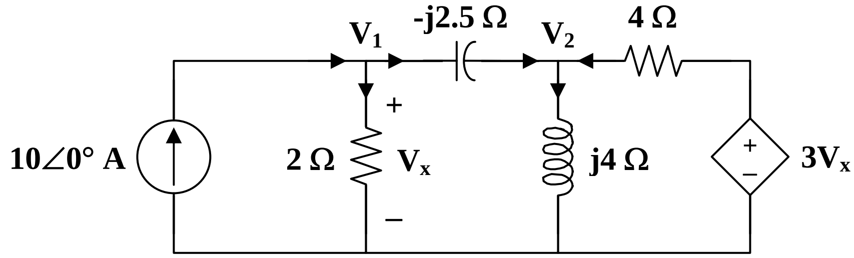
CHAPTER 10

P.P.10.1 $10\sin(2t) \longrightarrow 10\angle 0°, \quad \omega = 2$

$2\text{ H} \longrightarrow j\omega L = j4$

$0.2\text{ F} \longrightarrow \dfrac{1}{j\omega C} = \text{-}j2.5$

Hence, the circuit in the frequency domain is as shown below.



At node 1, $10 = \dfrac{\mathbf{V}_1}{2} + \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{\text{-}j2.5}$

$100 = (5 + j4)\,\mathbf{V}_1 - j4\mathbf{V}_2$ (1)

At node 2, $\dfrac{\mathbf{V}_2}{j4} = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{\text{-}j2.5} + \dfrac{3\mathbf{V}_x - \mathbf{V}_2}{4}$ where $\mathbf{V}_x = \mathbf{V}_1$

$\text{-}j2.5\mathbf{V}_2 = j4(\mathbf{V}_1 - \mathbf{V}_2) + 2.5(3\mathbf{V}_1 - \mathbf{V}_2)$

$0 = \text{-}(7.5 + j4)\,\mathbf{V}_1 + (2.5 + j1.5)\,\mathbf{V}_2$ (2)

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 5 + j4 & \text{-}j4 \\ \text{-}(7.5 + j4) & 2.5 + j1.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

where $\Delta = (5 + j4)(2.5 + j.15) - (\text{-}j4)(\text{-}(7.5 + j4)) = 22.5 - j12.5 = 25.74\angle \text{-}29.05°$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \frac{\begin{bmatrix} 2.5 + j1.5 & j4 \\ 7.5 + j4 & 5 + j4 \end{bmatrix}}{22.5 - j12.5} \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\mathbf{V}_1 = \frac{2.5 + j1.5}{22.5 - j12.5}(100) = \frac{2.915\angle 30.96°}{25.74\angle \text{-}29.05°}(100) = 11.32\angle 60.01°$$

$$\mathbf{V}_2 = \frac{7.5 + j4}{22.5 - j12.5}(100) = \frac{8.5\angle 28.07°}{25.74\angle \text{-}29.05°}(100) = 33.02\angle 57.12°$$

In the time domain,

$$v_1(t) = \underline{\mathbf{11.32\ sin(2t + 60.01°)\ V}}$$
$$v_2(t) = \underline{\mathbf{33.02\ sin(2t + 57.12°)\ V}}$$

P.P.10.2 The only non-reference node is a supernode.

$$\frac{15 - \mathbf{V}_1}{4} = \frac{\mathbf{V}_1}{j4} + \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{2}$$
$$15 - \mathbf{V}_1 = -j\mathbf{V}_1 + j4\mathbf{V}_2 + 2\mathbf{V}_2$$
$$15 = (1 - j)\mathbf{V}_1 + (2 + j4)\mathbf{V}_2 \qquad (1)$$

The supernode gives the constraint of

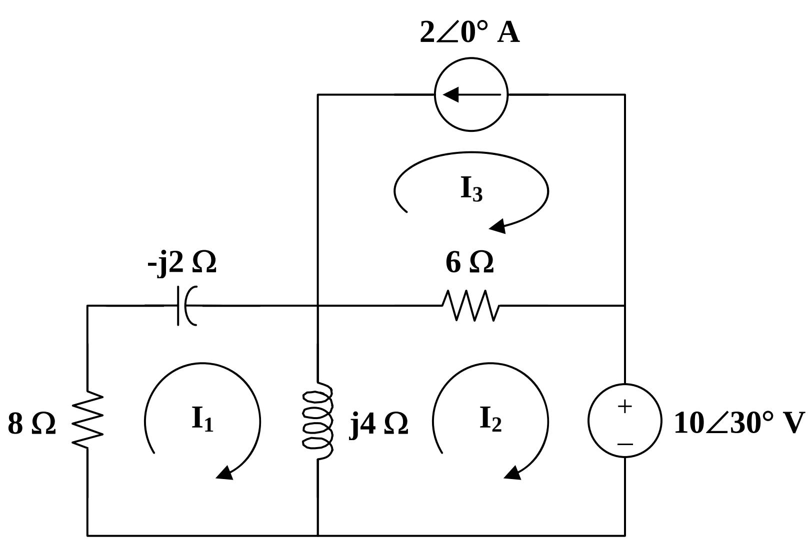
$$\mathbf{V}_1 = \mathbf{V}_2 + 20\angle 60° \qquad (2)$$

