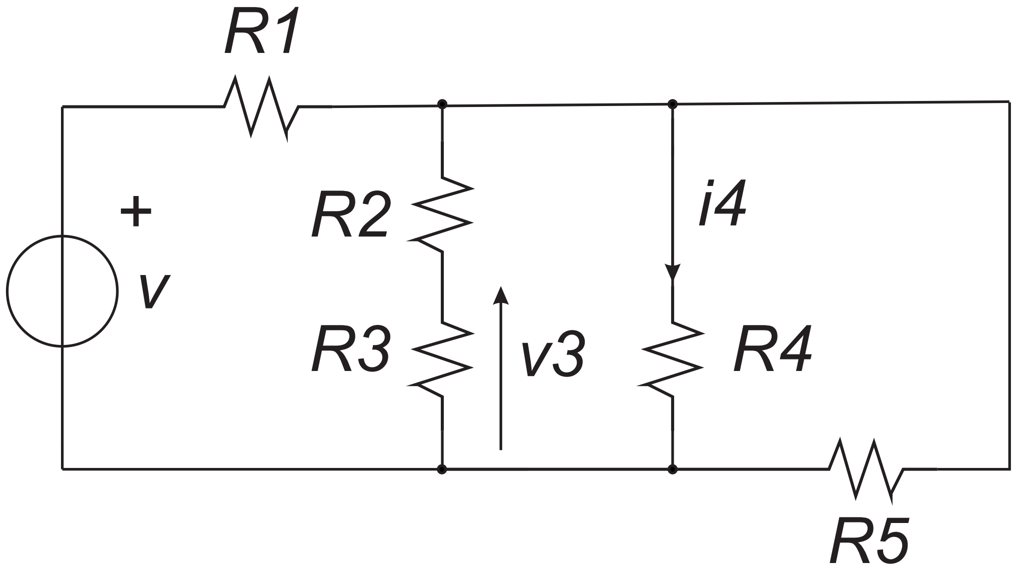
Current *i1* can be expressed in terms of the overall current *i*

