CHAPTER 8

POLAR COORDINATES

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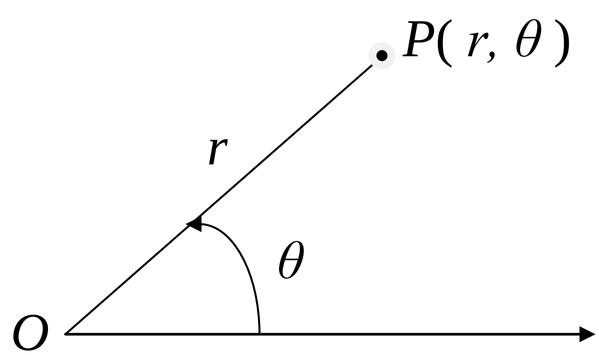
8.1 Polar Coordinates System

Definition:

The polar coordinates of point P is written as an ordered pair (r,θ), that is $P(r,\theta)$ where

r - distance from origin to P

θ - angle from polar axis to the line OP



Note:

(i) θ is <u>positive in anticlockwise</u> direction, and it is <u>negative in clockwise</u> direction.

(ii) Polar coordinate of a point is not unique.

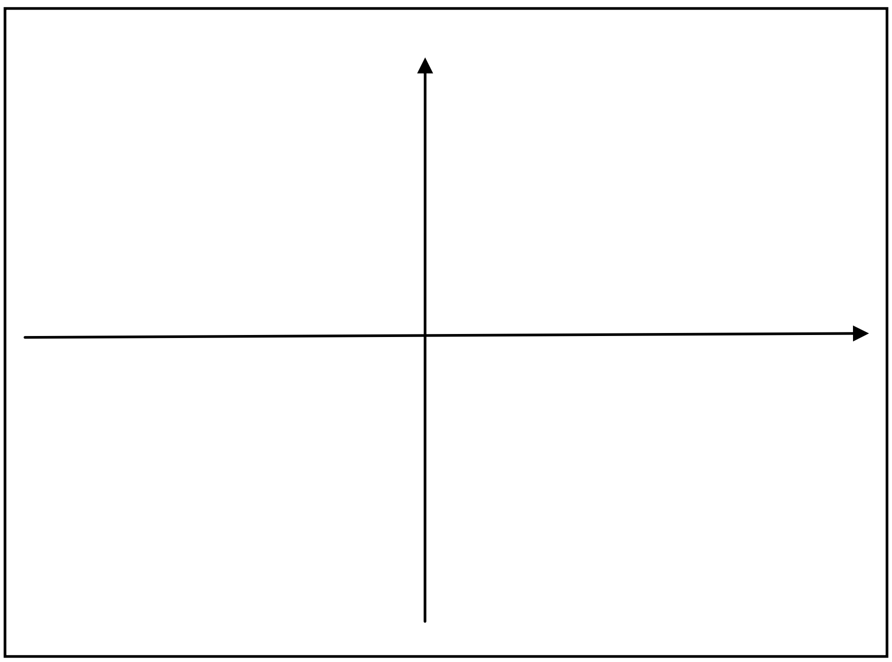
(iii) A point $(-r,\theta)$ is in the opposite direction of point (r,θ).

Example 1: Plot the following set of points in the same diagram:

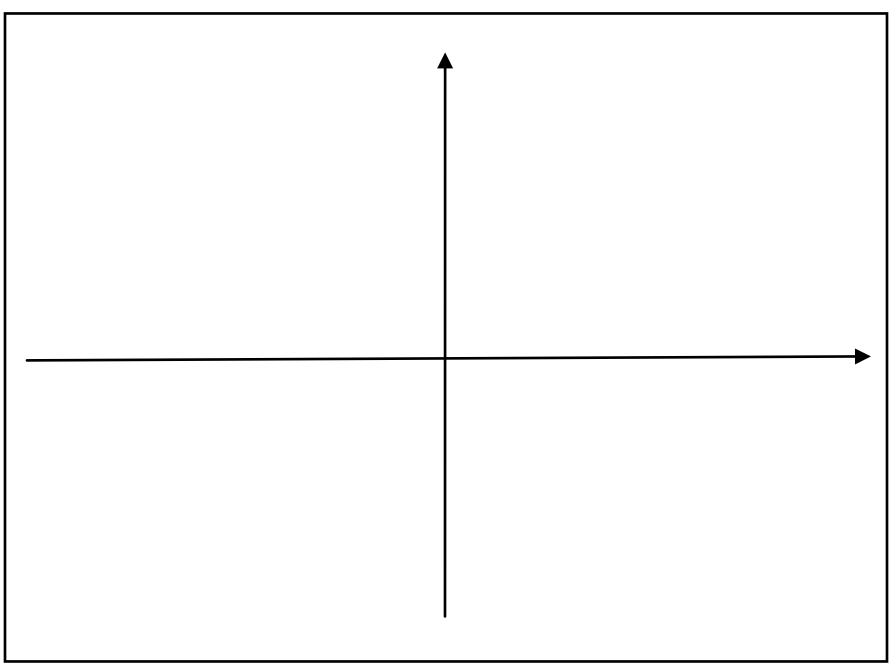
(a) $(3, 225^\circ)$, $(1, 225^\circ)$, $(-3, 225^\circ)$

(b) $\left(2, \frac{\pi}{3}\right)$, $\left(2, -\frac{\pi}{3}\right)$, $\left(-2, \frac{\pi}{3}\right)$

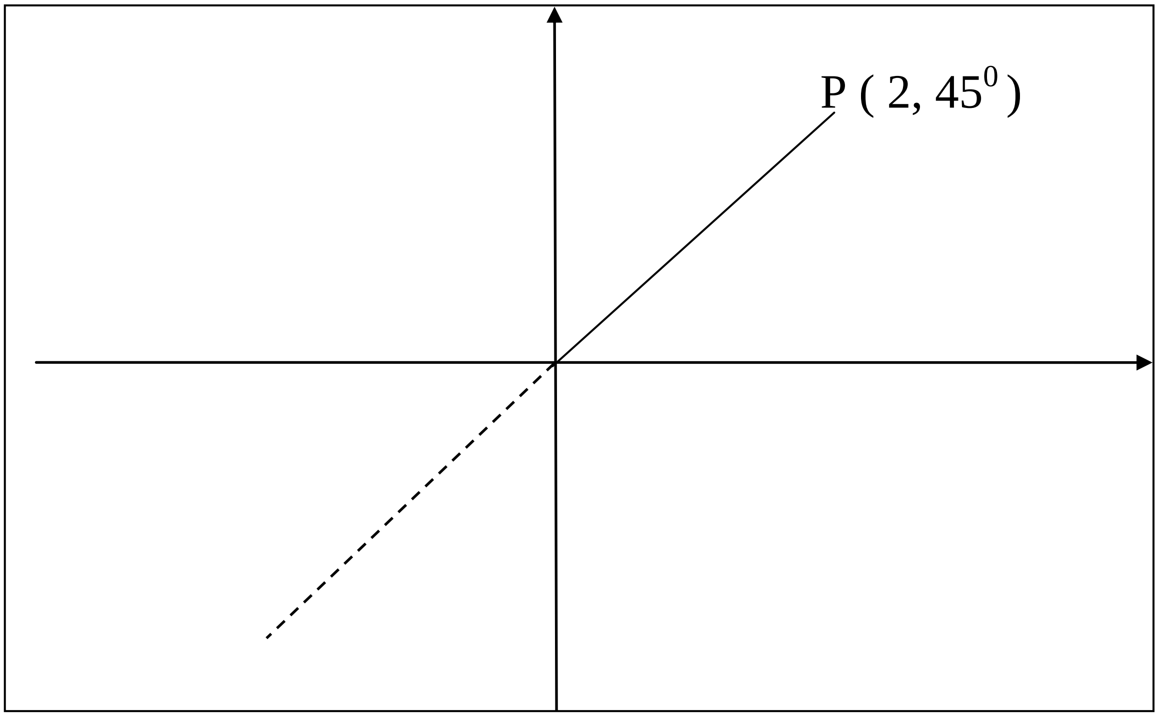
a)



b)



