



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

M. Afendi M. Piah, PhD

www.utm.my

Circuit Theory (SKEE 1023)

Assoc. Prof. Dr. Mohamed Afendi Mohamed Piah

**Institute of High Voltage & High Current
Faculty of Electrical Engineering, UTM**



Circuit Theory

Topics

- ❖ **Methods of Circuit Analysis:** Nodal Analysis; Node Voltages; Mesh Analysis; Loop Currents; Circuit Analysis with Voltage and Current Independent and Dependent Sources.

Circuit Theory

METHODS OF CIRCUIT ANALYSIS

Introduction

- ❑ Two powerful techniques for circuit analysis, ie;
 - Nodal analysis : based on application of KCL.
 - Mesh analysis : based on application of KVL.
- ❑ Analyze any linear circuit by obtaining a set of **simultaneous equations** that are then solved to obtain the required values of **current** or **voltage**.

Nodal Analysis

- ❑ Provide a general procedure for analyzing circuits using **node voltages** as the **circuit variables**.
- ❑ Interested in finding the node voltages.

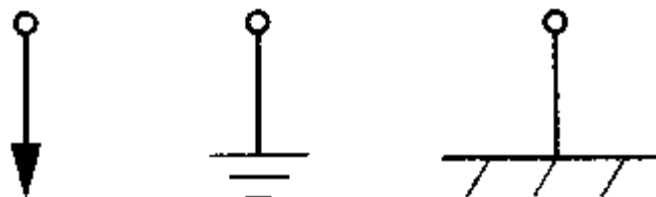
Circuit Theory

Steps to Determine Node Voltages (Nodal Analysis)

1. Select a node as the reference node. Assign voltages $v_1, v_2, \dots, V_{n-1}$ to the remaining $n - 1$ nodes.
 2. Apply KCL to each of the $n - 1$ nonreference nodes.
 3. Solve the resulting simultaneous equations to obtain the unknown node voltages.
- ❑ The reference (datum) node is commonly called the ground (assumed to have zero potential).

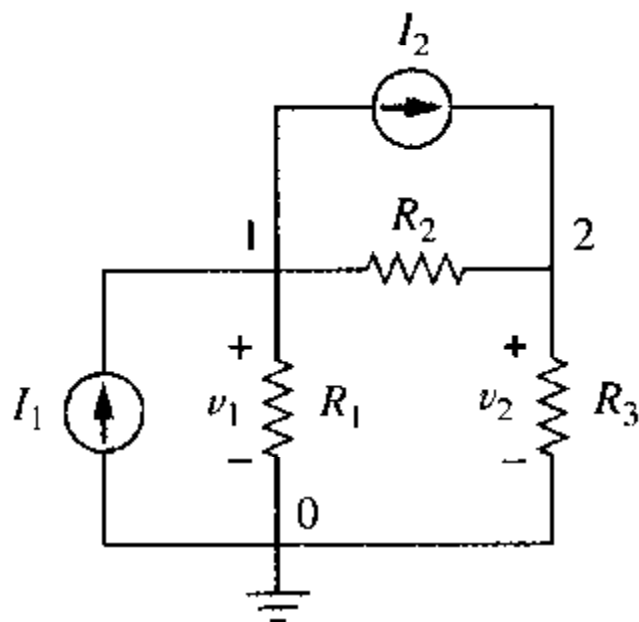
Symbols :

- Common ground
- Ground
- Chassis ground

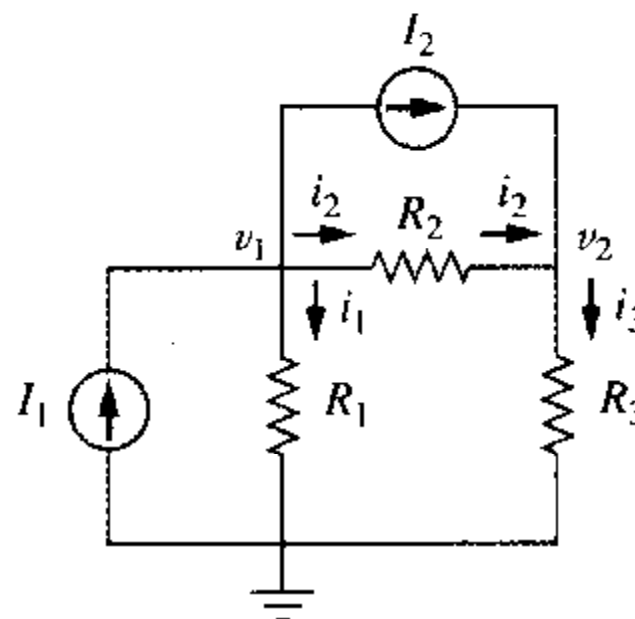


Circuit Theory

Method of Nodal Analysis



(a)

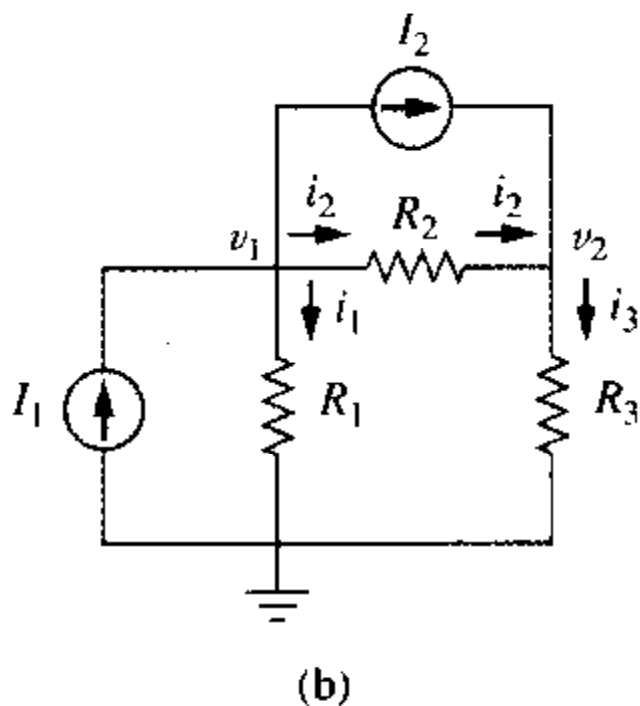


(b)