$i1= v1/R1 = v/R1$ from (*): $v=\dfrac{R1R2}{R1+R2}i$

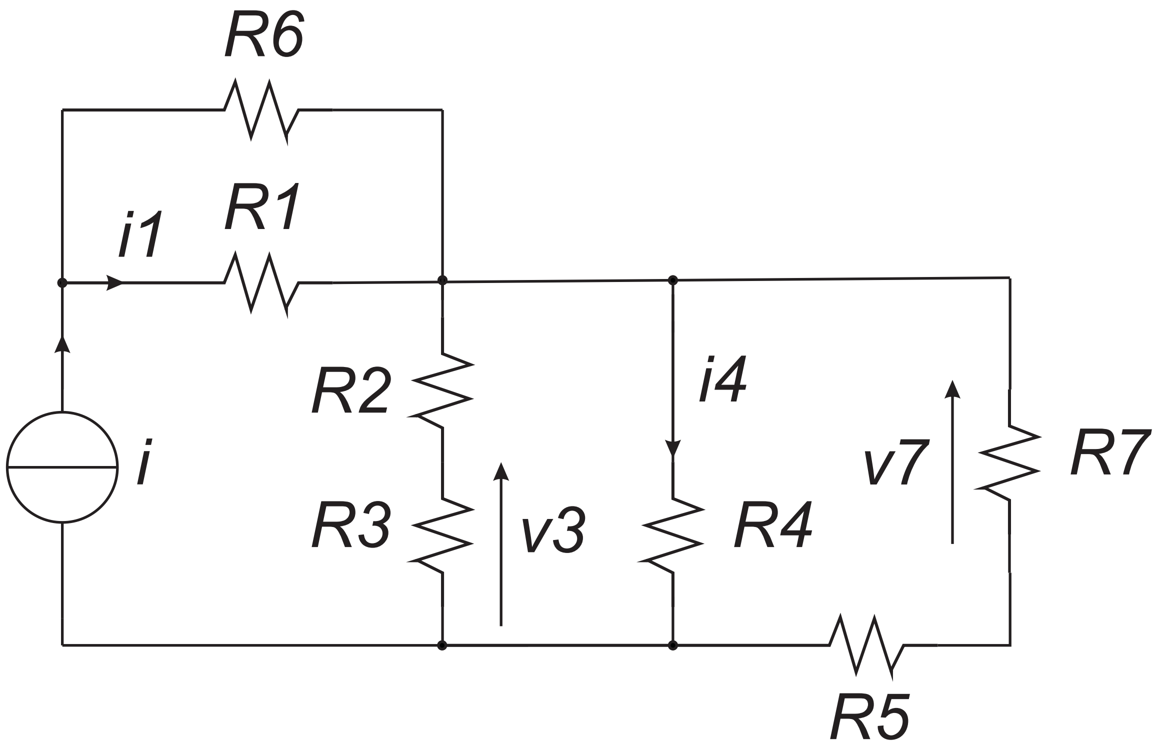
$$\boxed{i1 = \frac{R2}{R1+R2}\, i}$$

Current Divider Rule

Exercises

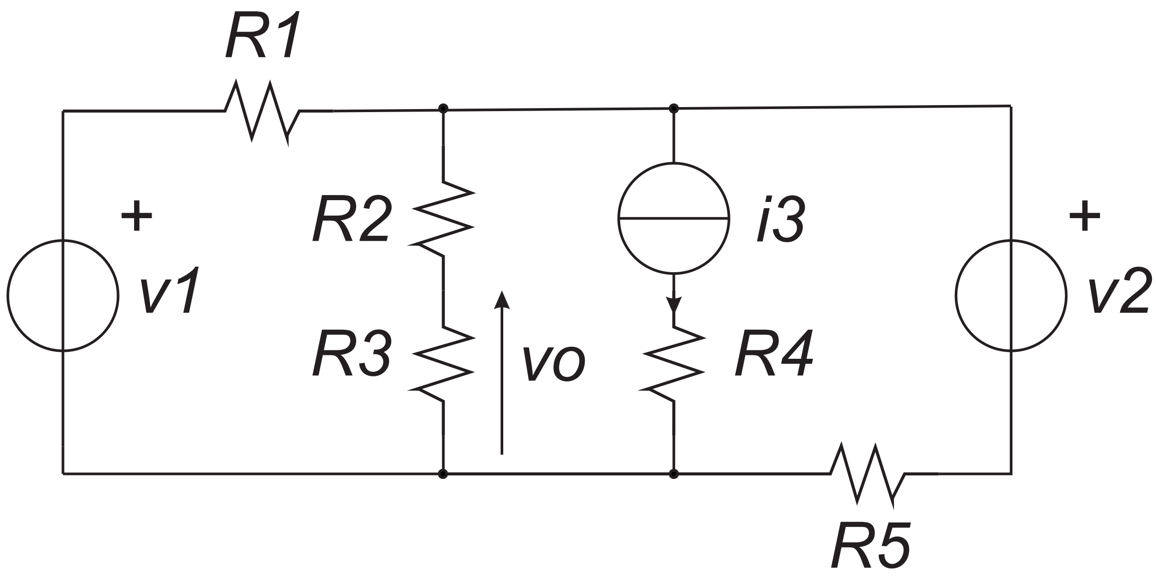


Ex. 1: Evaluate *v3* and *i4*



Ex.2: Evaluate *i1, v3, i4* and *v7*

Superposition Principle



In any **linear** circuit which includes more that one independent source, each output can be expressed as the sum of the contributions of each source.