Substituting (2) into (1) gives

$$15 = (1 - j)(20\angle 60°) + (3 + j3)\mathbf{V}_2$$
$$\mathbf{V}_2 = \frac{15 - (1 - j)(20\angle 60°)}{3 + j3} = \frac{14.327\angle 210.72°}{4.243\angle 45°} = 3.376\angle 165.7°$$
$$\mathbf{V}_1 = \mathbf{V}_2 + 20\angle 60° = (-3.272 + j0.8327) + (10 + j17.32)$$
$$\mathbf{V}_1 = 6.728 + j18.154$$

Therefore, $\mathbf{V}_1 = \underline{\mathbf{19.36\angle 69.67°\ V}}$, $\mathbf{V}_2 = \underline{\mathbf{3.376\angle 165.7°\ V}}$

P.P.10.3 Consider the circuit below.



For mesh 1, $(8 - j2 + j4)\mathbf{I}_1 - j4\mathbf{I}_2 = 0$

$$(8 + j2)\mathbf{I}_1 = j4\mathbf{I}_2 \qquad (1)$$

For mesh 2, $(6+j4)\mathbf{I}_2 - j4\mathbf{I}_1 - 6\mathbf{I}_3 + 10\angle 30° = 0$

For mesh 3, $\mathbf{I}_3 = -2$

Thus, the equation for mesh 2 becomes

$$(6+j4)\mathbf{I}_2 - j4\mathbf{I}_1 = -12 - 10\angle 30° \qquad (2)$$

From (1), $$\mathbf{I}_2 = \frac{8+j2}{j4}\mathbf{I}_1 = (0.5 - j2)\mathbf{I}_1 \qquad (3)$$

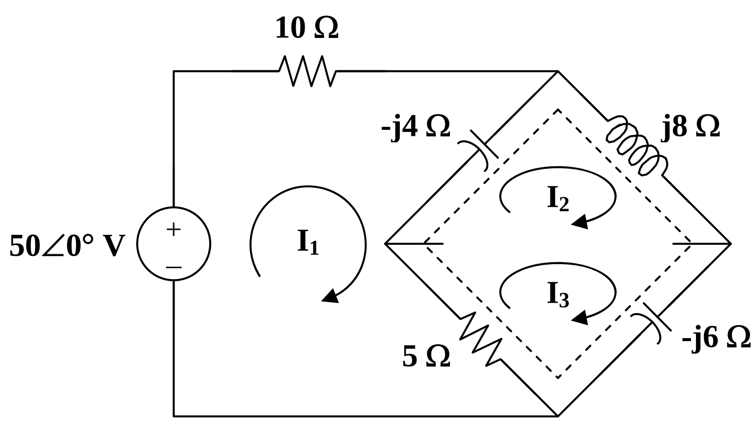
Substituting (3) into (2),

$$(6+j4)(0.5-j2)\mathbf{I}_1 - j4\mathbf{I}_1 = -12 - 10\angle 30°$$
$$(11 - j14)\mathbf{I}_1 = -(20.66 + j5)$$
$$\mathbf{I}_1 = \frac{-(20.66+j5)}{11-j14}$$

Hence, $$\mathbf{I}_o = -\mathbf{I}_1 = \frac{20.66+j5}{11-j14} = \frac{21.256\angle 13.6°}{17.8\angle -51.84°}$$

$$\mathbf{I}_o = \underline{\mathbf{1.194\angle 65.44°\ A}}$$

P.P.10.4 Meshes 2 and 3 form a supermesh as shown in the circuit below.



For mesh 1, $$-50 + (15 - j4)\mathbf{I}_1 - (-j4)\mathbf{I}_2 - 5\mathbf{I}_3 = 0$$
$$(15 - j4)\mathbf{I}_1 + j4\mathbf{I}_2 - 5\mathbf{I}_3 = 50 \qquad (1)$$

For the supermesh, $$(j8 - j4)\mathbf{I}_2 + (5 - j6)\mathbf{I}_3 - (5 - j4)\mathbf{I}_1 = 0 \qquad (2)$$

Also, $$\mathbf{I}_3 = \mathbf{I}_2 + 2 \qquad (3)$$

Eliminating $\mathbf{I}_3$ from (1) and (2)

$$(15 - j4)\mathbf{I}_1 + (-5 + j4)\mathbf{I}_2 = 60 \qquad (4)$$
$$(-5 + j4)\mathbf{I}_1 + (5 - j2)\mathbf{I}_2 = -10 + j12 \qquad (5)$$