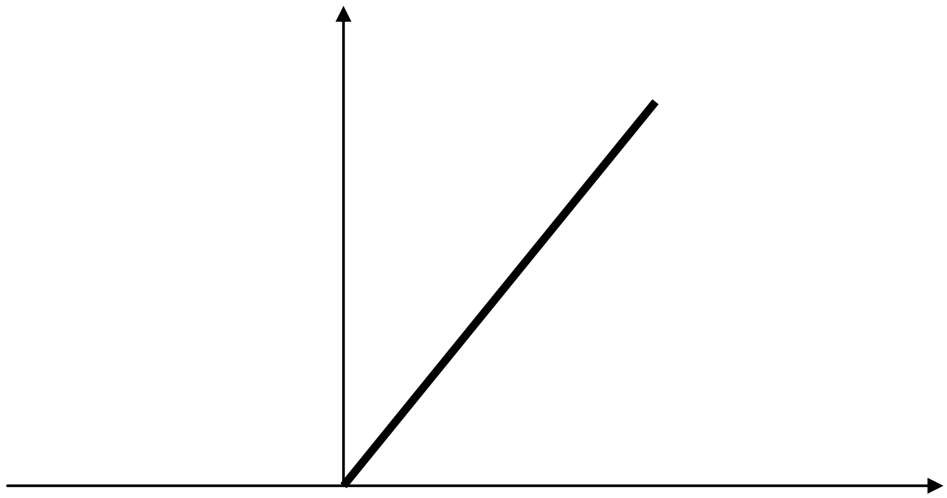
For every point $P(r,\theta)$ in $0 \le \theta \le 2\pi$, there exist 3 more coordinates that represent the point P.



Example 2:

Find all possible polar coordinates of the points whose polar coordinates are given as the following:
(a) $Q(2,-60^\circ)$ (b) $R(-1,225^\circ)$

8.2 Relationship between Cartesian and Polar Coordinates



1) Polar ⟹ Cartesian

$$x = r\cos\theta \qquad y = r\sin\theta$$

2) Cartesian ⟹ Polar

$$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}$$

Example 3: Find the Cartesian coordinates of the points whose polar coordinates are given as

(a) $\left(1, \frac{7\pi}{4}\right)$

(b) $\left(-4, \frac{2\pi}{3}\right)$

(c) $\left(2, -30^\circ\right)$

Example 4: Find all polar coordinates of the points whose rectangular coordinates are given as

(a) $(11,5)$

(b) $(0,2)$

(c) $(-4,-4)$

8.3 Forming polar equations from Cartesian equations and vice-versa.

To change the equation in Cartesian coordinates to polar coordinates, and conversely, use equation

$$x = r\cos\theta \qquad y = r\sin\theta \qquad r = \sqrt{x^2 + y^2}$$

Example 5: Express the following rectangular equations in polar equations.

(a) $y = x^2$ (b) $x^2 + y^2 = 16$ (c) $xy = 1$

Example 6: Express the following polar equations in rectangular equations.

(a) $r = 2\sin\theta$

(b) $r = \dfrac{3}{4\cos\theta + 5\sin\theta}$

(c) $r = 4\cos\theta + 4\sin\theta$

(d) $r = \tan\theta\sec\theta$

8.4 Graph Sketching of Polar Equations

There are two methods to sketch a graph of $r = f(\theta)$:

(1) Form a table for r and θ. $(0 \le \theta \le 2\pi)$.

From the table, plot the (r, θ) points.

(2) Symmetry test of the polar equation.