Circuit Theory

At node 1 and node 2, apply KCL gives;



$$I_1 = I_2 + i_1 + i_2 \quad \text{and} \quad I_2 + i_2 = i_3$$

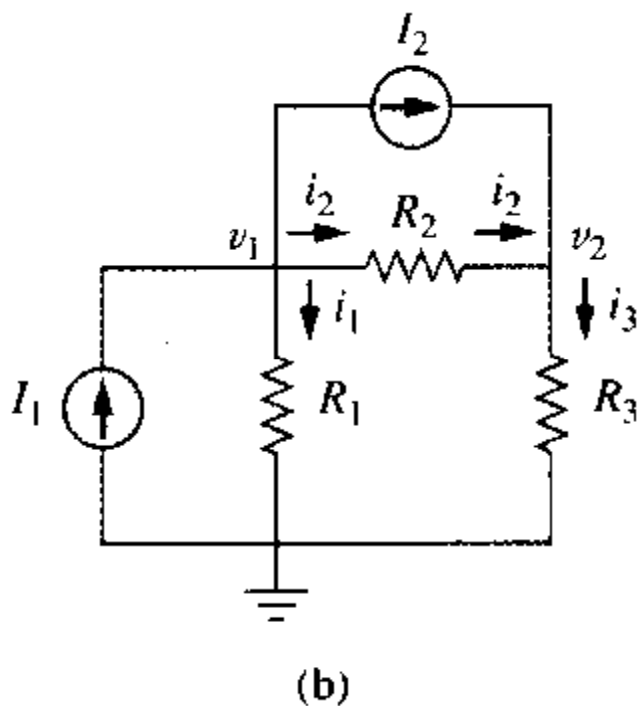
$$i_1 = \frac{v_1 - 0}{R_1} = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} = G_2 (v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} = G_3 v_2$$



Circuit Theory



At node 1

$$I_1 = I_2 + \left(\frac{1}{R_1} + \frac{1}{R_2} \right) v_1 - \frac{1}{R_2} v_2$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} \right) v_1 - \frac{1}{R_2} v_2 = I_1 - I_2$$

or

$$(G_1 + G_2) v_1 - G_2 v_2 = I_1 - I_2$$

$$I_2 = -\frac{1}{R_2} v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} \right) v_2$$

$$-\frac{1}{R_2} v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} \right) v_2 = I_2$$

or

$$-G_2 v_1 + (G_2 + G_3) v_2 = I_2 \quad 7$$

Circuit Theory

In matrix form;

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

or

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

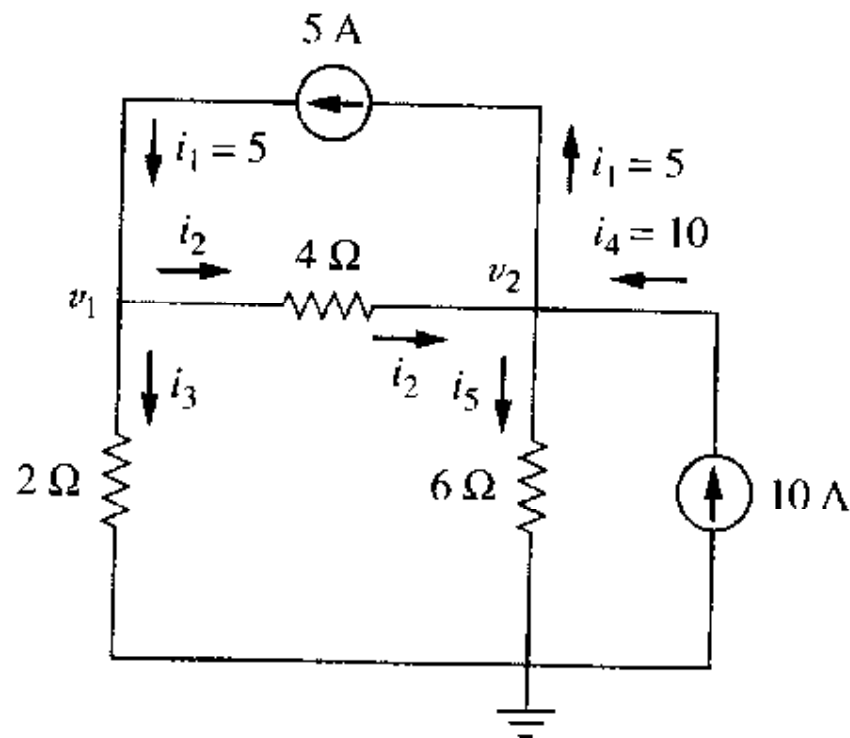
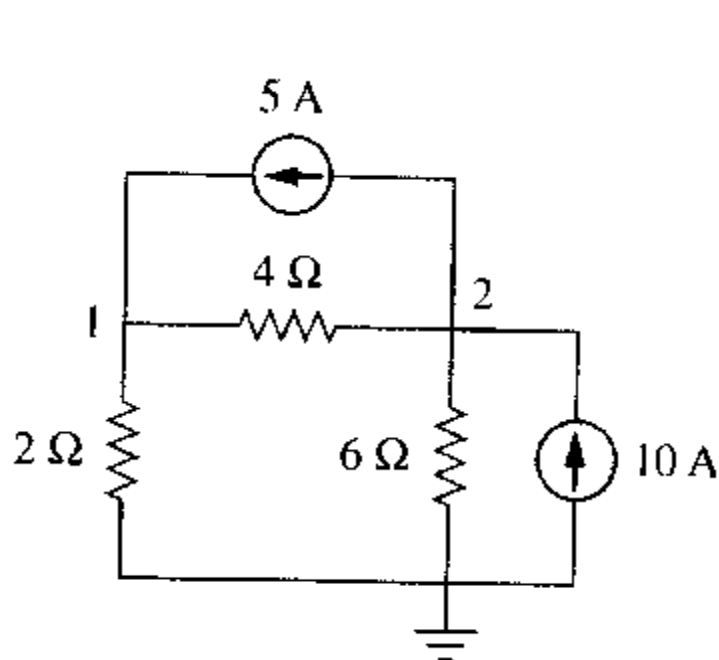
Then, determine the values of v_1 and v_2 .



Circuit Theory

Example 2.1

Calculate the node voltages in the circuit.



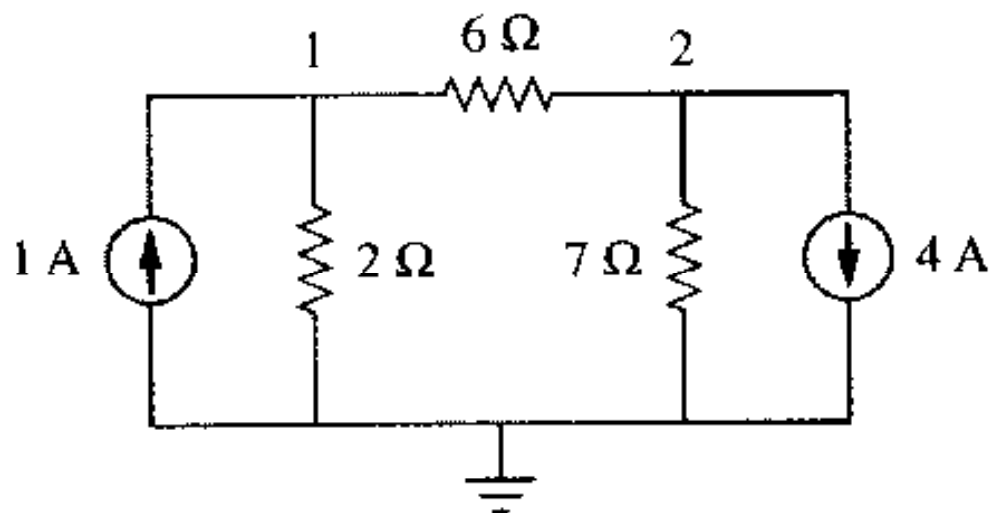


Circuit Theory

Problem 2.1

Obtain the node voltages in the circuit.

