P/s: This is only involved Integration and Improper Integral. Please find more exercises on Series.

1. Evaluate each of the following:

(a) $\int x^2 \cosh(3x)\,dx$

(b) $\int \frac{1}{\sqrt{x^2-4x-5}}\,dx$

2. Determine the convergence of each of the following improper integrals.

(a) $\int_1^\infty \frac{1}{x^2+2}\,dx$

(b) $\int_{-2}^{2} \frac{1}{(x+1)^3}\,dx$

3. (a) Evaluate $\int x^2 \sin^{-1} x\,dx$

(b) Find the constants A, B and C such that,

$$\frac{2x+6}{(x^2+1)(1-x)} = \frac{Ax+B}{x^2+1} + \frac{C}{1-x}$$

Hence, evaluate

$$\int_{-1}^{0} \frac{2x+6}{(x^2+1)(1-x)}\,dx$$

4. (a) Determine if the integral $\int_{-\infty}^{\infty} x\,e^{-x^2}\,dx$ convergent or divergent.

(b) Evaluate $\int_0^1 \sinh^2(4x-3)dx$.

Answers:

1 a) $\frac{x^2}{3}\sinh(3x)-\frac{2x}{9}\cosh(3x)+\frac{2}{27}\sinh(3x)+C$

b) $\cosh^{-1}\left(\frac{x-2}{3}\right)+C$

2. a) 0.675511 (converges)

b) This integral diverges.

3. a) $\frac{x^3}{3}\sin^{-1}x+\frac{1}{6}\left[2\left(1-x^2\right)^{\frac{1}{2}}-\frac{2}{3}\left(1-x^2\right)^{\frac{3}{2}}\right]+C$

b) $\frac{\pi}{2}+2\ln 2$

4.a) $\int_{-\infty}^{\infty} xe^{-x^2}dx=-1+1=0,\quad \text{converges}$

b) 12.8338