The polar equations is symmetrical

(a) about the x-axis if $f(-\theta) = f(\theta)$

- consider θ in range $[0, 180^0]$ only

(b) about the y-axis if $f(\pi - \theta) = f(\theta)$

- consider θ in range $[0, 90^0]$ **and** $[270^0, 360^0]$

(c) at the origin if $f(\pi + \theta) = f(\theta)$

- consider θ in range $[0, 180^0]$ **or** $[180^0, 360^0]$

* if symmetry at all, consider θ in range $[0, 90^0]$ **only.**

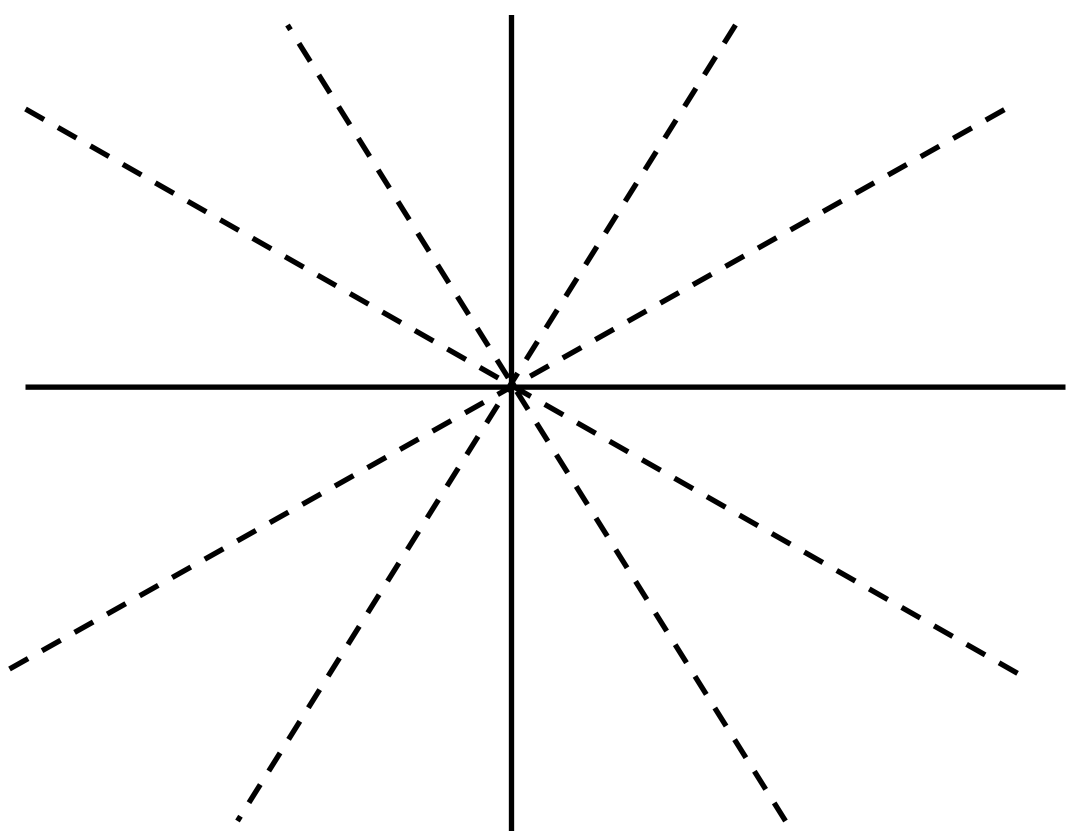
Example 7: Sketch the graph of $r = 2\sin\theta$

Solution: (Method 1)