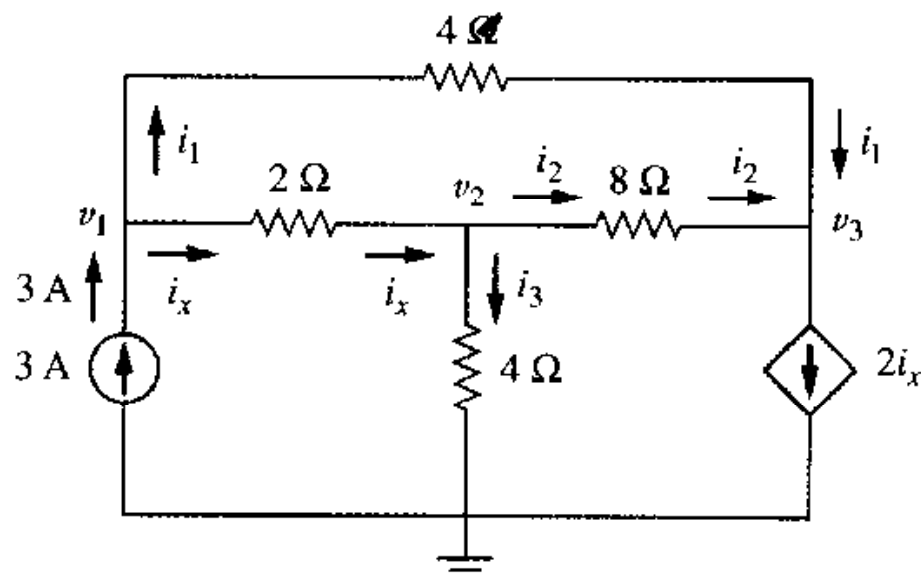
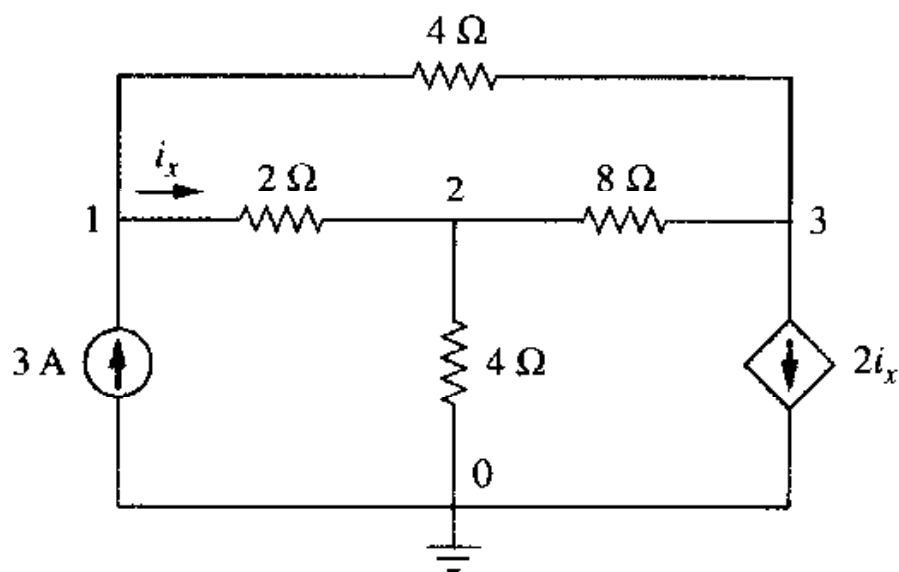
Ans: $v_1 = -2\text{ V}$, $v_2 = -14\text{ V}$



Circuit Theory

Example 2.2

Determine the voltages at the nodes.

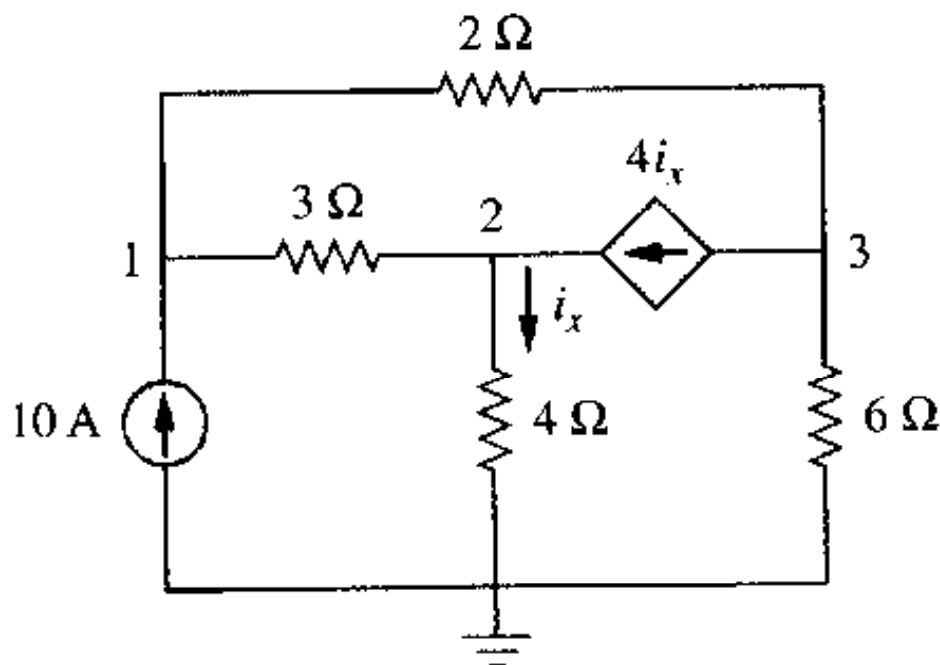


Circuit Theory

Problem 2.2

Determine the voltages at the nodes.

