February 5, 2006

CHAPTER 9

P.P.9.1 amplitude = **5**

phase = **-60°**

angular frequency (ω) = 4π = **12.57 rad/s**

period (T) = $\dfrac{2\pi}{\omega}$ = **0.5 s**

frequency (f) = $\dfrac{1}{T}$ = **2 Hz**

P.P.9.2 $i_1 = -4\sin(\omega t + 25°) = 4\cos(\omega t + 25° + 90°)$

$i_1 = 4\cos(\omega t + 115°)$, $\omega = 377$ rad/s

Compare this with

$i_2 = 5\cos(\omega t - 40°)$

indicates that the phase angle between i_1 and i_2 is

115° + 40° = 155°

Thus, **i_1 leads i_2 by 155°**

P.P.9.3 (a) (5 + j2)(-1 + j4) = -5 + j20 – j2 – 8 = -13 + j18

$5\angle 60°$ = 2.5 + j4.33

(5 + j2)(-1 + j4) – $5\angle 60°$ = -15.5 + j13.67

$[\,(5 + j2)(-1 + j4) - 5\angle 60\,]^*$ = **-15.5 – j13.67** = **$20.67\angle 221.41°$**

(b) $3\angle 40°$ = 2.298 + j1.928

10 + j5 + $3\angle 40°$ = 12.298 + j6.928 = $14.115\angle 29.39°$

-3 + j4 = $5\angle 126.87°$

$$\frac{10 + j5 + 3\angle 40°}{-3 + j4} = \frac{14.115\angle 29.39°}{5\angle 126.87°} = 2.823\angle -97.48°$$

$2.823\angle -97.48°$ = -0.3675 – j2.8

$10\angle 30°$ = 8.66 + j5

$$\frac{10 + j5 + 3\angle 40°}{-3 + j4} + 10\angle 30° = \mathbf{8.293 + j2.2}$$

P.P.9.4 (a) -cos(A) = cos(A – 180°) = cos(A + 180°)

Hence,

v = -7 cos(2t + 40°) = 7 cos(2t + 40° + 180°)

v = 7 cos(2t + 220°)

The phasor form is

V = $7\angle 220°$ V

(b) Since sin(A) = cos(A – 90°),

$i = 4\sin(10t + 10°) = 4\cos(10t+10° – 90°)$

$i = 4\cos(10t – 80°)$

The phasor form is

$\underline{\mathbf{I} = 4\angle\text{-}80°\ \text{A}}$

P.P.9.5 (a) Since $-1 = 1\angle\text{-}180° = 1\angle 180°$

$\mathbf{V} = -10\angle 30° = 10\angle(30°+180°) = 10\angle 210°$

The sinusoid is

$\underline{v(t) = \mathbf{10\cos(\omega t + 210°)\ V}}$

(b) $\mathbf{I} = j(5 – j12) = 12 + j5 = 13\angle 22.62°$

The sinusoid is

$\underline{i(t) = 13\cos(\omega t + 22.62°)\ \text{A}}$

P.P.9.6 Let $V = -10\sin(\omega t + 30°) + 20\cos(\omega t - 45°)$

Then, $V = 10\cos(\omega t + 30° + 90°) + 20\cos(\omega t - 45°)$

Taking the phasor of each term

$\mathbf{V} = 10\angle 120° + 20\angle\text{-}45°$

$\mathbf{V} = -5 + j8.66 + 14.14 – j14.14$

$\mathbf{V} = 9.14 – j5.48 = 10.66\angle\text{-}30.95°$

Converting **V** to the time domain

$v(t) =$ **$\underline{10.66\cos(\omega t – 30.95°)\text{V}}$**

P.P.9.7 Given that

$$2\frac{dv}{dt} + 5v + 10\int v\,dt = 20\cos(5t - 30°)$$

we take the phasor of each term to get

$$2j\omega\,\mathbf{V} + 5\,\mathbf{V} + \frac{10}{j\omega}\mathbf{V} = 20\angle\text{-}30°, \qquad \omega = 5$$