Here is the complete table

θ	0	30	60	90	120	150	180	210
$r = 2\sin\theta$	0	1.0	1.732	2	1.732	1	0	-1.0

θ	240	270	300	330	360
$r = 2\sin\theta$	-1.732	-2	-1.732	-1	0



Method 2

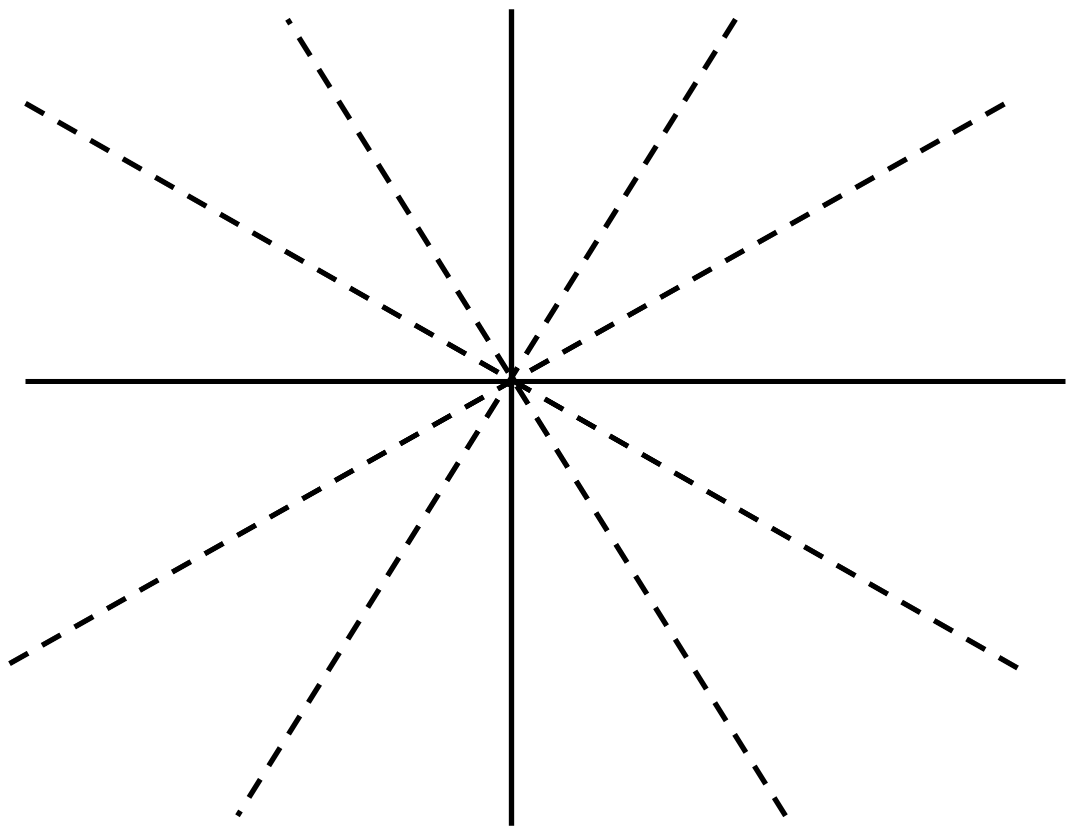
Symmetrical test for $f(\theta) = 2\sin\theta$

Symmetry	Symmetrical test
About x-axis	$f(-\theta) = f(\theta)$?
About y-axis	$f(\pi - \theta) = f(\theta)$?
About origin	$f(\pi + \theta) = f(\theta)$?