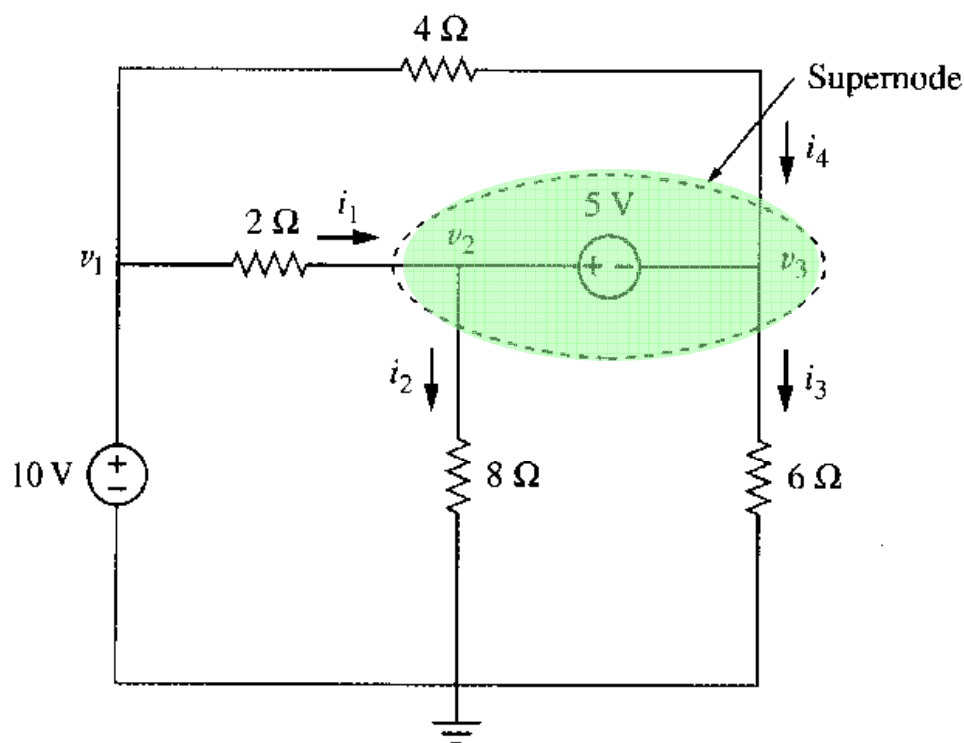
Ans : $v_1 = 80 \text{ V}$, $v_2 = -64 \text{ V}$, $v_3 = 156 \text{ V}$



Circuit Theory

Nodal Analysis with Voltage Sources

- How voltage sources affect nodal analysis. Consider the following two possibilities.



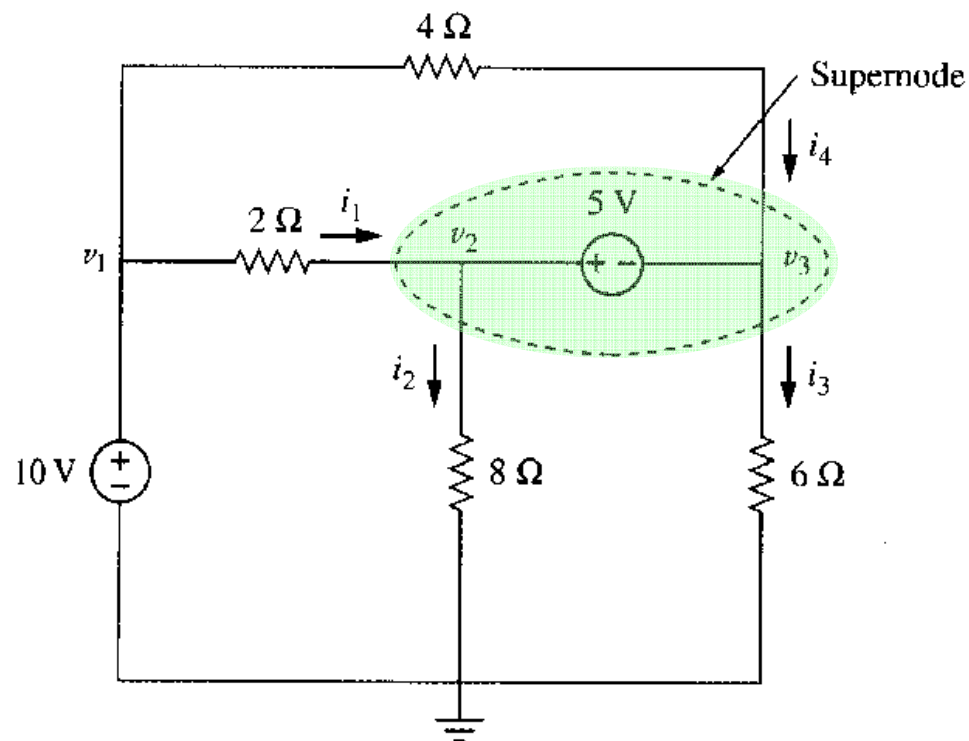
Case 1:

- If a voltage source is connected between the reference node and a nonreference node; $\Rightarrow v_1 = 10$ V.

Case 2:

- If the voltage source (dependent or independent) is connected between two nonreference nodes. The nonreference nodes form a *generalized node* or **supernode**.

Circuit Theory



- Nodes 2 and 3 form a supernode.
- There is no way knowing the current through a voltage source.
- KCL must be satisfied at a supernode, i.e.

$$i_1 + i_4 = i_2 + i_3$$

or

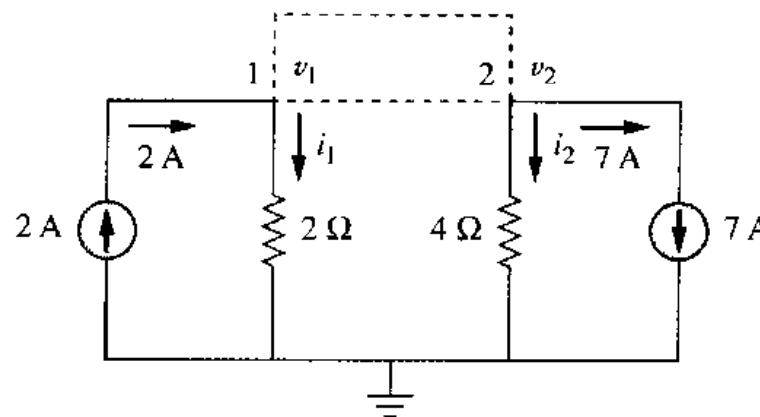
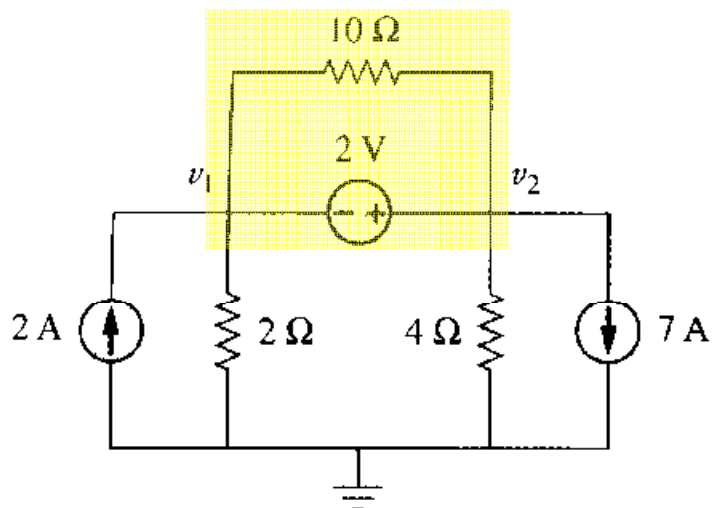
$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2}{8} + \frac{v_3}{6}$$

$$\text{and } v_2 - v_3 = 5$$

Circuit Theory

Example 2.3

For the circuit shown below, find the node voltages.