$\mathbf{V}\,[j10 + 5 – j(10/5)] = \mathbf{V}\,(5 + j8) = 20\angle\text{-}30°$

$$\mathbf{V} = \frac{20\angle -30°}{5 + j8} = \frac{20\angle -30°}{9.434\angle 58°}$$

$\mathbf{V} = 2.12\angle\text{-}88°$

Converting **V** to the time domain

$v(t) =$ **$\underline{2.12\cos(5t – 88°)\text{V}}$**

P.P.9.8 For the capacitor,

$\mathbf{V} = \mathbf{I} / (j\omega C)$, where $\mathbf{V} = 6\angle\text{-}30°$, $\omega = 100$

$\mathbf{I} = j\omega C\,\mathbf{V} = (j100)(50\text{x}10^{-6})(6\angle\text{-}30°)$

$\mathbf{I} = 30\angle 60°$ mA

$i(t) =$ **$\underline{30\cos(100t + 60°)\ \text{mA}}$**

P.P.9.9 $\mathbf{V}_s = 5\angle 0°$, $\omega = 10$

$\mathbf{Z} = 4 + j\omega L = 4 + j2$

$$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \frac{5\angle 0°}{4 + j2} = \frac{5(4 - j2)}{16 + 4} = 1 - j0.5 = 1.118\angle\text{-}26.57°$$

$\mathbf{V} = j\omega L\, \mathbf{I} = j2\, \mathbf{I} = (2\angle 90°)(1.118\angle\text{-}26.57°) = 2.236\angle 63.43°$

Therefore, $v(t) = \underline{\mathbf{2.236 \sin(10t + 63.43°)\ V}}$

$i(t) = \underline{\mathbf{1.118 \sin(10t - 26.57°)\ A}}$

P.P.9.10

Let $\mathbf{Z}_1$ = impedance of the 2-mF capacitor in series with the 20-Ω resistor

$\mathbf{Z}_2$ = impedance of the 4-mF capacitor

$\mathbf{Z}_3$ = impedance of the 2-H inductor in series with the 50-Ω resistor

$$\mathbf{Z}_1 = 20 + \frac{1}{j\omega C} = 20 + \frac{1}{j(10)(2 \times 10^{-3})} = 20 - j50$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C} = \frac{1}{j(10)(4 \times 10^{-3})} = \text{-}j25$$

$$\mathbf{Z}_3 = 50 + j\omega L = 50 + j(10)(2) = 50 + j20$$

$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \,\|\, \mathbf{Z}_3 = \mathbf{Z}_1 + \mathbf{Z}_2\mathbf{Z}_3 / (\mathbf{Z}_2 + \mathbf{Z}_3)$

$$\mathbf{Z}_{in} = 20 - j50 + \frac{\text{-}j25 x (50 + j20)}{\text{-}j25 + 50 + j20}$$

$\mathbf{Z}_{in} = 20 - j50 + 12.38 - j23.76$

$\mathbf{Z}_{in} = \underline{\mathbf{32.38 - j73.76\ \Omega}}$

P.P.9.11 In the frequency domain,

the voltage source is $\mathbf{V}_s = 10\angle 75°$

the 0.5-H inductor is $j\omega L = j(10)(0.5) = j5$

the $\frac{1}{20}$-F capacitor is $\frac{1}{j\omega C} = \frac{1}{j(10)(1/20)} = \text{-}j2$

Let $\mathbf{Z}_1$ = impedance of the 0.5-H inductor in parallel with the 10-Ω resistor

and $\mathbf{Z}_2$ = impedance of the (1/20)-F capacitor

$$\mathbf{Z}_1 = 10 \,\|\, j5 = \frac{(10)(j5)}{10 + j5} = 2 + j4 \qquad \text{and} \qquad \mathbf{Z}_2 = \text{-}j2$$