From (4) and (5),

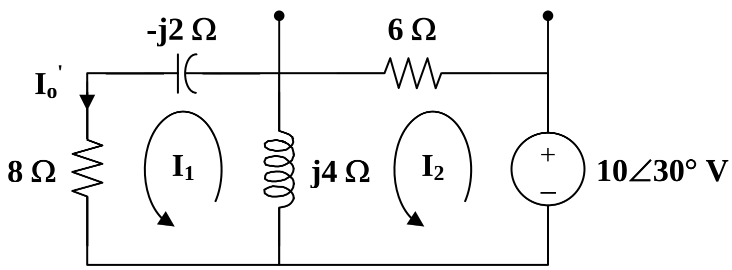
$$\begin{bmatrix} 15 - j4 & -5 + j4 \\ -5 + j4 & 5 - j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 60 \\ -10 + j12 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 15 - j4 & -5 + j4 \\ -5 + j4 & 5 - j2 \end{vmatrix} = 58 - j10 = 58.86\angle -9.78°$$

$$\Delta_1 = \begin{vmatrix} 60 & -5 + j4 \\ -10 + j12 & 5 - j2 \end{vmatrix} = 298 - j20 = 298.67\angle -3.84°$$

Thus, $\mathbf{I}_o = \mathbf{I}_1 = \dfrac{\Delta_1}{\Delta} = \underline{\mathbf{5.074\angle 5.94°\ A}}$

P.P.10.5 Let $\mathbf{I}_o = \mathbf{I}_o^{'} + \mathbf{I}_o^{''}$, where $\mathbf{I}_o^{'}$ and $\mathbf{I}_o^{''}$ are due to the voltage source and current source respectively. For $\mathbf{I}_o^{'}$ consider the circuit in Fig. (a).



(a)

For mesh 1, $(8 + j2)\mathbf{I}_1 - j4\mathbf{I}_2 = 0$

$$\mathbf{I}_2 = (0.5 - j2)\mathbf{I}_1 \qquad (1)$$

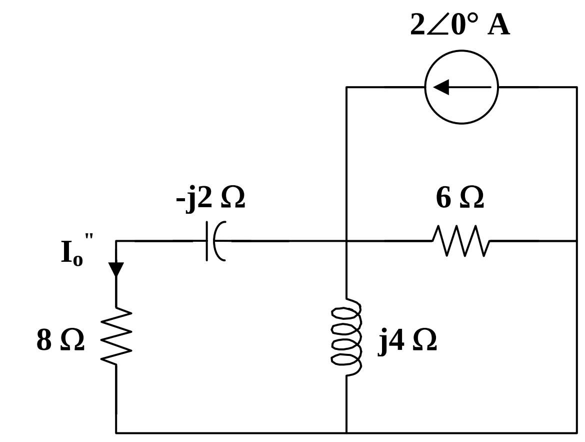
For mesh 2, $(6 + j4)\mathbf{I}_2 - j4\mathbf{I}_1 - 10\angle 30° = 0 \qquad (2)$

Substituting (1) into (2),

$$(6 + j4)(0.5 - j2)\mathbf{I}_1 - j4\mathbf{I}_1 = 10\angle 30°$$

$$\mathbf{I}_o^{'} = \mathbf{I}_1 = \frac{10\angle 30°}{11 - j14} = 0.08 + j0.556$$

For $\mathbf{I}_o''$ consider the circuit in Fig. (b).



(b)

Let $\mathbf{Z}_1 = 8 - j2\ \Omega$, $\mathbf{Z}_2 = 6 \parallel j4 = \dfrac{j24}{6 + j4} = 1.846 + j2.769\ \Omega$

$$\mathbf{I}_o'' = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(2) = \frac{(2)(1.846 + j2.769)}{9.846 + j0.77} = 0.4164 + j0.53$$

Therefore, $\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = 0.4961 + j1.086$

$\mathbf{I}_o = \underline{\mathbf{1.1939\angle 65.45°\ A}}$

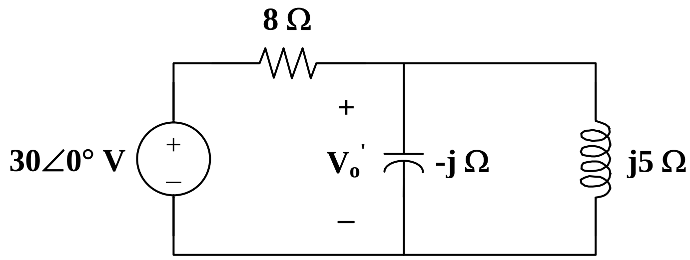
P.P.10.6 Let $v_o = v_o' + v_o''$, where v_o' is due to the voltage source and v_o'' is due to the current source. For v_o', we remove the current source.

$$30\sin(5t) \longrightarrow 30\angle 0°, \quad \omega = 5$$

$$0.2\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(5)(0.2)} = -j$$

$$1\text{ H} \longrightarrow j\omega L = j(5)(1) = j5$$

The circuit in the frequency domain is shown in Fig. (a).