Applying KCL to the supernode;

$$2 = i_1 + i_2 + 7$$

where,

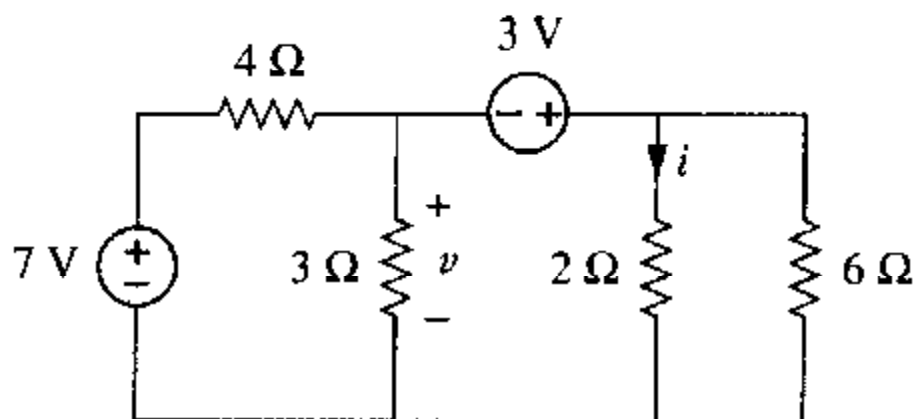
$$i_1 = \frac{v_1}{2} ; i_2 = \frac{v_2}{4} \quad \text{and} \quad v_2 - v_1 = 2$$

Circuit Theory

Problem 2.3

Find v and i .

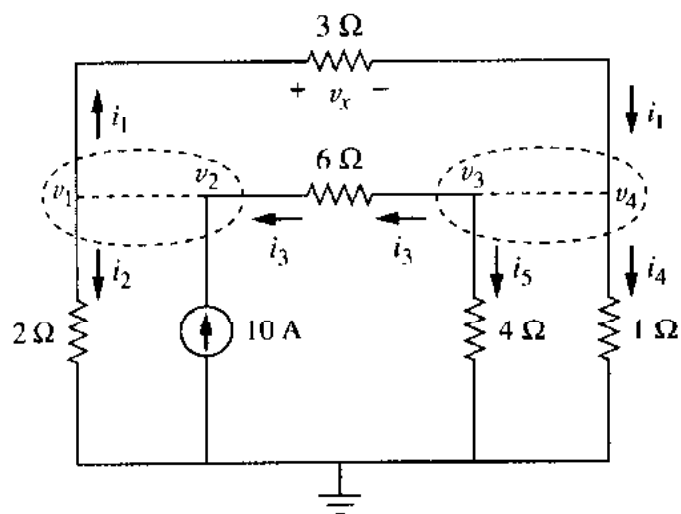
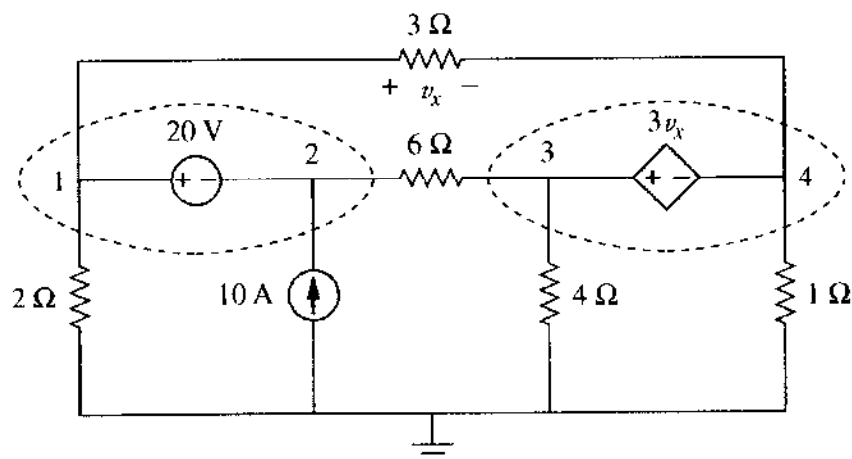
Ans: -0.2 V , 1.4 A



Circuit Theory

Example 2.4

Find the node voltages in the circuit.



Solution

KCL at supernode (1-2)

$$10 + i_3 = i_1 + i_2$$

$$\text{where, } i_3 = \frac{v_3 - v_2}{6}; \quad i_1 = \frac{v_1 - v_4}{3}; \quad i_2 = \frac{v_1}{2}$$

KCL at supernode (3-4)

$$i_1 = i_3 + i_4 + i_5$$

where, i_1 , and i_3 same as above and;

$$i_4 = \frac{v_4}{1}; \quad i_5 = \frac{v_3}{4}$$

Supportive equations :

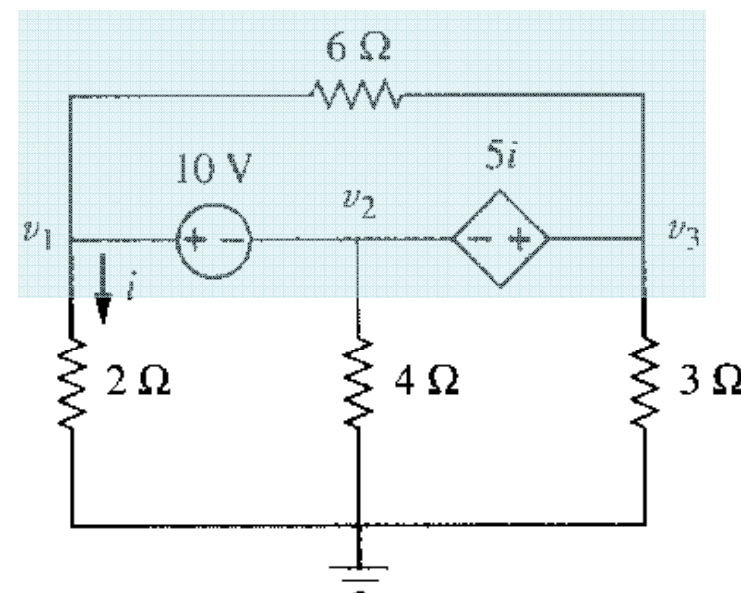
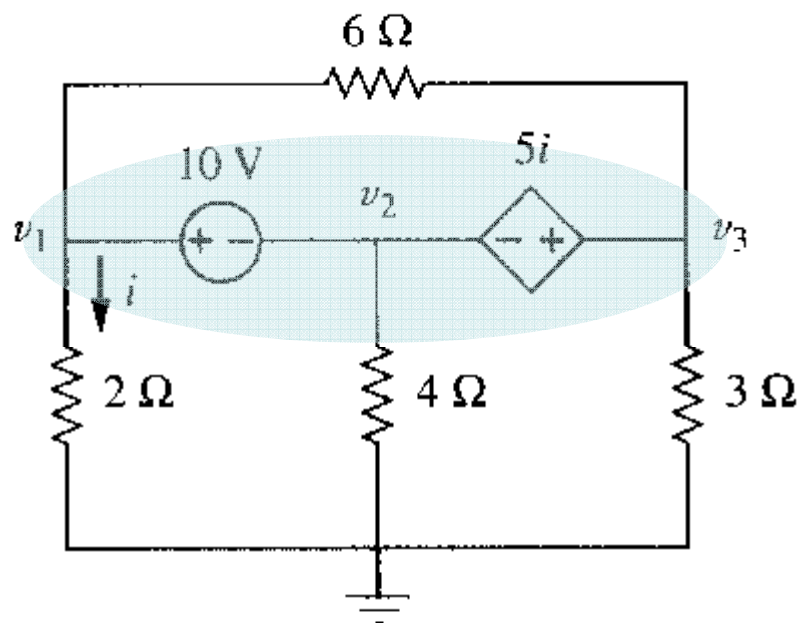
$$v_1 - v_2 = 20 \quad \text{and} \quad v_3 - v_4 = 3v_x = 3(v_1 - v_4)$$