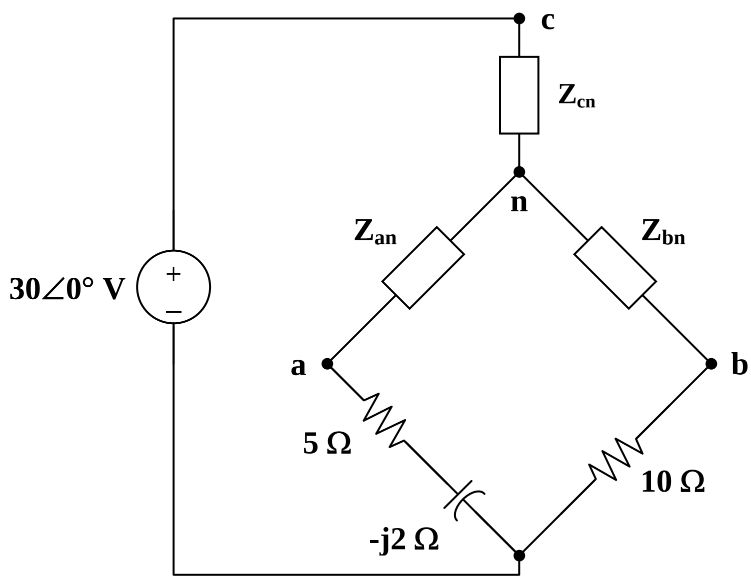
$\mathbf{V}_o = \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)\, \mathbf{V}_s$

$$\mathbf{V}_o = \frac{-j2}{2 + j4 - j2}(10\angle 75°) = \frac{-j(10\angle 75°)}{1 + j} = \frac{10\angle(75° - 90°)}{\sqrt{2}\angle 45°}$$

$\mathbf{V}_o = 7.071\angle\text{-}60°$

$v_o(t) = \underline{\mathbf{7.071 \cos(10t - 60°)\ V}}$

P.P.9.12 We need to find the equivalent impedance via a delta-to-wye transformation as shown below.



$$\mathbf{Z}_{an} = \frac{j4(8+j5)}{j4+8+j5-j3} = \frac{4(-5+j8)}{8+j6} = 0.32 + j3.76$$

$$\mathbf{Z}_{bn} = \frac{-j3(8+j5)}{8+j6} = \frac{3(5-j8)(8-j6)}{100} = -0.24 - j2.82$$

$$\mathbf{Z}_{cn} = \frac{j4(-j3)}{8+j6} = \frac{12(8-j6)}{100} = 0.96 - j0.72$$

The total impedance from the source terminals is

$$\mathbf{Z} = \mathbf{Z}_{cn} + (\mathbf{Z}_{an} + 5 - j2) \parallel (\mathbf{Z}_{bn} + 10)$$

$$\mathbf{Z} = \mathbf{Z}_{cn} + (5.32 + j1.76) \parallel (9.76 - j2.82)$$

$$\mathbf{Z} = \mathbf{Z}_{cn} + \frac{(5.32+j1.76)(9.76-j2.82)}{(5.32+j1.76)+(9.76-j2.82)}$$