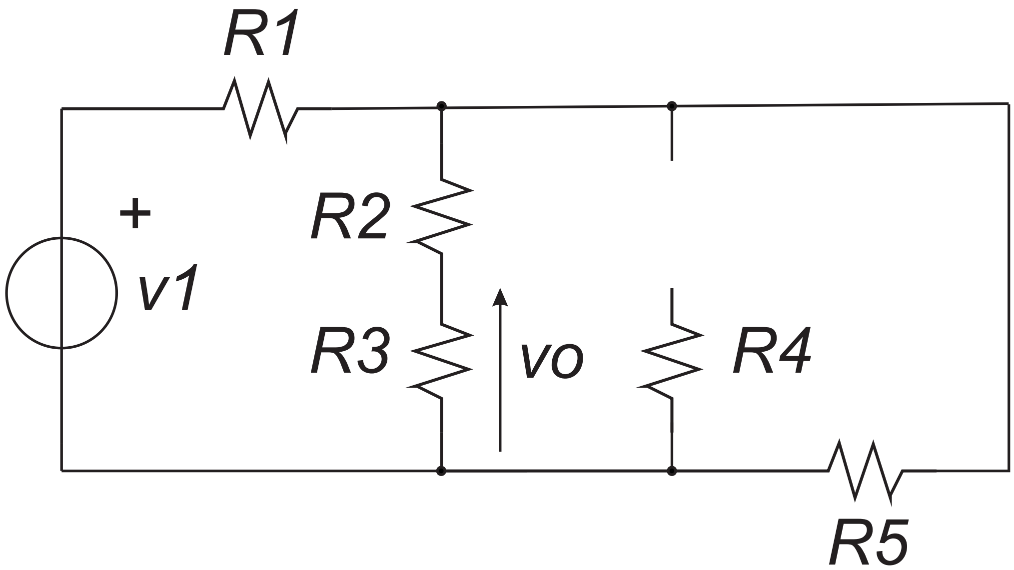
$$vo=vo'+vo''+vo'''$$

The contribution of a source to the output is the value of the output in the circuit when the considered source is active, while the other sources are turned off.

a **voltage source**, when it is turned off, is equivalent to a **short circuit**
a **current source**, when it is turned off, is equivalent to an **open circuit**

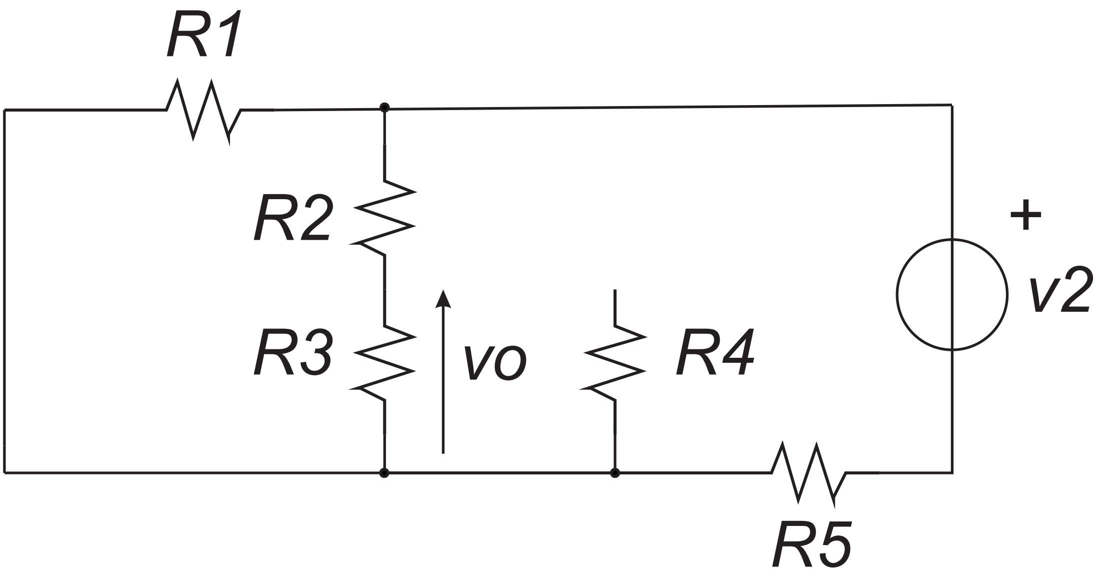
Superposition Principle

Contribution of v1 (vo')



$$vo' = \frac{(R2+R3)//R5}{R1+(R2+R3)//R5} \; \frac{R3}{R2+R3} \; v1$$

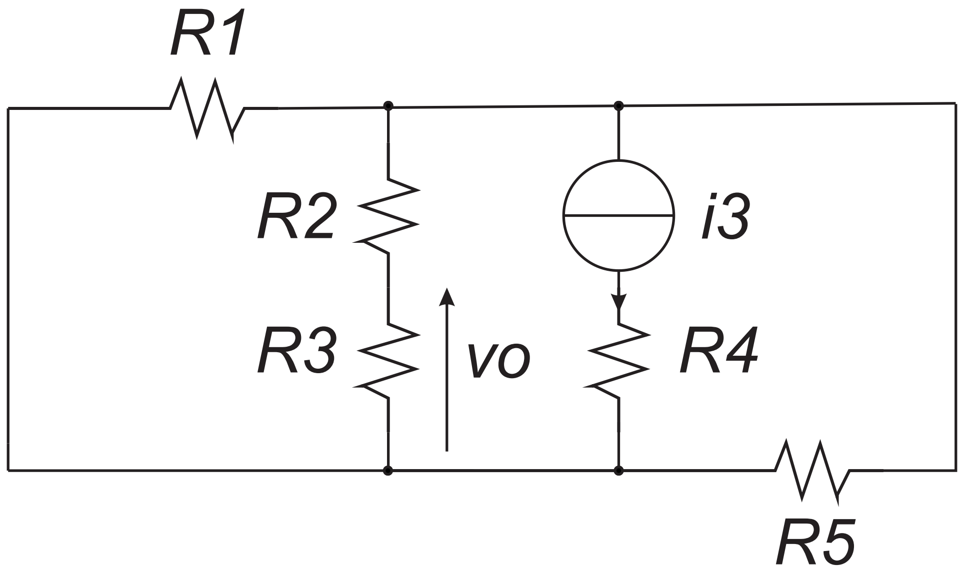
Contribution of v2 (vo")