Circuit Theory

Problem 2.4

Find v_1 , v_2 and v_3 in the circuit using nodal analysis.

Ans: $v_1 = 3.043$ V, $v_2 = -6.956$ V, $v_3 = 0.6522$



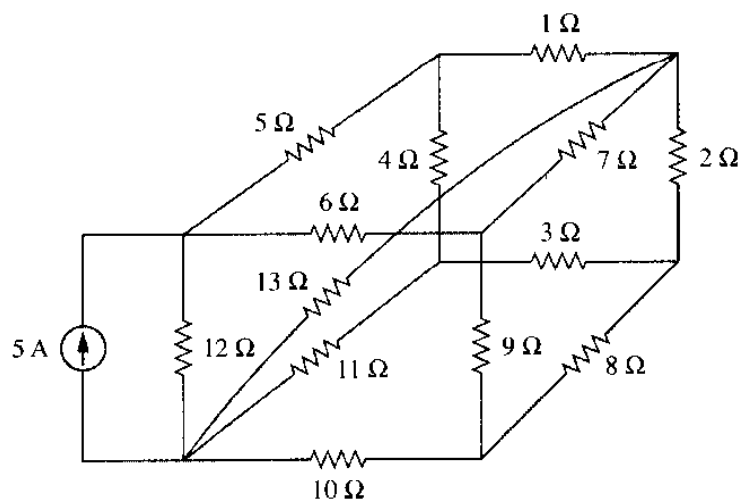
Circuit Theory

Mesh Analysis

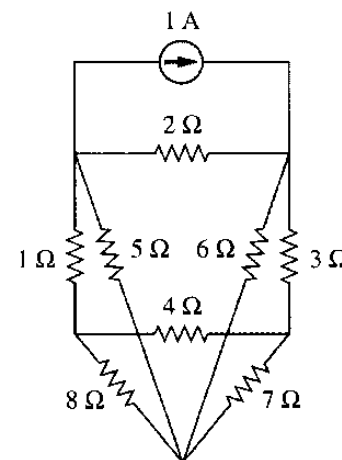
- ❑ Another general procedure for analyzing circuits, using **mesh current** as the **circuit variables**.
- ❑ Definitions: **Loop** is a closed path with no node passed more than once. **Mesh** is a loop that does not contain any other loop within it.
- ❑ Mesh analysis **applies KVL** to find **unknown currents**.
- ❑ Only applicable to a circuit that is *planar (two-dimensional)*. Nonplanar circuits can be handled using nodal analysis.

Circuit Theory

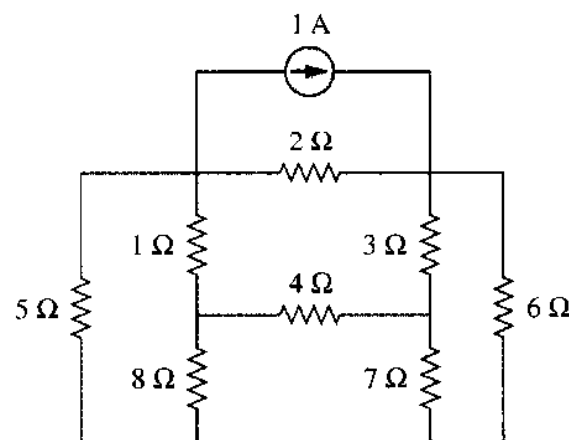
Types of circuit



A nonplanar circuit

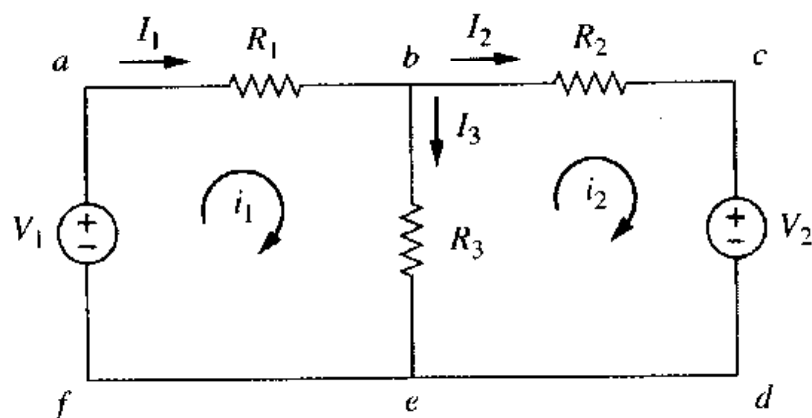


A planar circuit



The same circuit redrawn

Circuit Theory

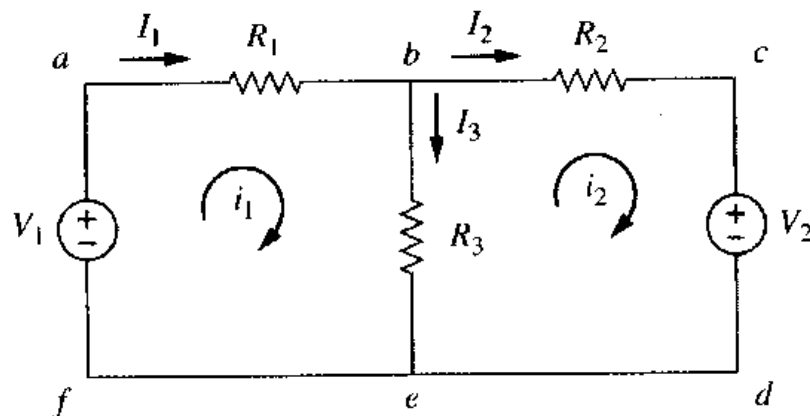


- Paths *abefa* and *bcdeb* are meshes. i_1 and i_2 are mesh current.
- In mesh analysis, we are interested in applying KVL to find the mesh current.

Steps to Determine Mesh Currents

- ❑ Assign mesh currents $i_1, i_2, \dots, i_n$ to the n meshes. Assume mesh currents flow clockwise.
- ❑ Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh current.
- ❑ Solve the resulting n simultaneous equations to get the mesh currents.