$$\mathbf{Z} = 0.96 - j0.72 + 3.744 + j0.4074$$

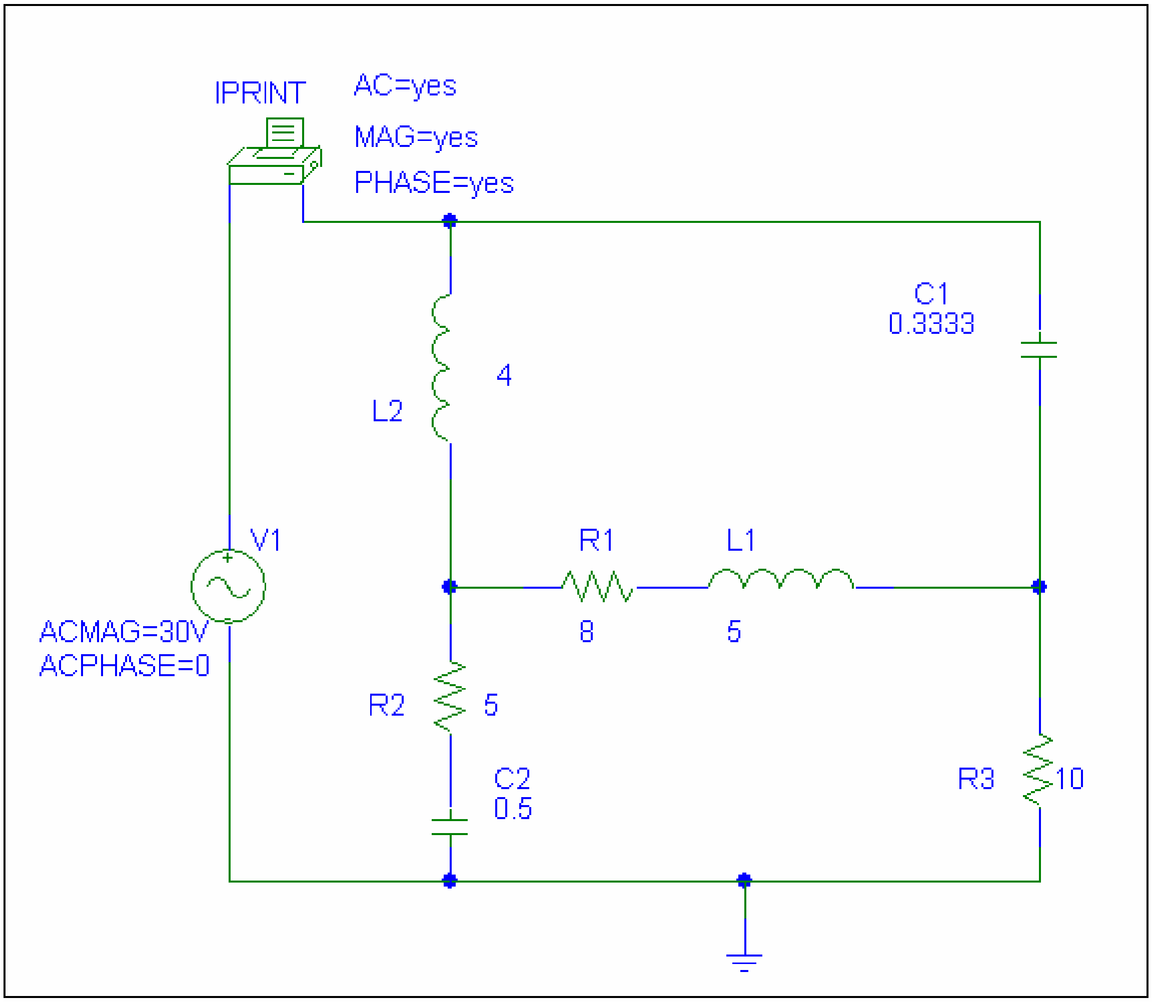
$$\mathbf{Z} = 4.704 - j0.3126 = 4.714\angle{-3.802°}$$

Therefore,

$$\mathbf{I} = \mathbf{V}/\mathbf{Z} = \frac{30\angle 0°}{4.714\angle -3.802°}$$

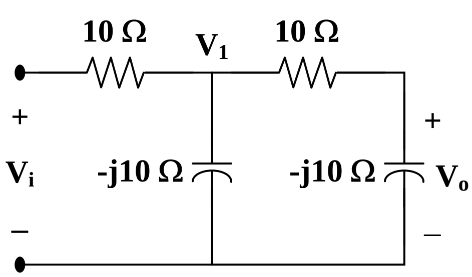
$$\mathbf{I} = \underline{\mathbf{6.364\angle 3.802°\ A}}$$

Let us now check this using PSpice. The solution produces the magnitude of I = 6.364E+00, and the phase angle = 3.803E+00, which agrees with the above answer.