$$vo'' = \frac{(R2+R3)//R1}{R5+(R2+R3)//R1} \; \frac{R3}{R2+R3} \; v2$$

Superposition Principle

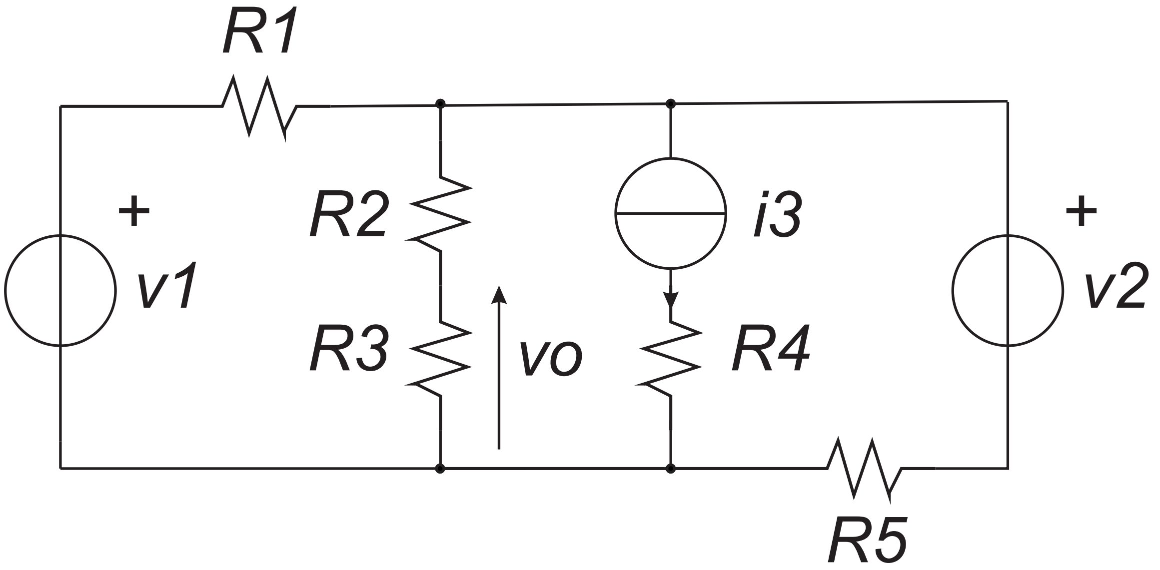
Contribution of i3 (vo''')