Circuit Theory



In matrix form;

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

For mesh 1

$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

or

$$(R_1 + R_3) i_1 - R_3 i_2 = V_1$$

For mesh 2

$$R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

or

$$-R_3 i_1 + (R_2 + R_3) i_2 = -V_2$$

- Notice that the branch currents are different from the mesh currents unless the mesh is isolated. i for a mesh current and I for a branch current.

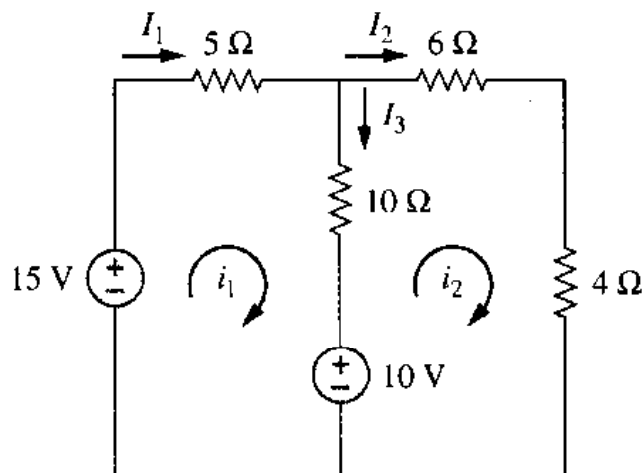
$$I_1 = i_1 ; \quad I_2 = i_2 ; \quad I_3 = i_1 - i_2$$



Circuit Theory

Example 2.5

Find the branch currents I_1 , I_2 and I_3 using mesh analysis.



In matrix form;

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Ans: $i_1 = 1 \text{ A}$; $i_2 = 1 \text{ A}$

Solution

Apply KVL for mesh 1

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1$$

Apply KVL for mesh 2

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$-i_1 + 2i_2 = 1$$

Therefore, the branch currents are;

$$I_1 = i_1 = 1 \text{ A}; I_2 = i_2 = 1 \text{ A}; I_3 = i_1 - i_2 = 0 \text{ A}$$

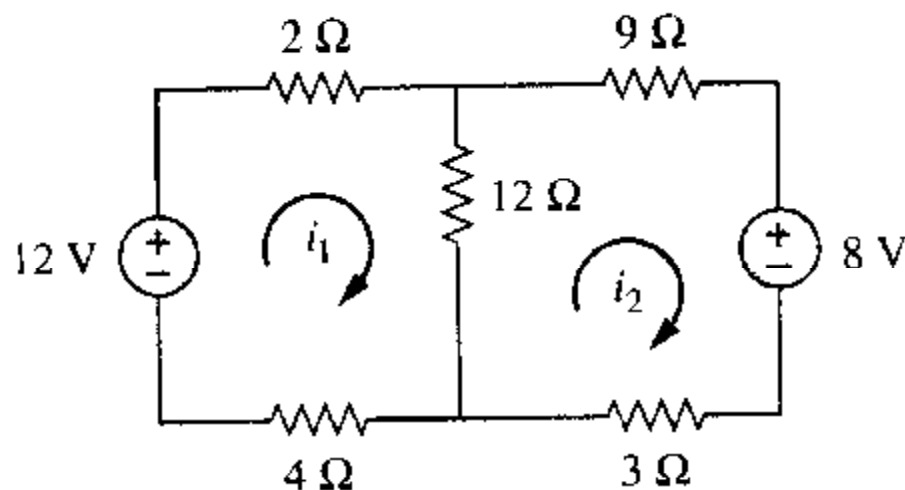


Circuit Theory

Problem 2.5

Calculate the mesh current i_1 and i_2 .

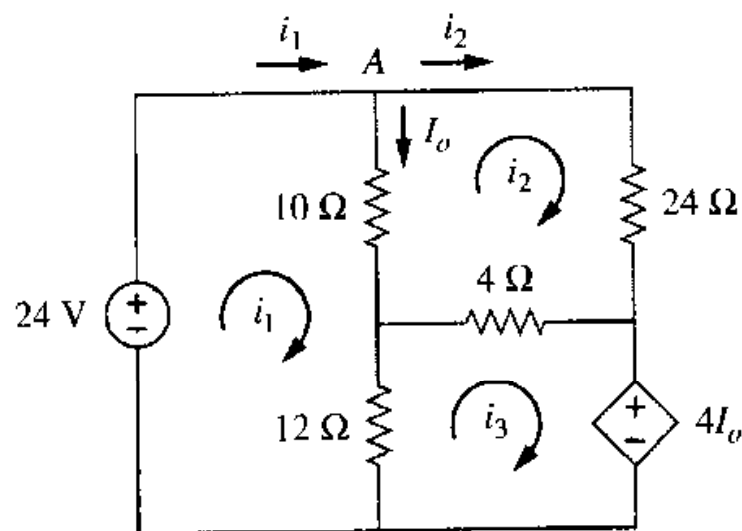
Ans: $i_1 = 2/3$ A, $i_2 = 0$ A



Circuit Theory

Example 2.6

Use mesh analysis to find the current I_o in the circuit.



Apply KVL for mesh 3

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

and $I_o = i_1 - i_2$; so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$\text{or } -i_1 - i_2 + 2i_3 = 0$$

Solution

Apply KVL for mesh 1

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12$$

Apply KVL for mesh 2

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0$$

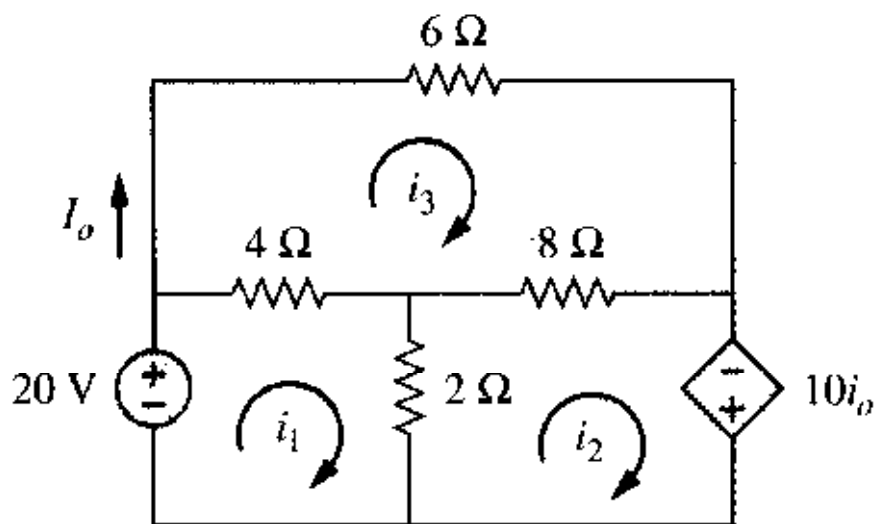
$$\Rightarrow \text{Ans: } I_o = i_1 - i_2 = 2.25 - 0.75 = 1.5 \text{ A}$$

Circuit Theory

Problem 2.6

Using mesh analysis, find I_o in the circuit.