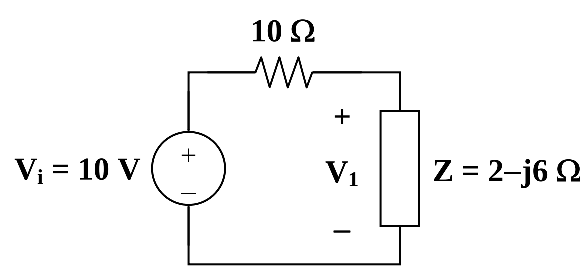


P.P.9.13 To show that the circuit in Fig. (a) meets the requirement, consider the equivalent circuit in Fig. (b).

$$\mathbf{Z} = -j10 \parallel (10 - j10) = \frac{-j10(10 - j10)}{10 - j20} = \frac{-j(10 - j10)}{1 - j2} = 2 - j6\ \Omega$$



(a)



(b)

$$\mathbf{V}_1 = \frac{2 - j6}{10 + 2 - j6}(10) = \frac{10}{3}(1 - j)$$

$$\mathbf{V}_o = \frac{-j10}{10 - j10}\mathbf{V}_1 = \left(\frac{-j}{1 - j}\right)\left(\frac{10}{3}\right)(1 - j) = -j\frac{10}{3}$$

$$\mathbf{V}_o = \frac{10}{3}\angle -90°$$

This implies that the RC circuit provides a 90° lagging phase shift.

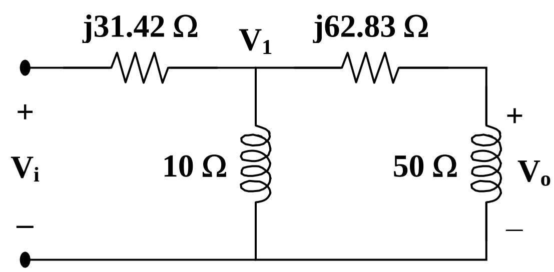
The output voltage is $\frac{10}{3}$ = **3.333 V**

P.P.9.14

the 1-mH inductor is $j\omega L = j(2\pi)(5\times10^{3})(1\times10^{-3}) = j31.42$

the 2-mH inductor is $j\omega L = j(2\pi)(5\times10^{3})(2\times10^{-3}) = j62.83$

Consider the circuit shown below.



$$\mathbf{Z} = 10 \parallel (50 + j62.83) = \frac{(10)(50 + j62.83)}{60 + j62.83}$$

$$\mathbf{Z} = 9.205 + j0.833 = 9.243\angle 5.17°$$

$$\mathbf{V}_1 = \mathbf{Z} / (\mathbf{Z} + j31.42)\,\mathbf{V}_i = \frac{9.243\angle 5.17°}{9.205 + j32.253}(1) = 0.276\angle -68.9°$$

$$\mathbf{V}_o = \frac{50}{50 + j62.83}\mathbf{V}_1 = \frac{50(0.276\angle -68.9°)}{80.297\angle 51.49°} = 0.172\angle -120.4°$$

Therefore,

magnitude = **0.172**

phase = **120.4°**

phase shift is **lagging**

P.P.9.15 $\mathbf{Z}_x = (\mathbf{Z}_3 / \mathbf{Z}_1)\, \mathbf{Z}_2$

$\mathbf{Z}_3 = 12\text{ k}\Omega$

$\mathbf{Z}_1 = 4.8\text{ k}\Omega$

$\mathbf{Z}_2 = 10 + j\omega L = 10 + j(2\pi)(6\times10^{6})(0.25\times10^{-6}) = 10 + j9.425$

$$\mathbf{Z}_x = \frac{12k}{4.8k}(10 + j9.425) = 25 + j23.5625\ \Omega$$

$R_x = 25, \quad X_x = 23.5625 = \omega L_x$

$$L_x = \frac{X_x}{2\pi f} = \frac{23.5625}{2\pi(6\times10^{6})} = 0.625\ \mu\text{H}$$

i.e. **a 25-Ω resistor in series with a 0.625-μH inductor**.