(a)

Note that $-j \| j5 = -j1.25$

By voltage division,

$$\mathbf{V}_o' = \frac{-j1.25}{8 - j1.25}(30) = 4.631\angle -81.12°$$

Thus, $v_o' = 4.631\sin(5t - 81.12°)$

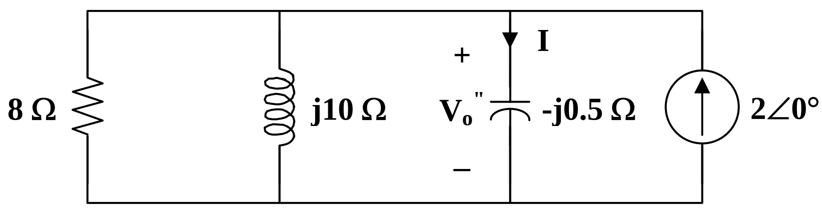
For v_o'', we remove the voltage source.

$$2\cos(10t) \longrightarrow 2\angle 0°, \quad \omega = 10$$

$$0.2\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.2)} = -j0.5$$

$$1\text{ H} \longrightarrow j\omega L = j(10)(1) = j10$$

The corresponding circuit in the frequency domain is shown in Fig (b).



(b)

Let $\mathbf{Z}_1 = -j0.5$, $\mathbf{Z}_2 = 8 \| j10 = \frac{j80}{8 + j10} = 4.878 + j3.9$

By current division,

$$\mathbf{I} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(2)$$

$$\mathbf{V}_o'' = \mathbf{I}(-j0.5) = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(2)(-j0.5) = \frac{-j(4.877 + j3.9)}{4.878 + j3.4}$$

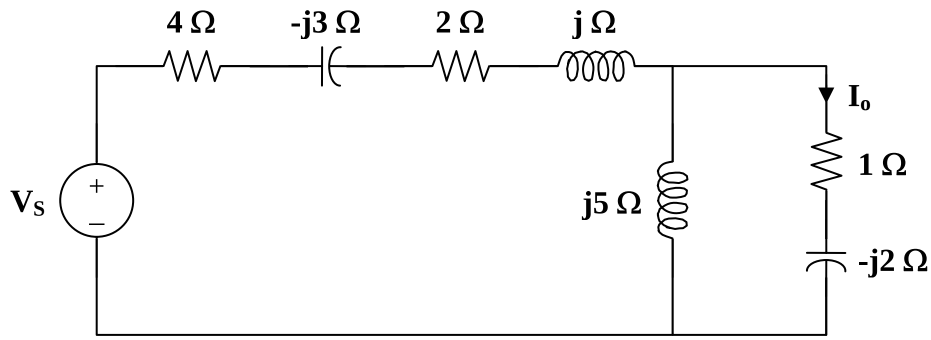
$$\mathbf{V}_o'' = \frac{6.245\angle -51.36°}{5.94\angle 34.88°} = 1.051\angle -86.24°$$

Thus, $v_o'' = 1.051\cos(10t - 86.24°)$

Therefore, $v_o = v_o' + v_o''$

$v_o =$ **<u>4.631 sin(5t – 81.12°) + 1.051 cos(10t – 86.24°) V</u>**

P.P.10.7 If we transform the current source to a voltage source, we obtain the circuit shown in Fig. (a).

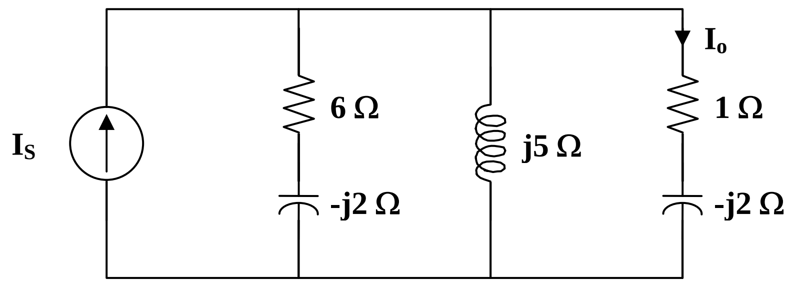


(a)

$$\mathbf{V}_s = \mathbf{I}_s \mathbf{Z}_s = (j4)(4 - j3) = 12 + j16$$

We transform the voltage source to a current source as shown in Fig. (b).

Let $\mathbf{Z} = 4 - j3 + 2 + j = 6 - j2$. Then, $\mathbf{I}_s = \dfrac{\mathbf{V}_s}{\mathbf{Z}} = \dfrac{12 + j16}{6 - j2} = 1.5 + j3$.



