UNIVERSITI TEKNOLOGI MALAYSIA
SSCE 1693 ENGINEERING MATHEMATICS I
TUTORIAL 8: POLAR & COMPLEX NUMBER

POLAR

1. Sketch the following curves.

   (a) $x = 2t^2, \quad y = 4t.$ (b) $x = 3 + 2t^2, \quad y = -4t.$

   (c) $x = 3\cos\theta, \quad y = 3\sin\theta.$ (d) $x = 2 + 3\cos\theta, \quad y = 3\sin\theta - 1.$

2. Find the Cartesian equation for each of the following parametric equations.

   (a) $x = 4t, \quad y = \dfrac{4}{t}.$ (b) $x = 2 - m, \quad y = m^2 + 4.$

   (c) $x = 2 - 5\cos\theta, \quad y = 1 - 3\sin\theta.$ (d) $x = 3t^2, \quad y = 6t.$

3. Plot the following points.

   (a) $(3,\, 150°).$ (b) $(4,\, \frac{1}{2}\pi).$ (c) $(1,\, -120°).$

   (d) $(2,\, -\frac{7}{6}\pi).$ (e) $(2,\, \pi).$ (f) $(1,\, 2).$

4. Express the points in Question 3 in Cartesian coordinates.

5. Express the following equations in polar equations.

   (a) $x + y = 1.$ (b) $xy = a^2.$ (c) $x = 4.$

   (d) $y = 2.$ (e) $x^2 + y^2 = 2x.$ (f) $x^2 = y.$

6. Find the points of intersection between each of the following pairs of curves.

   (a) $r = 2\cos 2\theta$ and $r = 2\sin\theta.$ (b) $r = 1$ and $r^2 = 2\cos 2\theta.$

   (c) $r = a$ and $r = 2a\sin\theta.$ (d) $r = 1 + \cos\theta$ and $r = 3\cos\theta.$

7. By plotting the points $(r,\, \theta)$, for θ between 0 to 2π, sketch the graphs of the following polar equations.

   (a) $r = 3.$ (b) $r = \frac{1}{3}\pi.$ (c) $r = 1 - \cos\theta.$

   (d) $r = 2 + \cos\theta.$ (e) $r = 2\sin\theta.$ (f) $r = 3\sin 2\theta.$

8. Test the symmetries of the following equations. Hence, sketch the graphs.

   (a) $r = 2 + 3\cos\theta.$ (b) $r = 3\cos 2\theta.$ (c) $r = 1 - \sin\theta.$

   (d) $r = 2 - 3\sin\theta.$ (e) $r = a\sin 3\theta.$ (f) $r^2 = 2a^2\cos 2\theta.$

COMPLEX NUMBER

9. Find the complex numbers w and v such that
$$z^2 - 4 - 2i\,z = (z + i\,v)^2 - wz\,i,$$
for all z.

10. If $z = x + iy$, find the real values of x and y that satisfy the equation
$$\frac{2z}{1+i} - \frac{2z}{i} = \frac{5}{2+i}.$$

11. Solve the following equations and sketch the roots on an Argand diagram.

(a) $z^3 + 8i = 0.$ (b) $z^6 = -1.$ (c) $z^4 + 81 = 0.$ (d) $9z^2 + 18 = 0.$

12. Prove that the modulus of $2 + \cos\theta + i\sin\theta$ is $(5 + 4\cos\theta)^{1/2}$. Hence show that the modulus of
$$\frac{2 + \cos\theta + i\sin\theta}{2 + \cos\theta - i\sin\theta}$$
is unity.

13. Find all the roots of the following equations.

(a) $z^2 - 4iz - 4 - 2i = 0.$ (b) $3z^2 - (2 + 11i)z + 3 - 5i = 0.$

(c) $z^2 + (1 - i)z + 3 - 5i = 4 - 7i.$ (d) $z^2 + (1 + 3i)z + 2 = 0.$

14. Without using calculator, find the complex number w such that $w^2 = -2 + 2\sqrt{3}i$. Hence, solve
$$z^2 + \left(8 - 4\sqrt{3}i\right)z + \left(12 - 24\sqrt{3}\,i\right) = 0.$$

15. Find in the form of $r\,\mathrm{cis}\,\theta$ the complex numbers z that satisfy the equation
$$\frac{1 + z^2}{1 - z^2} = i.$$

16. Find the three roots of the equation $z^3 = 1$. Show that if α and β are the two roots with non–zero imaginary parts, then $\alpha^2 = \beta$ and $\beta^2 = \alpha$.

17. Find the solutions of the equation
$$z^4 + 16z^2 + 100 = 0,$$
in the form $a + i\,b$.

18. Find the three roots of the equation
$$8x^3 = (2 - x)^3,$$
expressing each in the form $a + i\,b$.

19. Write down the solutions of the equation $w^4 = 16$. Hence deduce that the solutions of the equations $(z+2)^4 = 16(z-1)^4$.

20. A complex number z is such that $z + z^{-1} = \sqrt{3}$. Find the value of $z^4 + z^{-4}$ and determine the set of values for n so that $z^n + z^{-n} = 0$.

21. Use De Moivre's Theorem to write the following complex numbers in the form of $a + i\,b$.

(a) $(-1+i)^{29}$. (b) $(-1-i)^{12}$. (c) $(1-i)^{17}$.

(d) $(2+2i)^{36}$. (e) $(\sqrt{3}-i)^{13}$. (f) $(-\sqrt{3}+i)^{15}$.

22. Find all the roots of each of the following complex numbers.

(a) $(2\sqrt{3}-2i)^{\frac{1}{2}}$. (b) $i^{\frac{1}{3}}$. (c) $(-4+4i)^{\frac{1}{5}}$. (d) $256^{\frac{1}{4}}$.

23. Find the roots following equations in the form $r\text{cis}\,\theta$ and sketch its on an Argand diagram.

(a) $z^2 - \sqrt{5}\,i = 0$. (b) $z^3 + 8i = 0$. (c) $z^5 + 32 = 0$. (d) $z^6+1 = \sqrt{3}\,i$.

24. Express $\dfrac{1 + i\tan\theta}{1 - i\tan\theta}$ in terms $r(\cos\theta + i\,\sin\theta)$. Hence, obtain the cube roots of

$$\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}}.$$

25. By using De Moivre's Theorem, prove that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$$

Hence, find all the roots of the equation

$$16x^5 - 20x^3 + 5x = 1.$$

26. By using De Moivre's Theorem prove that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$

Hence, find all the roots of the equation

$$64x^6 - 96x^4 + 36x^2 - 3 = 0.$$

27. If $x + iy = \cos(u + iv)$, where x, y, u, v are real, prove that

(a) $(1+x)^2 + y^2 = (\cosh v + \cos u)^2$. (b) $(1-x)^2 + y^2 = (\cosh v - \cos u)^2$.

28. Express $\sin ix$ and $\cos ix$ in terms of $\sinh x$ and $\cosh x$ respectively.
If $x + iy = \tan(u + iv)$, prove that

(a) $\coth 2v = \dfrac{x^2+y^2+1}{2y}$. (b) $\cot 2u = \dfrac{1-x^2-y^2}{2x}$.