Ans: -5 A

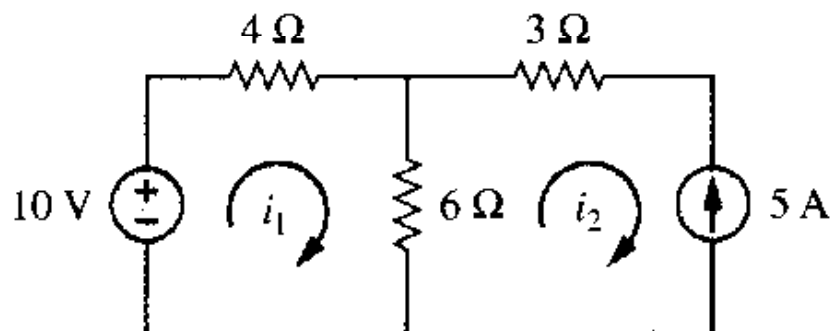


Circuit Theory

Mesh Analysis with Current Sources

- ❑ The presence of the current sources reduces the number of equations. The number of equations (based on KVL) that need to be write is $n - m$, where n : number of meshes and m : number of current sources.
- ❑ Consider the following 2 possible cases:

Case 1: When current source exists only in one mesh.

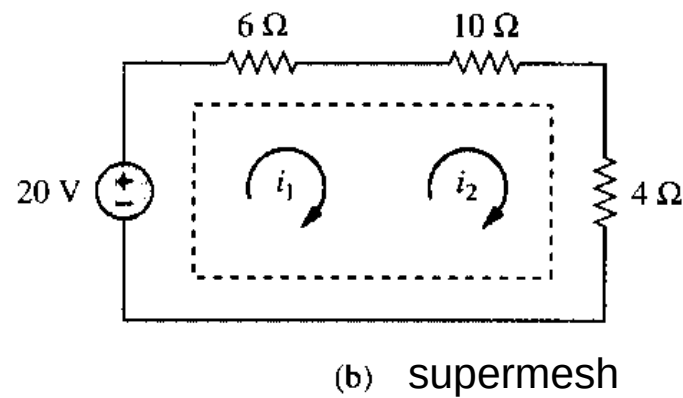
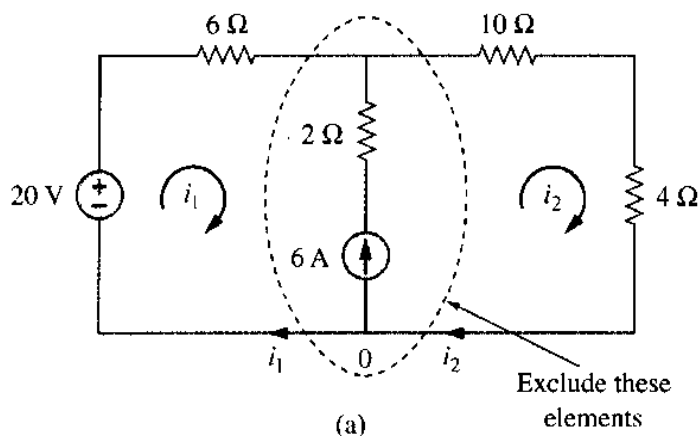


In the figure, $n - m = 2 - 1 = 1$; therefore,
KVL at mesh 1: $-10 + 4i_1 + 6(i_1 - i_2) = 0$
and $i_2 = -5$ (supportive equation)

Circuit Theory

www.utm.my

Case 1: When current source exists between two meshes. We create a supermesh by excluding the current source and any elements connected in series with it.



KVL at supermesh gives: $-20 + 6i_1 + 10i_2 + 4i_2 = 0$

or $6i_1 + 14i_2 = 20 \Rightarrow (a)$

Supportive equation: $i_2 - i_1 = 6 \Rightarrow (b)$

Solving eqns (a) and (b), we get,

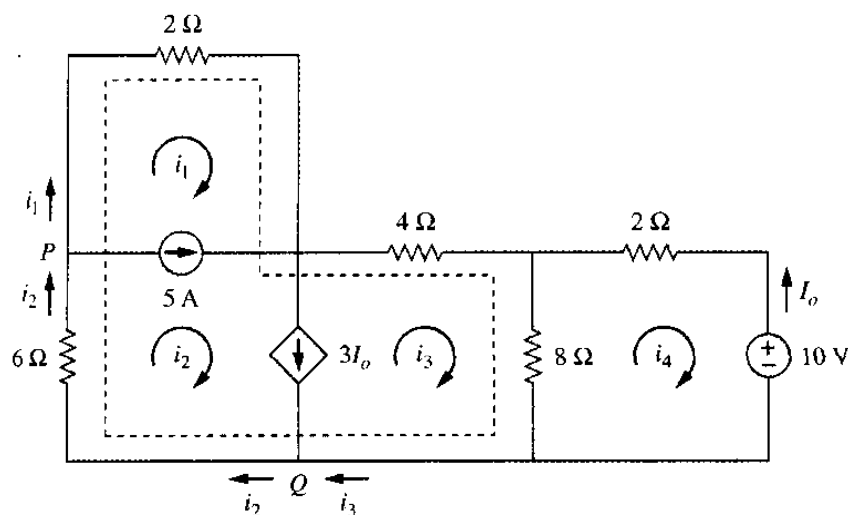
$i_1 = -3.2A$; $i_2 = 2.8A$

Circuit Theory

www.utm.my

Example 2.7

Find i_1 to i_4 using mesh analysis in the circuit.



Solution

Number of meshes, $n = 4$; Number of Current Sources, $m = 2$
 $\Rightarrow$ Number of equations need to be applied, $n - m = 4 - 2 = 2$

Meshes 1 & 2 form a supermesh. Also meshes 2 & 3 form another supermesh. The two supermeshes intersect and form a larger supermesh.

$\Rightarrow$ Applying KVL to the larger supermesh;

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$\text{or } i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \quad \Rightarrow (a)$$

$\Rightarrow$ Applying KVL in mesh 4;

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

$$\text{or } 5i_4 - 4i_3 = -5 \quad \Rightarrow (b)$$

Supportive equations;

$$\Rightarrow i_2 - i_1 = 5 \quad \Rightarrow (c)$$

$$\Rightarrow i_2 - i_3 = 3I_o = 3(-i_4) \quad \Rightarrow (d)$$

From eqns (a) to (d), we get,

$$i_1 = -7.5 \text{ A}, i_2 = -2.5 \text{ A}, i_3 = 3.93 \text{ A}, i_4 = 2.143 \text{ A}$$

Circuit Theory

Problem 2.7

Use mesh analysis to determine i_1 , i_2 and i_3 .

Ans: $i_1 = 3.474$ A; $i_2 = 0.4737$ A; $i_3 = 1.1052$ A