(b)

Note that $\mathbf{Z} \parallel j5 = \dfrac{(6 - j2)(j5)}{6 + j3} = \dfrac{10}{3}(1 + j)$.

By current division,

$$\mathbf{I}_o = \frac{\frac{10}{3}(1 + j)}{\frac{10}{3}(1 + j) + (1 - j2)}(1.5 + j3)$$

$$\mathbf{I}_o = \frac{-20 + j40}{13 + j4} = \frac{44.72\angle 116.56^\circ}{13.602\angle 17.1^\circ}$$

$$\mathbf{I}_o = \underline{\mathbf{3.288\angle 99.46^\circ\ A}}$$

P.P.10.8 When the voltage source is set equal to zero,

$$\mathbf{Z}_{th} = 10 + (\text{-j4}) \parallel (6 + j2)$$

$$\mathbf{Z}_{th} = 10 + \frac{(\text{-j4})(6 + j2)}{6 - j2}$$

$$\mathbf{Z}_{th} = 10 + 2.4 - j3.2$$

$$\mathbf{Z}_{th} = \underline{\mathbf{12.4 - j3.2\ \Omega}}$$

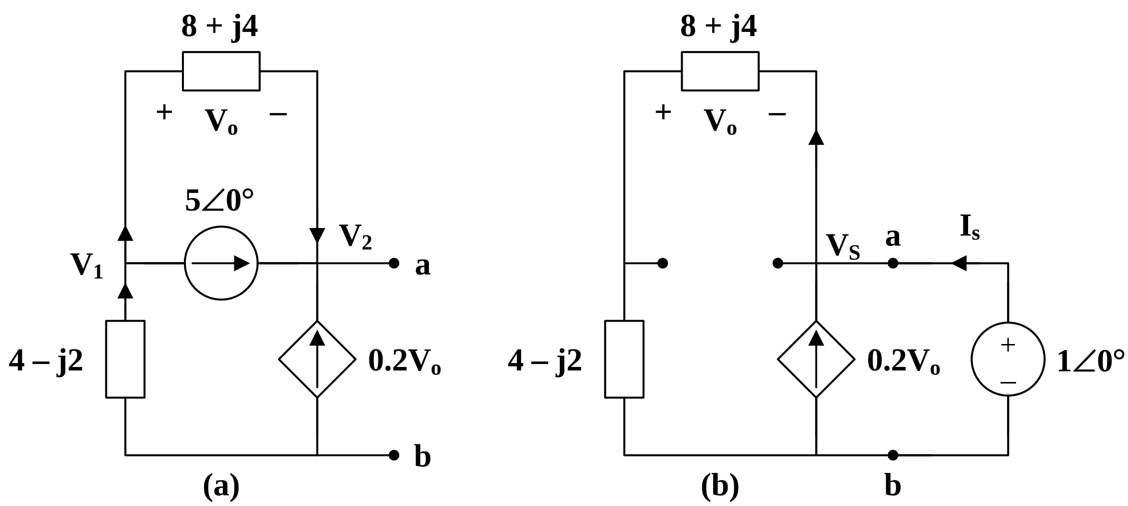
By voltage division,

$$\mathbf{V}_{th} = \frac{-\ j4}{6 + j2 - j4}(30\angle 20°) = \frac{(\text{-j4})(30\angle 20°)}{6 - j2}$$

$$\mathbf{V}_{th} = \frac{(4\angle \text{-}90°)(30\angle 20°)}{6.324\angle \text{-}18.43°}$$

$$\mathbf{V}_{th} = \underline{\mathbf{18.97\angle\text{-}51.57°\ V}}$$

P.P.10.9 To find $\mathbf{V}_{th}$, consider the circuit in Fig. (a).



At node 1, $\dfrac{0 - \mathbf{V}_1}{4 - j2} = 5 + \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4}$

$$-(2 + j)\mathbf{V}_1 = 50 + (1 - j0.5)(\mathbf{V}_1 - \mathbf{V}_2)$$

$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_1 \qquad (1)$$

At node 2, $5 + 0.2\mathbf{V}_o + \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4} = 0$, where $\mathbf{V}_o = \mathbf{V}_1 - \mathbf{V}_2$.

Hence, the equation for node 2 becomes

$$5 + 0.2(\mathbf{V}_1 - \mathbf{V}_2) + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4} = 0$$

$$\mathbf{V}_1 = \mathbf{V}_2 - \frac{50}{3 + j0.5} \qquad (2)$$

Substituting (2) into (1),

$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_2 + (50)\frac{3 + j0.5}{3 - j0.5}$$

$$0 = -50 - (2 + j)\mathbf{V}_2 + \frac{50}{37}(35 + j12)$$

$$\mathbf{V}_2 = \frac{-2.702 + j16.22}{2 + j} = 7.35\angle 72.9°$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = \underline{\mathbf{7.35\angle 72.9°\ V}}$$

To find $\mathbf{Z}_{th}$, we remove the independent source and insert a 1-V voltage source between terminals a-b, as shown in Fig. (b).