Since *r* symmetry at *y*-axis, consider θ in range

$[0, 90^0]$ **and** $[270^0, 360^0]$

θ	0	30	60	90	270	300	330	360
$r = 2\sin\theta$	0	1.0	1.732	2	-2	-1.732	-1	0

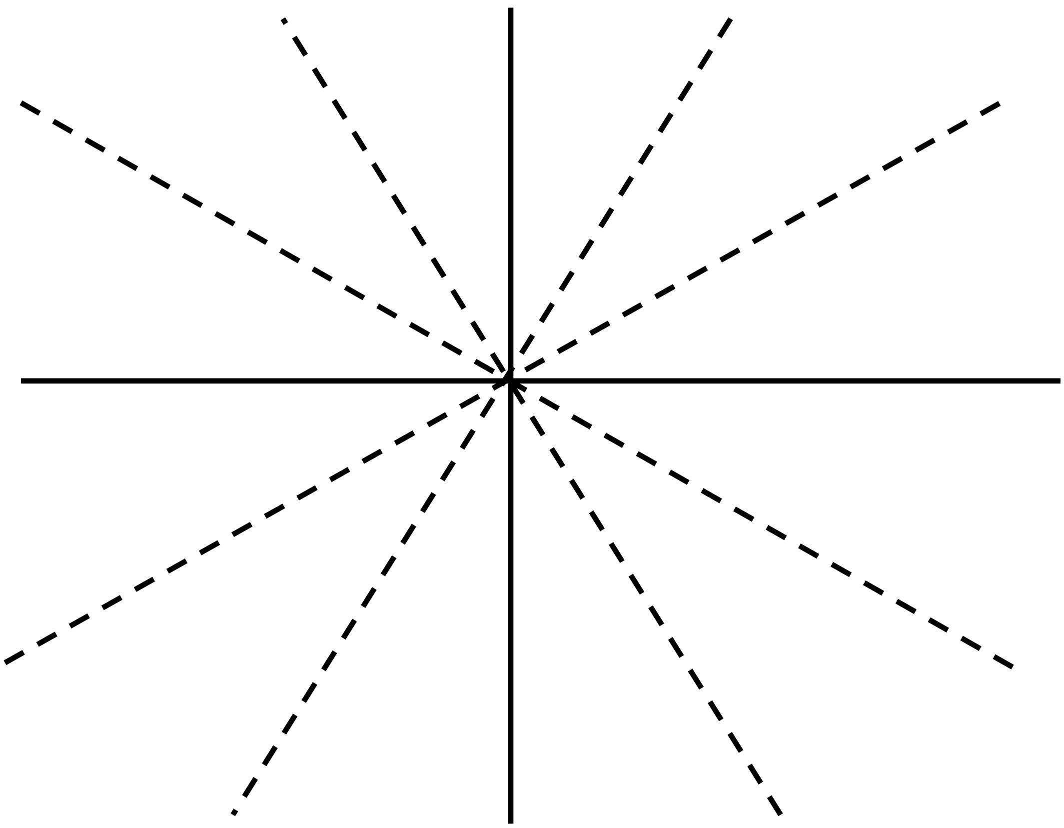


Example 8: Sketch the graph of $r = \frac{3}{2} - \cos\theta$

Symmetry	Symmetrical test
About x-axis	$f(-\theta) = f(\theta)$?
About y-axis	$f(\pi - \theta) = f(\theta)$?
About origin	$f(\pi + \theta) = f(\theta)$?

Since *r* symmetry at *x*-axis, consider θ in range [0, 180^0] only.

θ	0	30	60	90	120	150	180
$r = \frac{3}{2} - \cos\theta$							

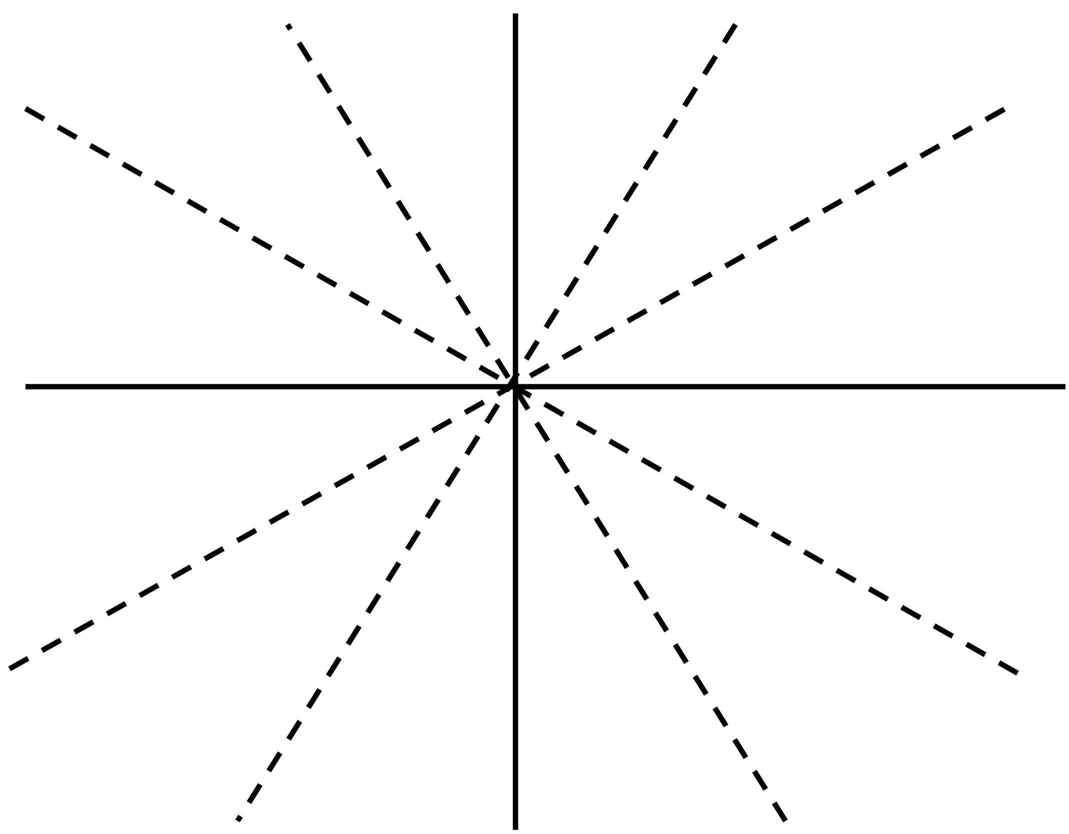


Example 9: Sketch the graph of $r = 2\sin^2\theta$

Symmetry	Symmetrical test
About x-axis	$f(-\theta) = f(\theta)$?
About y-axis	$f(\pi - \theta) = f(\theta)$?
About origin	$f(\pi + \theta) = f(\theta)$?

