Summary for chapter 1 : VECTORS

Basic Concept	Dot Product	Cross Product
Magnitude $\|\mathbf{v}\| = \sqrt{a^2 + b^2 + c^2}$ **Unit Vector** $\hat{\mathbf{u}}_{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$	**Definition** $\mathbf{u} \cdot \mathbf{v} = \langle d, e, f \rangle \cdot \langle a, b, c \rangle = d(a) + e(b) + f(c)$ **Angle Between 2 Vectors** $\theta = \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\right)$ **Types of Angle** θ is acute angle $\Leftrightarrow \mathbf{u} \cdot \mathbf{v} > 0$ θ is obtuse angle $\Leftrightarrow \mathbf{u} \cdot \mathbf{v} < 0$ $\theta = 90^\circ \Leftrightarrow \mathbf{u} \cdot \mathbf{v} = 0$ $\Leftrightarrow$ **u** and **v** are orthogonal $\Leftrightarrow$ **u** is perpendicular to **v**	**Definition** $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ d & e & f \\ a & b & c \end{vmatrix} = i(ec - bf) - j(dc - af) + k(db - ae)$ **Area of Parallelogram/Triangle** Area of parallelogram = $\|\mathbf{u} \times \mathbf{v}\|$ Area of triangle = $\frac{1}{2}\|\mathbf{u} \times \mathbf{v}\|$ **Note** **u** and **v** are parallel $\Leftrightarrow \mathbf{u} \times \mathbf{v} = 0$

	Line	Plane
Equation	**1. vector form** $< x, y, z >=< x_0, y_0, z_0 > +t < a, b, c >$ Eg: $< x, y, z >=< 2,-5,4 > +t < 3,-8,-1 >$ **2. parametric form** $x = x_0 + at,\ \ y = y_0 + bt,\ z = z_0 + ct$ Eg: $x = 5 + 2t, y = -2 + 2t, z = 3 - t$ **3. symmetrical form** $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$ Eg: $\frac{x+1}{3} = \frac{y-3}{4} = \frac{z+4}{2}$ Where: point on the line $= (x_0, y_0, z_0)$ $\mathbf{v} =< a, b, c >$	$ax + by + cy = d$ with $d = ax_0 + by_0 + cz_0$ where: point on the plane $= (x_0, y_0, z_0)$ normal vector, $\mathbf{N} =< a, b, c >$
Angle	**1. between 2 lines** $\cos\theta = \frac{\mathrm{u}\cdot\mathrm{v}}{\lvert\mathrm{u}\rvert\lvert\mathrm{v}\rvert}$ where ***u*** : vector parallel to line 1 ***v*** : vector parallel to line 2	**1. between 2 planes** $\cos\theta = \frac{N_1 \cdot N_2}{\lvert N_1\rvert\lvert N_2\rvert}$ where $\boldsymbol{N_1}$ = normal vector to plane 1 $\boldsymbol{N_2}$ = normal vector to plane 2 **2. between a line and a plane** $\sin\theta = \frac{V \cdot N}{\lvert V\rvert\lvert N\rvert}$ where ***V*** = vector parallel to line ***N*** = normal vector to plane

Intersection

<u>between 2 lines</u>

$$l_1 : x = x_1 + at \qquad l_2 : x = x_2 + ds$$
$$y = y_1 + bt \qquad y = y_2 + es$$
$$z = z_1 + ct \qquad z = z_2 + fs$$

$$\boldsymbol{v}_1 = \langle \mathrm{a}, \mathrm{b}, \mathrm{c} \rangle \qquad \boldsymbol{v}_2 = \langle d, e, f \rangle$$

	If YES	**If NOT**
1. Parallel	$\boldsymbol{v}_1 = \lambda \boldsymbol{v}_2$ Stop (conclude)	$\boldsymbol{v}_1 \neq \lambda \boldsymbol{v}_2$ Move to 2
2. Intersect	$x_1 + at = x_2 + ds$ $y_1 + bt = y_2 + es$ $z_1 + ct = z_2 + fs$ Find/get the value of s and t and make sure s and t satisfies all three equation Find/get the point of intersection (x, y, z)	The values of s and t not satisfy one of the three equation Move to 3
3. Skew	Stop (conclude)	

<u>between 2 planes</u>

To obtain the equation of a line of the intersection of 2 planes, we need

1. a vector parallel to the line
   $\Rightarrow \mathbf{N}_1 \times \mathbf{N}_2$
2. a point $(\mathrm{x}_0, \mathrm{y}_0, \mathrm{z}_0)$ on the line
   $\Rightarrow$ can be chosen by solving the equation of plane 1 and plane 2

Shortest distance	**<u>1. from a point to a line</u>** $d = \frac{\lvert \overline{v} \times \overline{PQ} \rvert}{\lvert \overline{v} \rvert}$ where V = vector parallel to line P = point **on** the line Q = point **to** the line **<u>2. between 2 skewed lines</u>** $D = \frac{\lvert \overline{N} \cdot \overline{PQ} \rvert}{\lvert \overline{N} \rvert}$ where ***N*** = ***U*** x ***V*** ***U*** = vector parallel to line 1 ***V*** = vector parallel to line 2 P = point on line 1 Q = point on line 2	**<u>1. from a point to a plane</u>** $D = \frac{\lvert ax_1 + by_1 + cz_1 - d \rvert}{\lvert \sqrt{a^2 + b^2 + c^2} \rvert}$ where $< a,b,c >$ = normal vector to plane (x_1,y_1,z_1) = point **to** the plane **<u>2. between 2 parallel planes</u>** $D = \frac{\lvert d_1 - d_2 \rvert}{\sqrt{a^2 + b^2 + c^2}}$