At node a, $\quad \mathbf{I}_s = -0.2\mathbf{V}_o + \dfrac{\mathbf{V}_s}{8 + j4 + 4 - j2}$

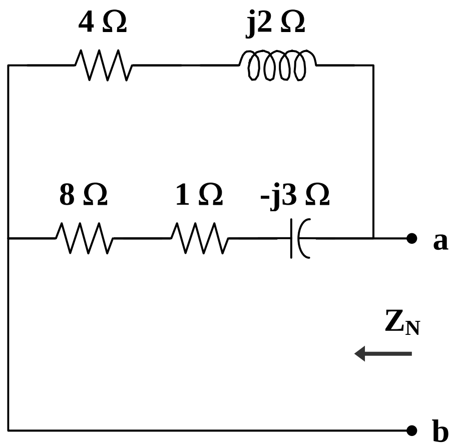
But, $\quad \mathbf{V}_s = 1 \quad$ and $\quad -\mathbf{V}_o = \dfrac{8 + j4}{8 + j4 + 4 - j2}\mathbf{V}_s$

So, $\quad \mathbf{I}_s = (0.2)\dfrac{8 + j4}{12 + j2} + \dfrac{\mathbf{1}}{12 + j2} = \dfrac{2.6 + j0.8}{12 + j2}$

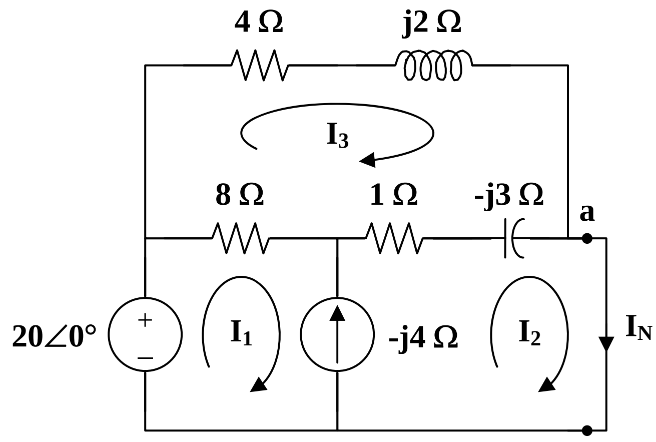
and $\quad \mathbf{Z}_{th} = \dfrac{\mathbf{V}_s}{\mathbf{I}_s} = \dfrac{1}{\mathbf{I}_s} = \dfrac{12 + j2}{2.6 + j0.8} = \dfrac{12.166\angle 9.46°}{2.72\angle 17.10°}$

$$\mathbf{Z}_{th} = \underline{\mathbf{4.473\angle{-7.64}°\ \Omega}}$$

P.P.10.10 To find $\mathbf{Z}_N$, consider the circuit in Fig. (a).



(a)



(b)

$$\mathbf{Z}_N = (4 + j2) \,||\, (9 - j3) = \frac{(4 + j2)(9 - j3)}{13 - j}$$

$$\mathbf{Z}_N = \underline{\mathbf{3.176 + j0.706\ \Omega}}$$

To find $\mathbf{I}_N$, short-circuit terminals a-b as shown in Fig. (b). Notice that meshes 1 and 2 form a supermesh.

For the supermesh, $\quad -20 + 8\mathbf{I}_1 + (1 - j3)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0 \qquad (1)$

Also, $\quad \mathbf{I}_1 = \mathbf{I}_2 + j4 \qquad (2)$

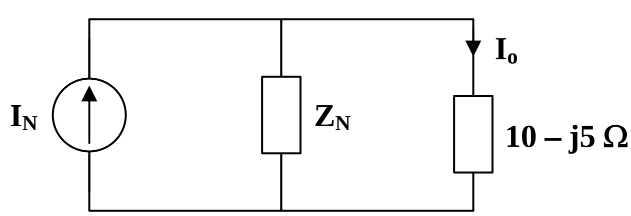
For mesh 3, $\quad (13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0 \qquad (3)$

Solving for $\mathbf{I}_2$, we obtain

$$\mathbf{I}_N = \mathbf{I}_2 = \frac{50 - j62}{9 - j3} = \frac{79.65\angle -51.11°}{9.487\angle -18.43°}$$

$$\mathbf{I}_N = \underline{\mathbf{8.396\angle -32.68°\ A}}$$

Using the Norton equivalent, we can find $\mathbf{I}_o$ as in Fig. (c).