Since r symmetry at ______________, consider θ in range ______________ only.

θ							
$r = 2\sin^2\theta$							



8.5 Intersection Of Curves In Polar Coordinates.

- In Cartesian, we normally either solve two simultaneous equations or plot the graph in order to find the point of intersection between two curves.

- But in polar, method for finding points of intersection must include both solving simultaneous equations and sketching the curves.

Example 10:
Find the points of intersection of the circle $r = 2\cos\theta$ and $r = 2\sin\theta$ for $0 \le \theta \le \pi$

Solution:

♦ Solve simultaneous equations

$$2\cos\theta = 2\sin\theta \quad \Rightarrow \quad \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

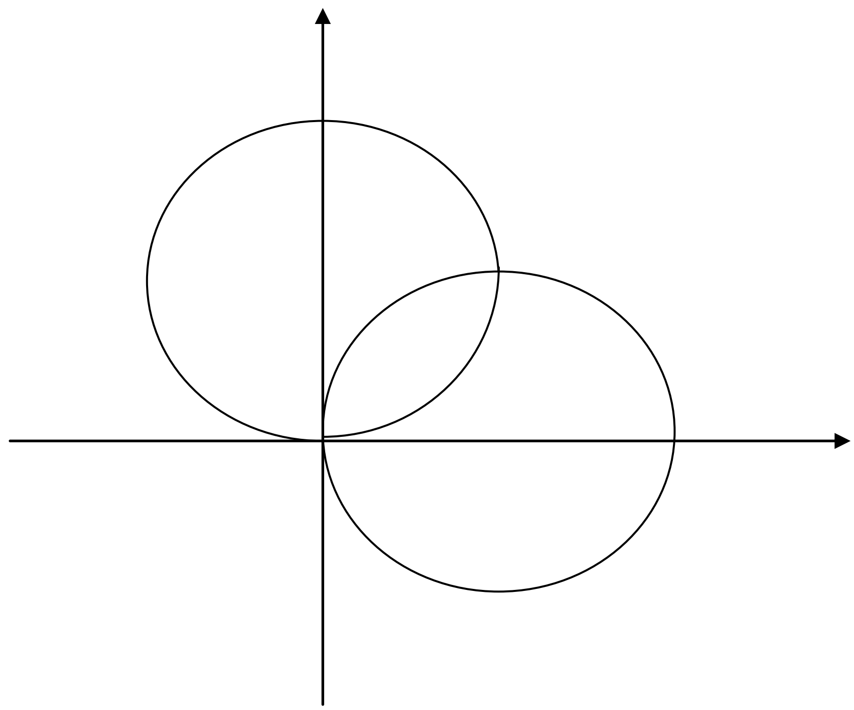
$$\text{when } \theta = \frac{\pi}{4}, \quad r = 2\cos\frac{\pi}{4} = \sqrt{2}$$

$$\text{when } \theta = \frac{5\pi}{4}, \quad r = 2\cos\frac{5\pi}{4} = -\sqrt{2}$$

So, point of intersection: $\left(\sqrt{2}, \frac{\pi}{4}\right)$

Note: $\left(\sqrt{2}, \frac{\pi}{4}\right)$ and $\left(-\sqrt{2}, \frac{5\pi}{4}\right)$ is the same point.

♦ Draw the curves:



From graph, (0,0) is also a point of intersection.

Conclusion: Points of intersection are $\left(\sqrt{2}, \frac{\pi}{4}\right)$ and (0,0).

Example 11:

Find the points of intersection of the curves

$r = \frac{3}{2} - \cos\theta$ and $\theta = \frac{2\pi}{3}$.

Solution:

♦ Solves simultaneous equations

♦ Draw the curves:

From the graph,

Conclusion: