FORMULA SSCE 1693

Trigonometric

$$\cos^2 x + \sin^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$\cot^2 x + 1 = \text{cosec}^2 x$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$
$$2 \sin x \cos y = \sin(x+y) + \sin(x-y)$$
$$2 \sin x \sin y = -\cos(x+y) + \cos(x-y)$$
$$2 \cos x \cos y = \cos(x+y) + \cos(x-y)$$

Hyperbolic

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{cosech}^2 x$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$
$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$
$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

Logarithm

$$a^x = e^{x \ln a}$$
$$\log_a x = \frac{\log_b x}{\log_b a}$$

Inverse Hyperbolic

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}), \ -\infty < x < \infty$$
$$\cosh^{-1} x = \ln(x + \sqrt{x^2-1}), \ x \geq 1$$
$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right), \ -1 < x < 1$$

Differentiations	Integrations
$\frac{d}{dx}[k] = 0$, k constant.	$\int k\,dx = kx + C$, k constant.
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$.
$\frac{d}{dx}[e^x] = e^x$.	$\int e^x dx = e^x + C$.
$\frac{d}{dx}[\ln\|x\|] = \frac{1}{x}$.	$\int \frac{dx}{x} = \ln\|x\| + C$.
$\frac{d}{dx}[\cos x] = -\sin x$.	$\int \sin x\,dx = -\cos x + C$.
$\frac{d}{dx}[\sin x] = \cos x$.	$\int \cos x\,dx = \sin x + C$.
$\frac{d}{dx}[\tan x] = \sec^2 x$.	$\int \sec^2 x\,dx = \tan x + C$.
$\frac{d}{dx}[\cot x] = -\text{cosec}^2 x$.	$\int \text{cosec}^2 x\,dx = -\cot x + C$.

Differentiations	Integrations
$\frac{d}{dx}[\sec x] = \sec x \tan x$.	$\int \sec x \tan x\,dx = \sec x + C$.
$\frac{d}{dx}[\text{cosec}\, x] = -\text{cosec}\, x \cot x$.	$\int \text{cosec}\, x \cot x\,dx = -\text{cosec}\, x + C$.
$\frac{d}{dx}[\cosh x] = \sinh x$.	$\int \sinh x\,dx = \cosh x + C$.
$\frac{d}{dx}[\sinh x] = \cosh x$.	$\int \cosh x\,dx = \sinh x + C$.
$\frac{d}{dx}[\tanh x] = \text{sech}^2 x$.	$\int \text{sech}^2 x\,dx = \tanh x + C$.
$\frac{d}{dx}[\coth x] = -\text{cosech}^2 x$.	$\int \text{cosech}^2 x\,dx = -\coth x + C$.
$\frac{d}{dx}[\text{sech}\, x] = -\text{sech}\, x \tanh x$.	$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$.
$\frac{d}{dx}[\text{cosech}\, x] = -\text{cosech}\, x \coth x$.	$\int \text{cosech}\, x \coth x\,dx = -\text{cosech}\, x + C$.

Differentiations of Inverse Functions
$\dfrac{d}{dx}[\sin^{-1} x] = \dfrac{1}{\sqrt{1-x^2}},\ \lvert x\rvert < 1.$
$\dfrac{d}{dx}[\cos^{-1} x] = \dfrac{-1}{\sqrt{1-x^2}},\ \lvert x\rvert < 1.$
$\dfrac{d}{dx}[\tan^{-1} x] = \dfrac{1}{1+x^2}.$
$\dfrac{d}{dx}[\cot^{-1} x] = \dfrac{-1}{1+x^2}.$
$\dfrac{d}{dx}[\sec^{-1} x] = \dfrac{1}{\lvert x\rvert\sqrt{x^2-1}},\ \lvert x\rvert > 1.$
$\dfrac{d}{dx}[\text{cosec}^{-1} x] = \dfrac{-1}{\lvert x\rvert\sqrt{x^2-1}},\ \lvert x\rvert > 1.$
$\dfrac{d}{dx}[\sinh^{-1} x] = \dfrac{1}{\sqrt{x^2+1}}.$
$\dfrac{d}{dx}[\cosh^{-1} x] = \dfrac{1}{\sqrt{x^2-1}},\ \lvert x\rvert > 1.$
$\dfrac{d}{dx}[\tanh^{-1} x] = \dfrac{1}{1-x^2},\ \lvert x\rvert < 1.$
$\dfrac{d}{dx}[\coth^{-1} x] = \dfrac{1}{1-x^2},\ \lvert x\rvert > 1.$
$\dfrac{d}{dx}[\text{sech}^{-1} x] = \dfrac{-1}{x\sqrt{1-x^2}},\ 0 < x < 1.$
$\dfrac{d}{dx}[\text{cosech}^{-1} x] = \dfrac{-1}{\lvert x\rvert\sqrt{1+x^2}},\ x \neq 0.$

Integrations Resulting in Inverse Functions
$\displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + C.$
$\displaystyle\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C.$
$\displaystyle\int \frac{dx}{\lvert x\rvert\sqrt{x^2-1}} = \sec^{-1}(x) + C.$
$\displaystyle\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1}(x) + C.$
$\displaystyle\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1}(x) + C,\ x > 0.$
$\displaystyle\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) + C,\ \lvert x\rvert < 1.$
$\displaystyle\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\left(\frac{x-1}{x+1}\right) + C,\ \lvert x\rvert > 1.$
$\displaystyle\int \frac{dx}{x\sqrt{1-x^2}} = -\text{sech}^{-1}(x) + C,\ \lvert x\rvert < 1.$
$\displaystyle\int \frac{dx}{\lvert x\rvert\sqrt{1+x^2}} = -\text{cosech}^{-1} x + C,\ x \neq 0.$