(c)

By current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + 10 - j5}\mathbf{I}_N = \frac{3.176 + j0.706}{13.176 - j4.294}(8.396\angle -32.68°)$$

$$\mathbf{I}_o = \frac{(3.254\angle 12.53°)(8.396\angle -32.68°)}{13.858\angle -18.05°}$$

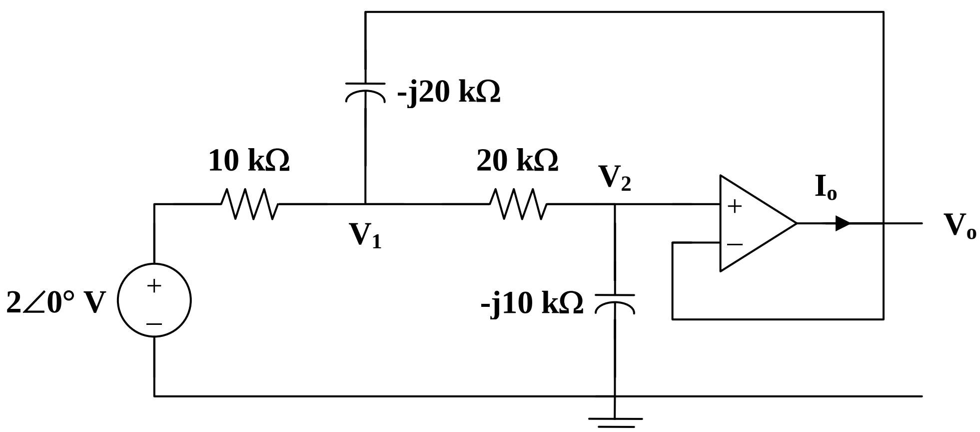
$$\mathbf{I}_o = \underline{\mathbf{1.971\angle -2.10°\ A}}$$

P.P.10.11

$$10\text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(5\times 10^3)(10\times 10^{-9})} = -j20\text{ k}\Omega$$

$$20\text{ nF} \longrightarrow \frac{1}{j\omega C_2} = \frac{1}{j(5\times 10^3)(20\times 10^{-9})} = -j10\text{ k}\Omega$$

Consider the circuit in the frequency domain as shown below.



As a voltage follower, $\mathbf{V}_2 = \mathbf{V}_o$

At node 1,
$$\frac{2 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$4 = (3 + j)\mathbf{V}_1 - (1 + j)\mathbf{V}_o \qquad (1)$$

At node 2,
$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{20} = \frac{\mathbf{V}_o - 0}{-j10}$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1) gives

$$4 = j6\mathbf{V}_o \qquad \text{or} \qquad \mathbf{V}_o = \frac{2}{3}\angle -90°$$

Hence, $v_o(t) = 0.667\cos(5000t - 90°)\ \text{V}$

$v_o(t) =$ **$0.667\sin(5000t)\ \text{V}$**

Now, $\mathbf{I}_o = \dfrac{\mathbf{V}_o - \mathbf{V}_1}{-j20k}$

But from (2) $\mathbf{V}_o - \mathbf{V}_1 = -j2\mathbf{V}_o = \dfrac{-4}{3}$

$$\mathbf{I}_o = \frac{-4/3}{-j20k} = -j66.66\ \mu\text{A}$$

Hence, $i_o(t) = 66.67\cos(5000t - 90°)\ \mu\text{A}$

$i_o(t) =$ **$66.67\sin(5000t)\ \mu\text{A}$**

P.P.10.12 Let $Z = R \,\|\, \dfrac{1}{j\omega C} = \dfrac{R}{1 + j\omega RC}$

$$\frac{V_s}{V_o} = \frac{R}{R + Z}$$

The loop gain is

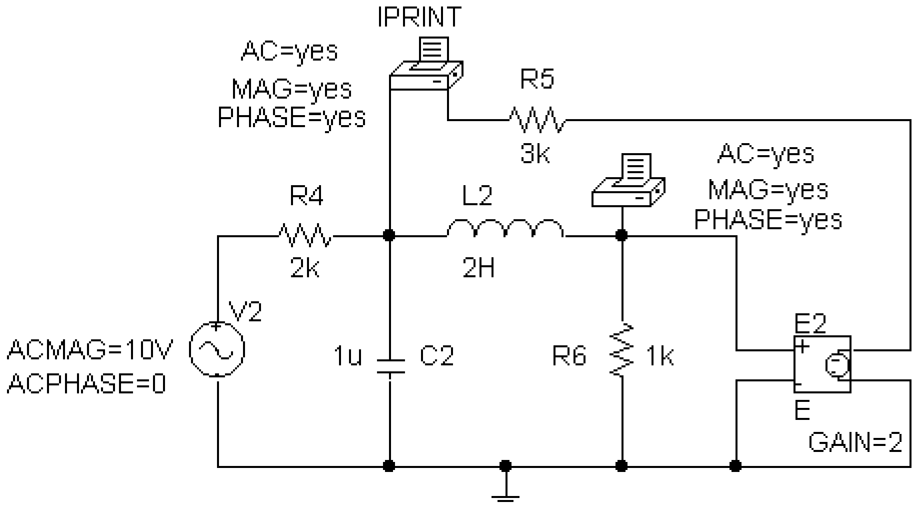
$$1/G = \frac{V_s}{V_o} = \frac{R}{R+Z} = \frac{R}{R + \dfrac{R}{1 + j\omega RC}} = \frac{1 + j\omega RC}{2 + j\omega RC}$$