$$vo''' = -R1//R5//(R2+R3)\,\frac{R3}{R2+R3}\,i3$$

Superposition Principle

Overall Circuit



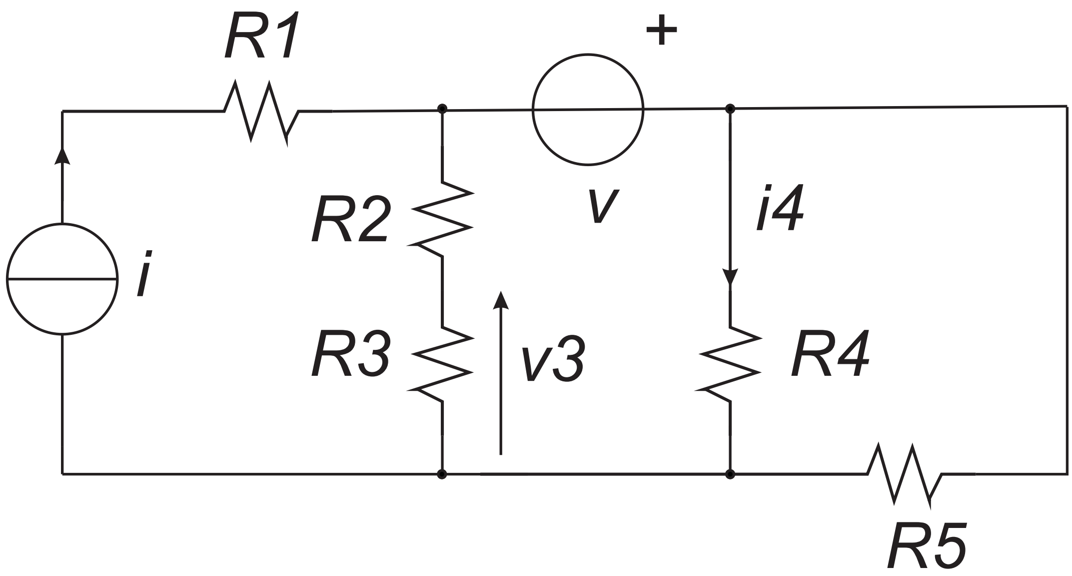
$$vo = vo' + vo'' + vo'''$$

$$vo = \frac{(R2+R3)//R5}{R1+(R2+R3)//R5} \frac{R3}{R2+R3} v1 +$$

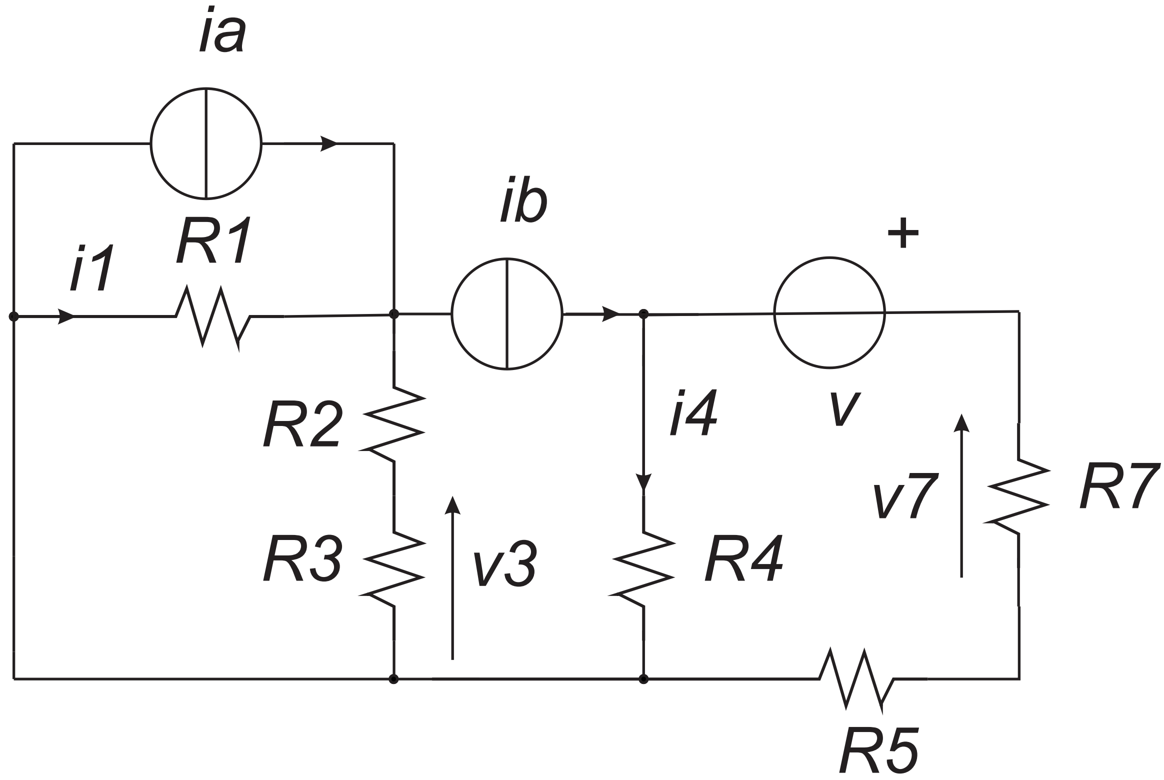
$$+ \frac{(R2+R3)//R1}{R5+(R2+R3)//R1} \frac{R3}{R2+R3} v2 +$$

$$- R1//R5//(R2+R3) \frac{R3}{R2+R3} i3$$

Exercises



Ex. 1: Evaluate *v3* and *i4*



Ex.2: Evaluate *i1, v3, i4* and *v7*