where $\omega RC = (1000)(10 \times 10^3)(1 \times 10^{-6}) = 10$

$$1/G = \frac{1 + j10}{2 + j10} = \frac{10.05\angle 84.29°}{10.2\angle 78.69°}$$

$G =$ **$1.0147\angle -5.6°$**

P.P.10.13 The schematic is shown below.



Since $\omega = 2\pi f = 3000 \text{ rad/s} \longrightarrow f = 477.465 \text{ Hz}$. Setup/Analysis/AC Sweep as Linear for 1 point starting and ending at a frequency of 447.465 Hz. When the schematic is saved and run, the output file includes

Frequency	IM(V_PRINT1)	IP(V_PRINT1)
4.775E+02	5.440E-04	-5.512E+01

Frequency	VM($N_0005)	VP($N_0005)
4.775E+02	2.683E-01	-1.546E+02

From the output file, we obtain

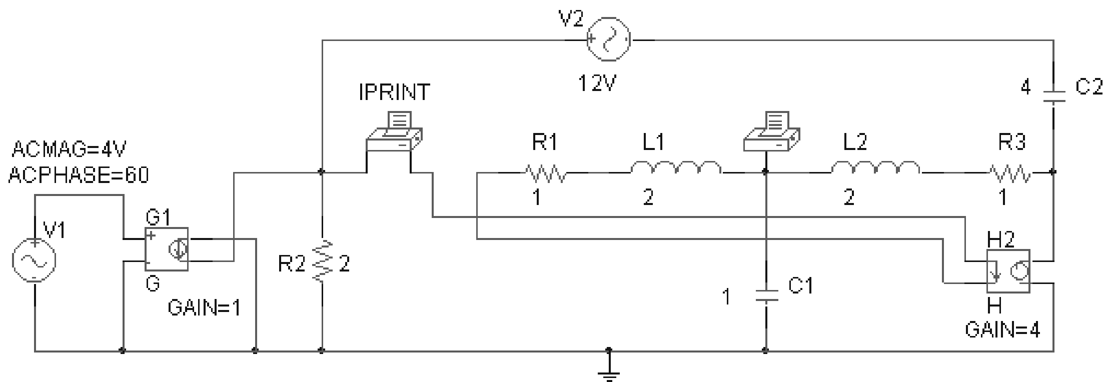
$\mathbf{V}_o = 0.2682\angle{-154.6°}\text{ V}$ and $\mathbf{I}_o = 0.544\angle{-55.12°}\text{ mA}$

Therefore,

$v_o(t) =$ **0.2682 cos(3000t – 154.6°) V**

$i_o(t) =$ **0.544 cos(3000t – 55.12°) mA**

P.P.10.14 The schematic is shown below.



We select ω = 1 rad/s and f = 0.15915 Hz. We use this to obtain the values of capacitances, where $C = 1/\omega X_c$, and inductances, where $L = X_L/\omega$. Note that IAC does not allow for an AC PHASE component; thus, we have used VAC in conjunction with G to create an AC current source with a magnitude and a phase. To obtain the desired output use Setup/Analysis/AC Sweep as Linear for 1 point starting and ending at a frequency of 0.15915 Hz. When the schematic is saved and run, the output file includes

Frequency	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	2.584E+00	1.580E+02

Frequency	VM($N_0004)	VP($N_0004)
1.592E-01	9.842E+00	4.478E+01

From the output file, we obtain

$\mathbf{V}_x = \underline{\mathbf{9.842\angle 44.78° \ V}}$ and $\mathbf{I}_x = \underline{\mathbf{2.584\angle 158° \ A}}$

P.P.10.15 $C_{eq} = \left(1 + \frac{R_2}{R_1}\right)C = \left(1 + \frac{10 \times 10^6}{10 \times 10^3}\right)(10 \times 10^{-9}) = \underline{\mathbf{10\ \mu F}}$

P.P.10.16 If $R = R_1 = R_2 = 2.5\ k\Omega$ and $C = C_1 = C_2 = 1\ nF$

$$f_o = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(2.5 \times 10^3)(1 \times 10^{-9})} = \underline{\mathbf{63.66\ kHz}}$$