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Circuit Theory (SKEE 1023)

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Topics

- Methods of Circuit Analysis: Nodal Analysis; Node Voltages; Mesh Analysis; Loop Currents; Circuit Analysis with Voltage and Current Independent and Dependent Sources.

METHODS OF CIRCUIT ANALYSIS

Introduction

- Two powerful techniques for circuit analysis, ie;
  - Nodal analysis : based on application of KCL.
  - Mesh analysis : based on application of KVL.
- Analyze any linear circuit by obtaining a set of simultaneous equations that are then solved to obtain the required values of current or voltage.

Nodal Analysis

- Provide a general procedure for analyzing circuits using node voltages as the circuit variables.
- Interested in finding the node voltages.

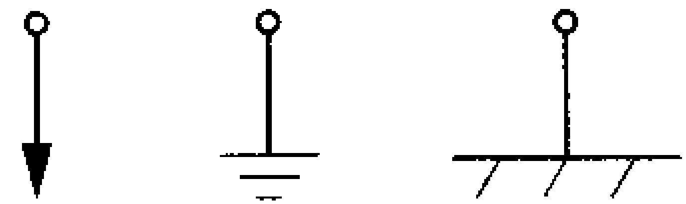
Steps to Determine Node Voltages (Nodal Analysis)

1. Select a node as the reference node. Assign voltages v_1, v_2, …. V_{n-1} to the remaining n – 1 nodes.
2. Apply KCL to each of the n – 1 nonreference nodes.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

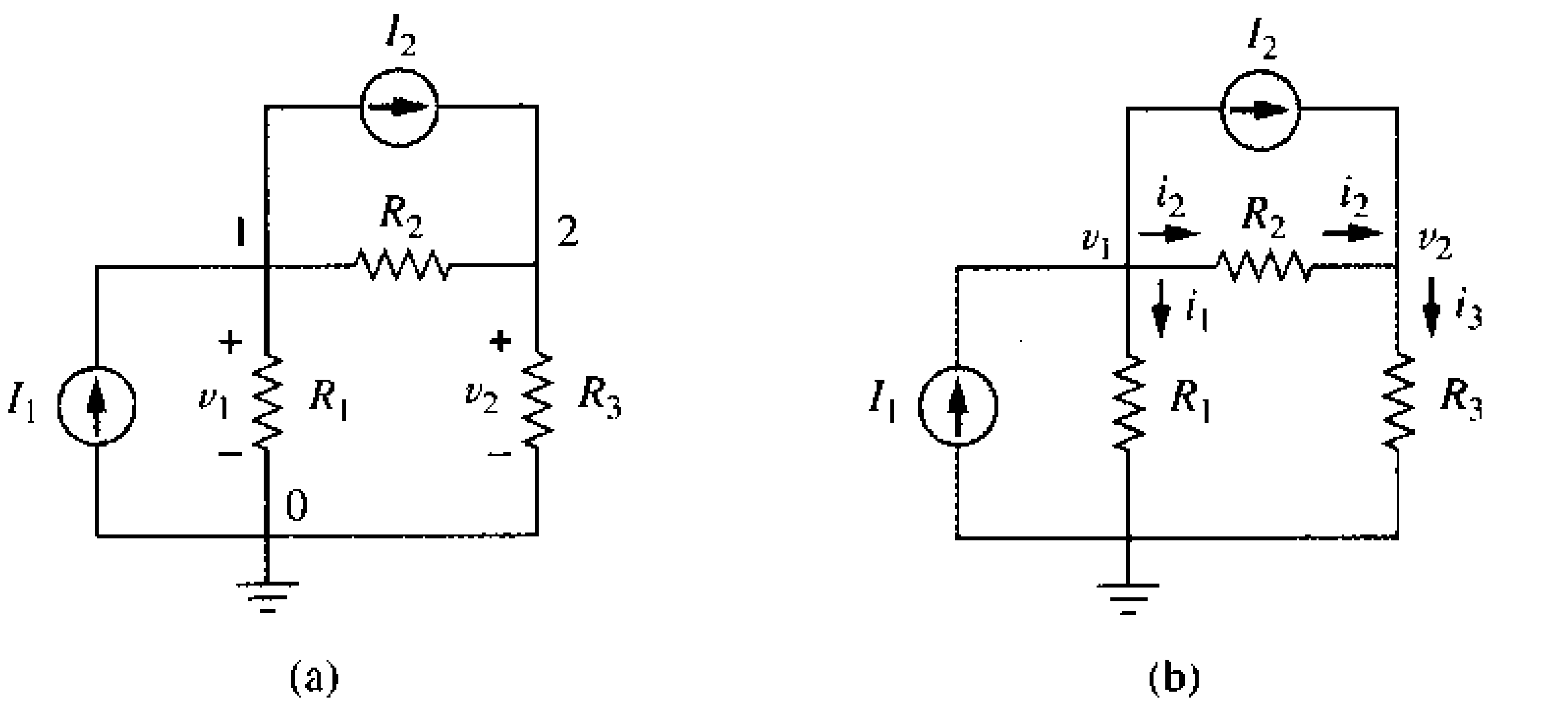
- The reference (datum) node is commonly called the ground (assumed to have zero potential).

Symbols :

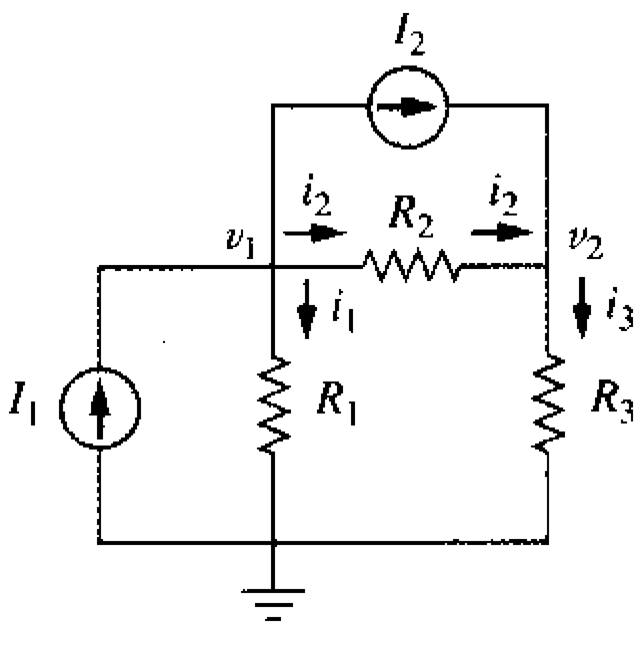
-Common ground
-Ground
-Chassis ground



Method of Nodal Analysis



At node 1 and node 2, apply KCL gives;



(b)

$$I_1 = I_2 + i_1 + i_2 \quad \text{and} \quad I_2 + i_2 = i_3$$

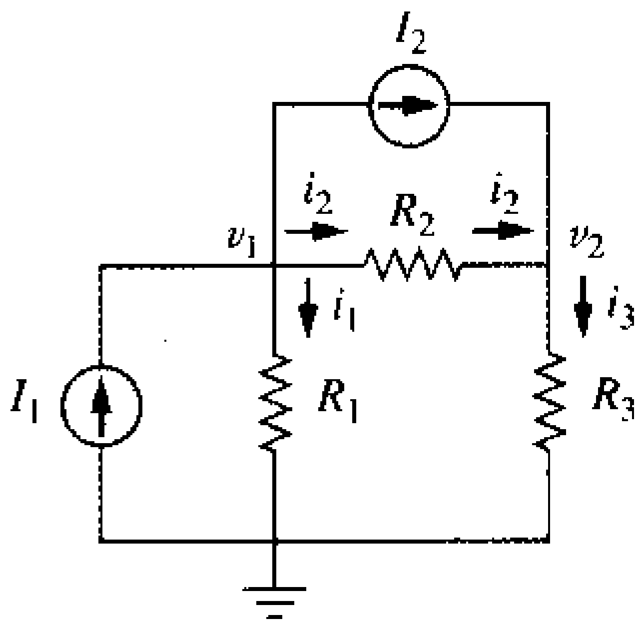
$$i_1 = \frac{v_1 - 0}{R_1} = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} = G_2(v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} = G_3 v_2$$

M. Afendi M. Piah, PhD

Circuit Theory



(b)

$$I_1 = I_2 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_1 - \frac{1}{R_2}v_2$$

At node 1

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_1 - \frac{1}{R_2}v_2 = I_1 - I_2$$

or

$$(G_1 + G_2)v_1 - G_2 v_2 = I_1 - I_2$$

$$I_2 = -\frac{1}{R_2}v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)v_2$$

At node 2

$$-\frac{1}{R_2}v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)v_2 = I_2$$

or

$$-G_2 v_1 + (G_2 + G_3)v_2 = I_2$$

In matrix form;

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} I_1-I_2 \\ I_2 \end{bmatrix}$$

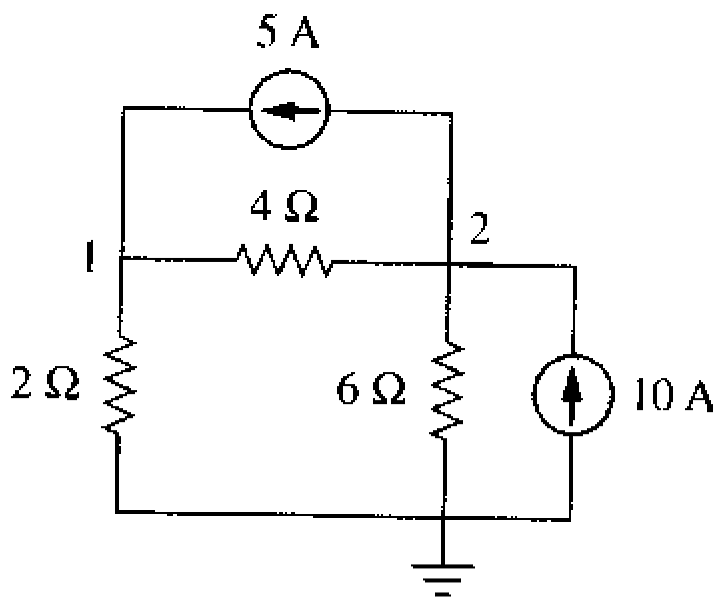
or

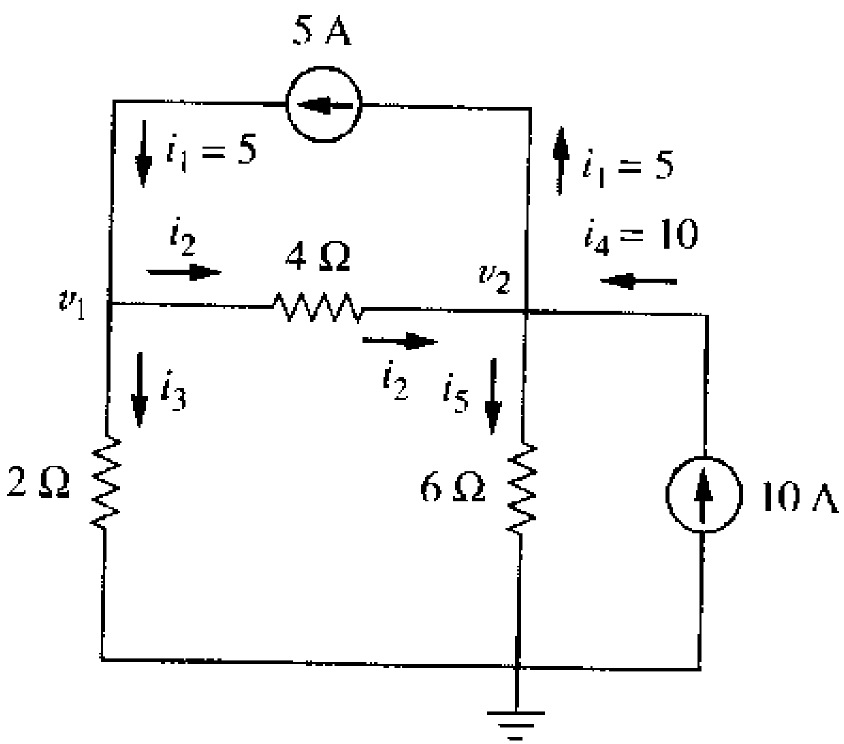
$$\begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} I_1-I_2 \\ I_2 \end{bmatrix}$$

Then, determine the values of v_1 and v_2.

Example 2.1

Calculate the node voltages in the circuit.

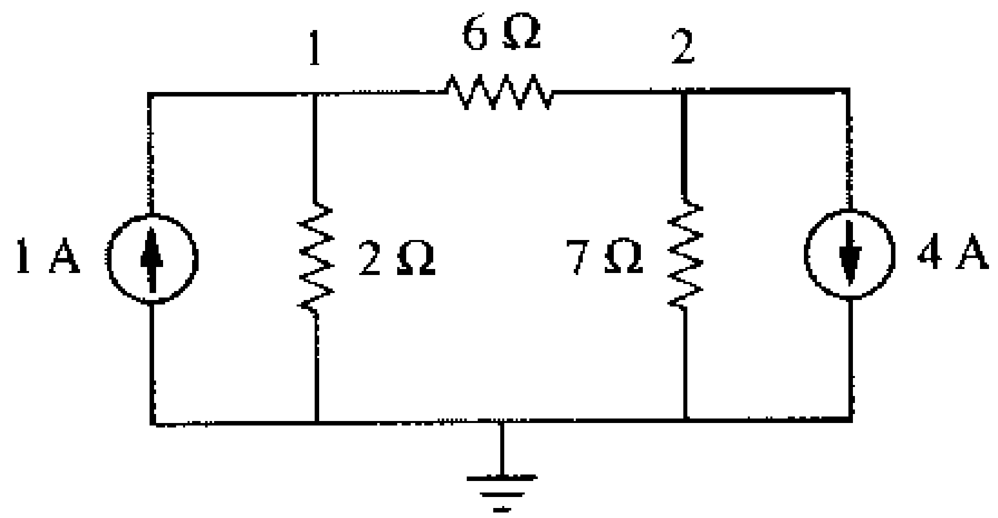




Problem 2.1

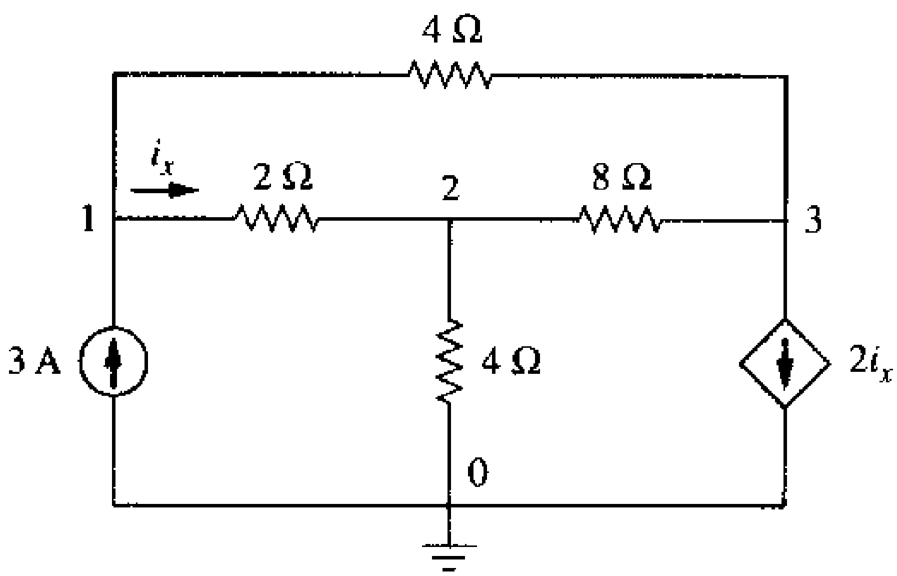
Obtain the node voltages in the circuit.

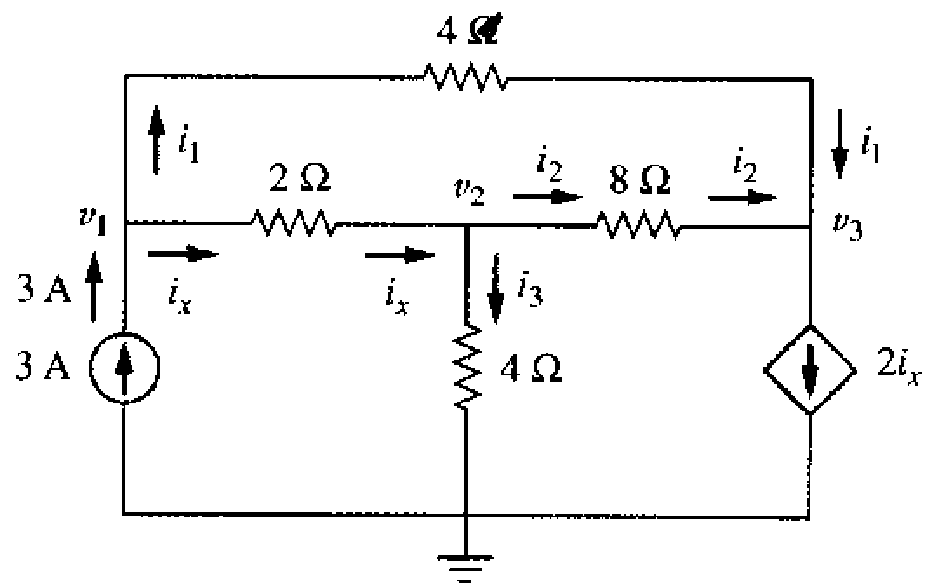
Ans: v_1 = -2 V, v_2 = -14 V



Example 2.2

Determine the voltages at the nodes.

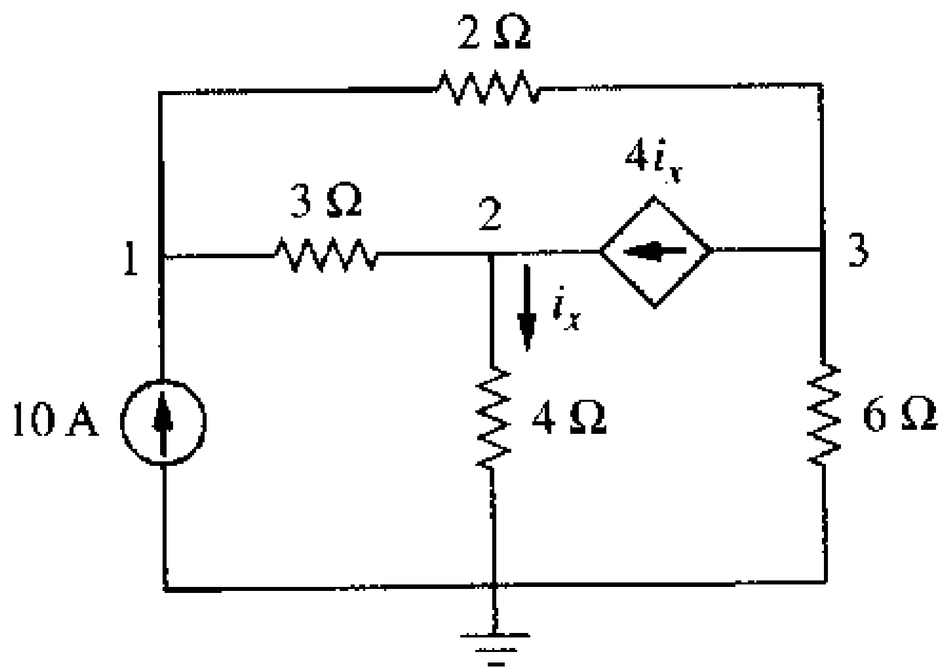




Problem 2.2

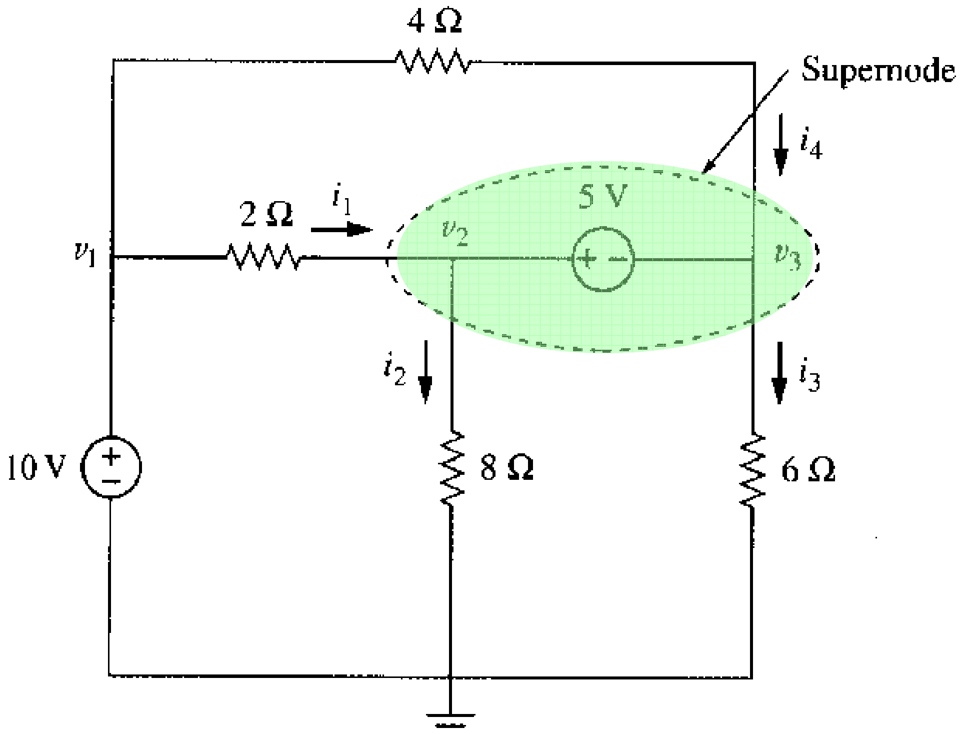
Determine the voltages at the nodes.

Ans : v_1 = 80 V, v_2 = -64 V, v_3 = 156 V



Nodal Analysis with Voltage Sources

- How voltage sources affect nodal analysis. Consider the following two possibilities.



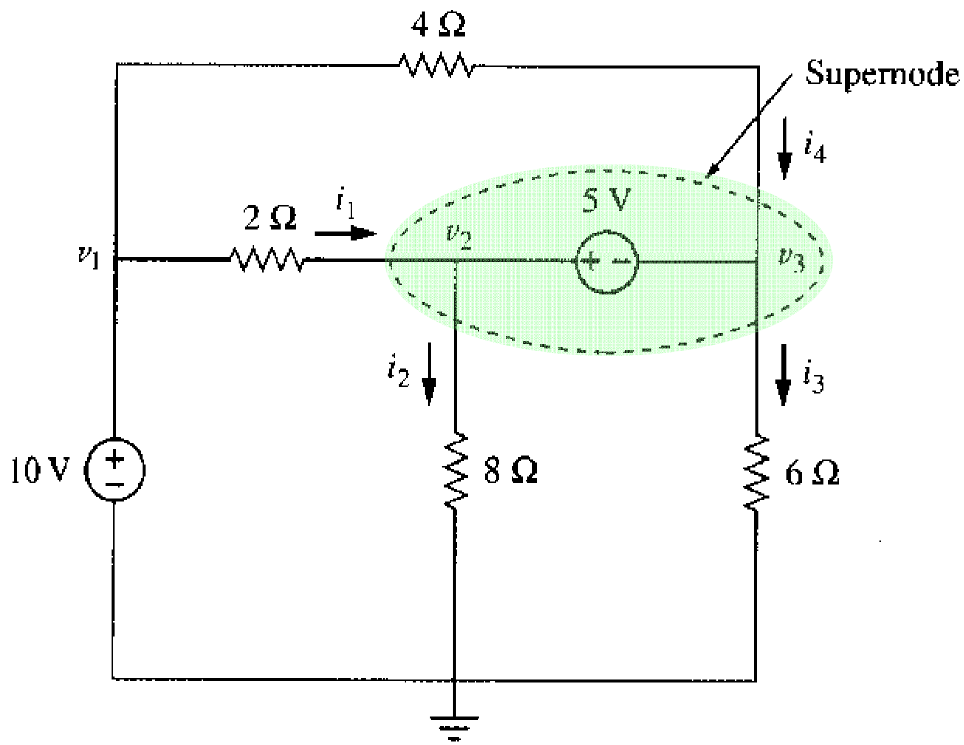
Case 1:

- If a voltage source is connected between the reference node and a nonreference node; ⇒ v_1 = 10 V.

Case 2:

- If the voltage source (dependent or independent) is connected between two nonreference nodes. The nonreference nodes form a *generalized node* or *supernode*.

Circuit Theory



- Nodes 2 and 3 form a supernode.
- There is no way knowing the current through a voltage source.
- KCL must be satisfied at a supernode, i.e.

$$i_1 + i_4 = i_2 + i_3$$

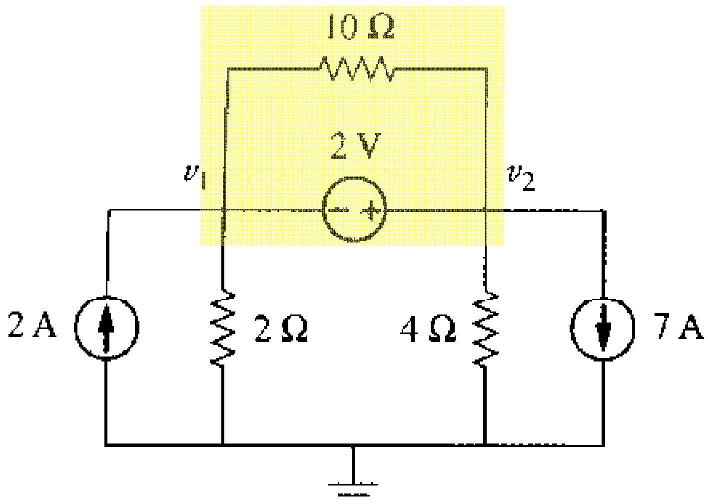
or

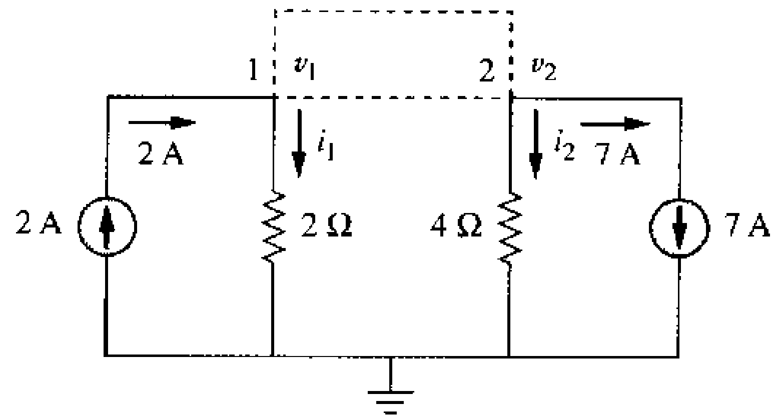
$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2}{8} + \frac{v_3}{6}$$

and $v_2 - v_3 = 5$

Example 2.3

For the circuit shown below, find the node voltages.





Applying KCL to the supernode;

$$2 = i_1 + i_2 + 7$$

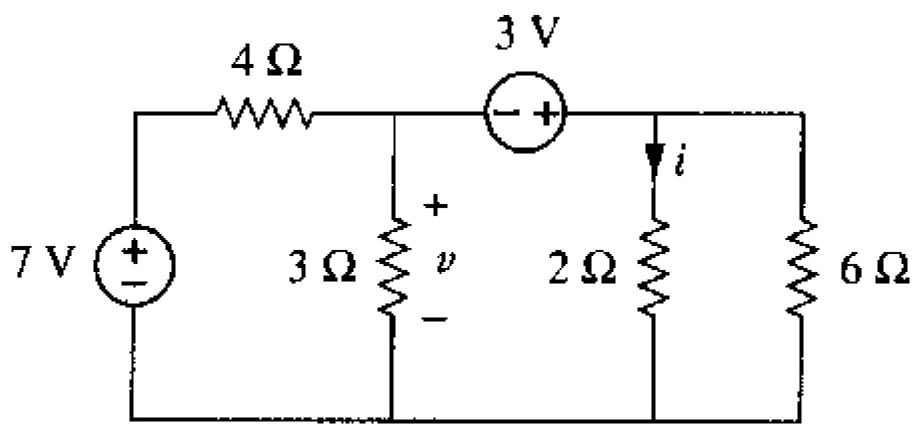
where,

$$i_1 = \frac{v_1}{2} \; ; \quad i_2 = \frac{v_2}{4} \quad \text{and} \quad v_2 - v_1 = 2$$

Problem 2.3

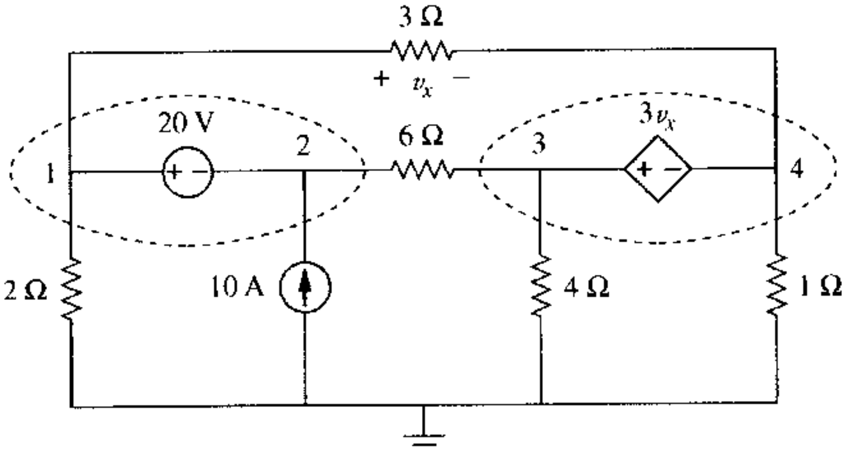
Find *v* and *i*.

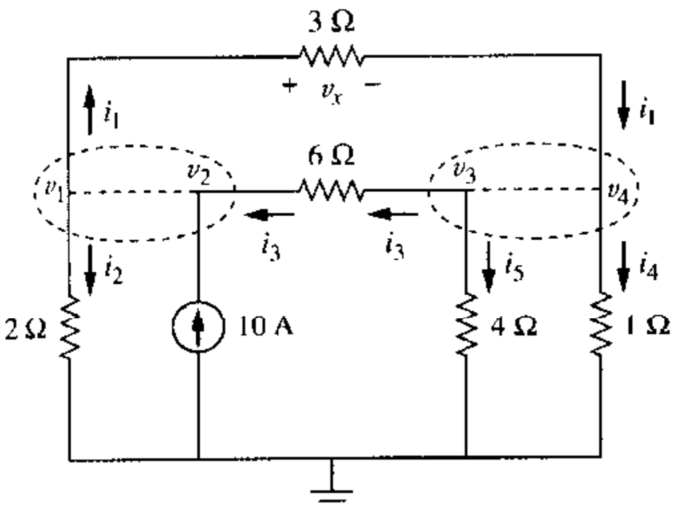
Ans: -0.2 V, 1.4 A



Example 2.4

Find the node voltages in the circuit.





Solution

KCL at supernode (1-2)

$$10 + i_3 = i_1 + i_2$$

$$\text{where, } i_3 = \frac{v_3 - v_2}{6}; \quad i_1 = \frac{v_1 - v_4}{3}; \quad i_2 = \frac{v_1}{2}$$

KCL at supernode (3-4)

$$i_1 = i_3 + i_4 + i_5$$

where, i_1, and i_3 same as above and;

$$i_4 = \frac{v_4}{1}; \quad i_5 = \frac{v_3}{4}$$

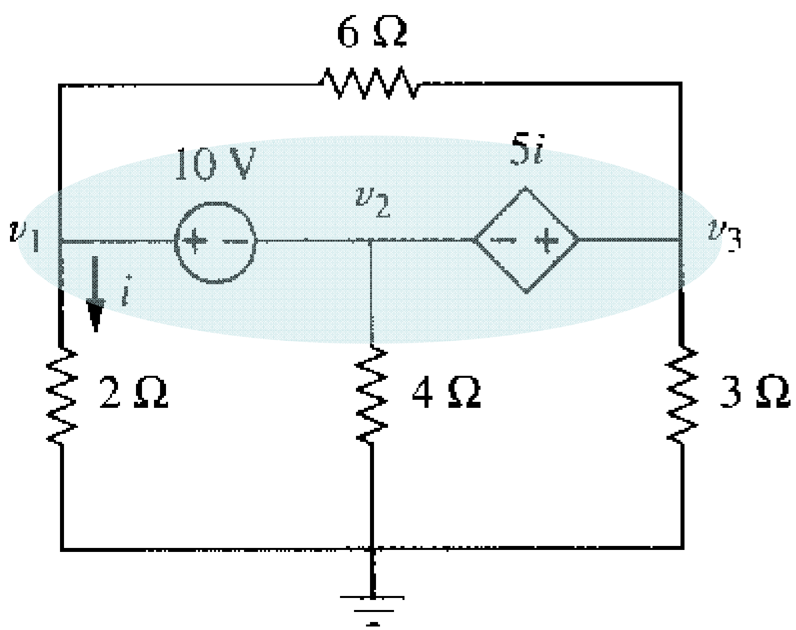
Supportive equations:

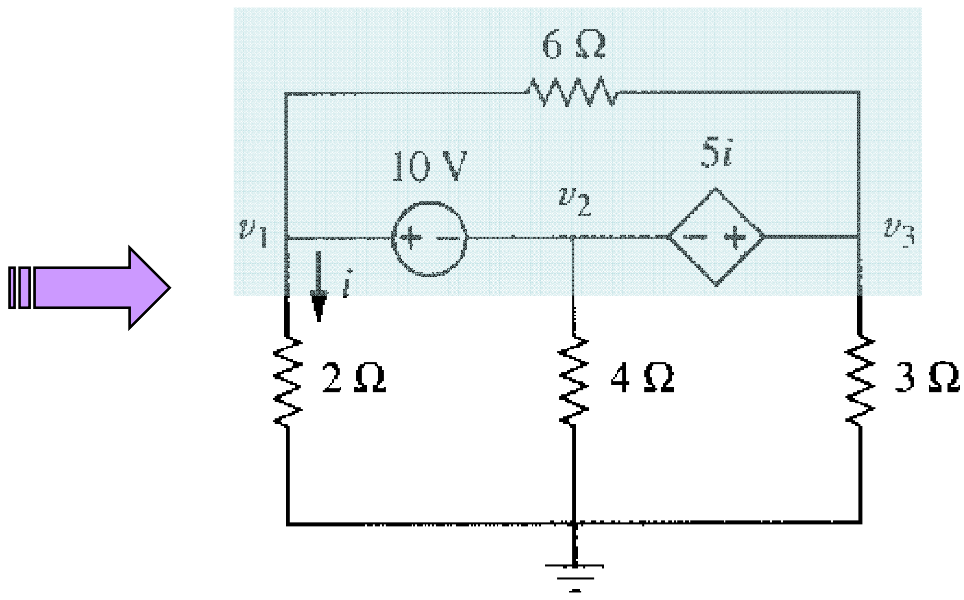
$$v_1 - v_2 = 20 \quad \text{and} \quad v_3 - v_4 = 3v_x = 3(v_1 - v_4)$$

Problem 2.4

Find v_1, v_2 and v_3 in the circuit using nodal analysis.

Ans: v_1 = 3.043 V, v_2 = -6.956 V, v_3 = 0.6522

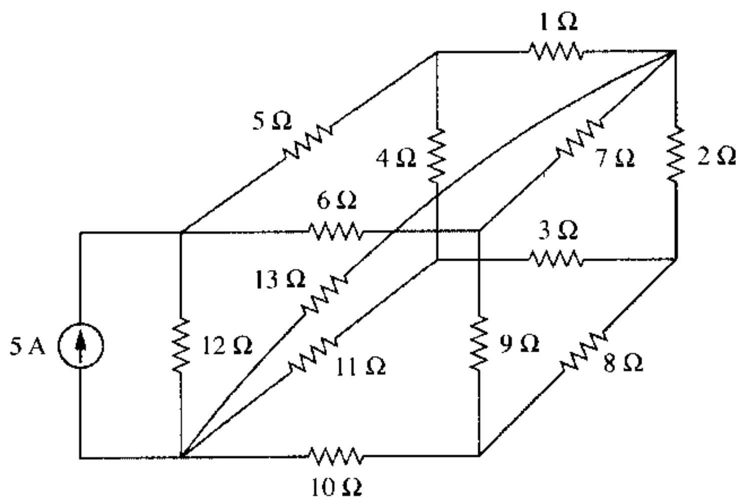




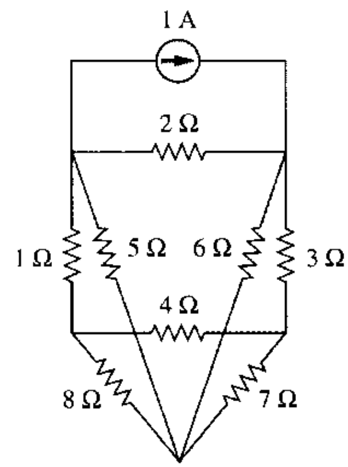
Mesh Analysis

- Another general procedure for analyzing circuits, using mesh current as the circuit variables.
- Definations: Loop is a closed path with no node passed more than once. Mesh is a loop that does not contain any other loop within it.
- Mesh analysis applies KVL to find unknown currents.
- Only applicable to a circuit that is *planar (two-dimensional)*. Nonplanar circuits can be handled using nodal analysis.

Types of circuit

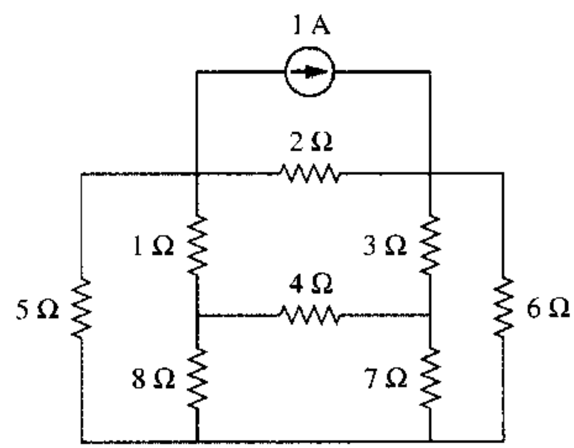


A nonplanar circuit

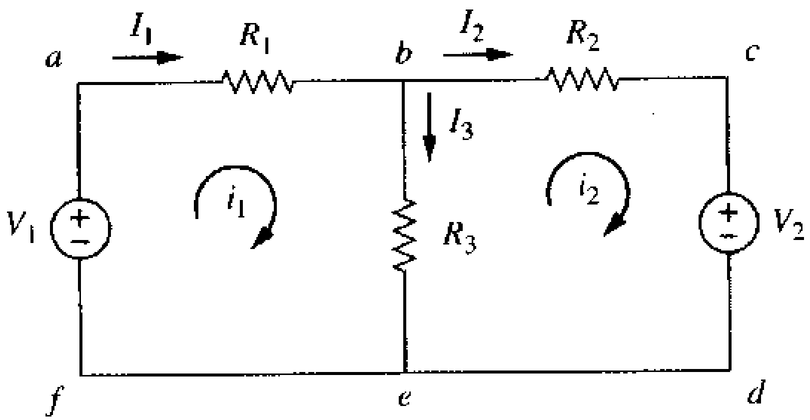


A planar circuit

The same
circuit redrawn



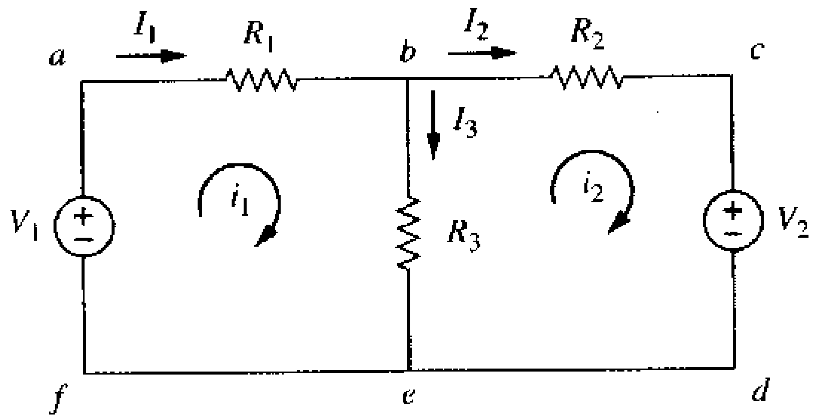
Circuit Theory



- Paths *abefa* and *bcdeb* are meshes. i_1 and i_2 are mesh current.
- In mesh analysis, we are interested in applying KVL to find the mesh current.

Steps to Determine Mesh Currents

- Assign mesh currents i_1, i_2, …. i_n to the n meshes. Assume mesh currents flow clockwise.
- Apply KVL to each of the *n* meshes. Use Ohm's law to express the voltages in terms of the mesh current.
- Solve the resulting *n* simultaneous equations to get the mesh currents.



For mesh 1

$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

or

$$(R_1 + R_3) i_1 - R_3 i_2 = V_1$$

For mesh 2

$$R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

or

$$-R_3 i_1 + (R_2 + R_3) i_2 = -V_2$$

In matrix form;

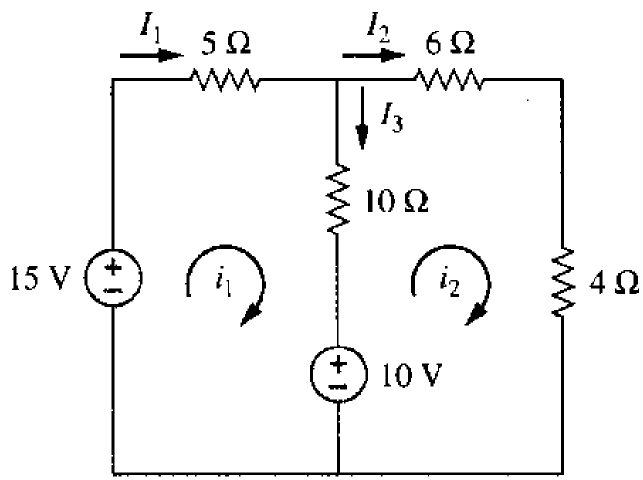
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

- Notice that the branch currents are different from the mesh currents unless the mesh is isolated. *i* for a mesh current and *I* for a branch current.

$$I_1 = i_1 \; ; \quad I_2 = i_2 \; ; \quad I_3 = i_1 - i_2$$

Example 2.5

Find the branch currents I_1, I_2 and I_3 using mesh analysis.



Solution

Apply KVL for mesh 1

$$-15+5i_1+10(i_1-i_2)+10=0$$

or

$$3i_1-2i_2=1$$

Apply KVL for mesh 2

$$6i_2+4i_2+10(i_2-i_1)-10=0$$

or

$$-i_1+2i_2=1$$

In matrix form;

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{Ans}: i_1=1\,\text{A};\quad i_2=1\,\text{A}$$

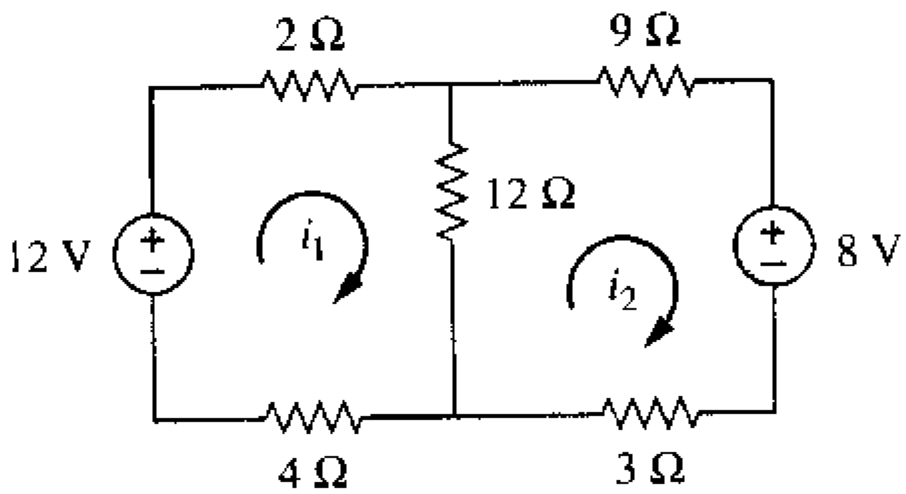
Therefore, the branch currents are;

$$I_1=i_1=1\,\text{A};\ I_2=i_2=1\,\text{A};\ I_3=i_1-i_2=0\,\text{A}$$

Problem 2.5

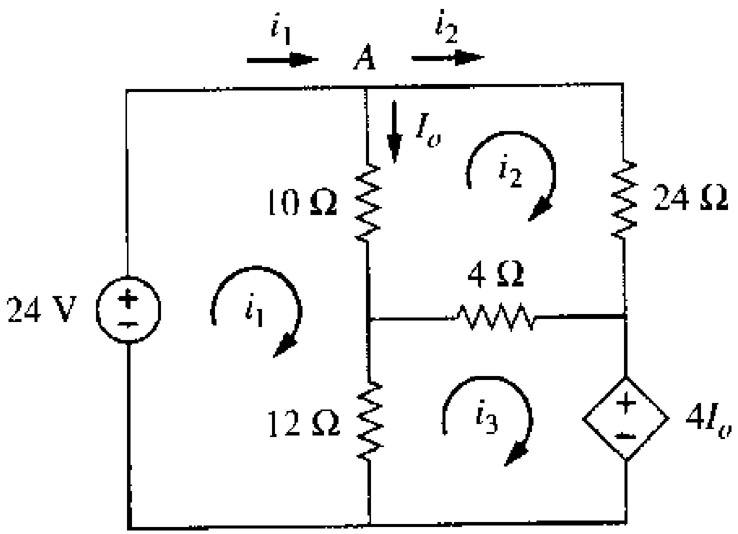
Calculate the mesh current i_1 and i_2.

Ans: $i_1 = 2/3$ A, $i_2 = 0$ A



Example 2.6

Use mesh analysis to find the current I_o in the circuit.



Solution

Apply KVL for mesh 1

$$-24+10(i_1-i_2)+12(i_1-i_3)=0$$

or

$$11i_1-5i_2-6i_3=12$$

Apply KVL for mesh 2

$$24i_2+4(i_2-i_3)+10(i_2-i_1)=0$$

or

$$-5i_1+19i_2-2i_3=0$$

Apply KVL for mesh 3

$$4I_0+12(i_3-i_1)+4(i_3-i_2)=0$$

and $I_0=i_1-i_2$; so that

$$4(i_1-i_2)+12(i_3-i_1)+4(i_3-i_2)=0$$

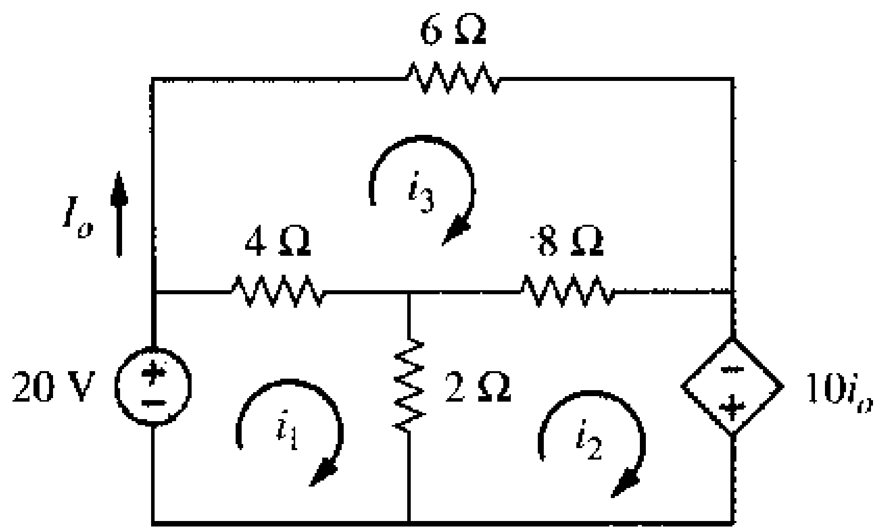
or $-i_1-i_2+2i_3=0$

Ans: $I_0=i_1-i_2=2.25-0.75=1.5\text{ A}$

Problem 2.6

Using mesh analysis, find I_0 in the circuit.

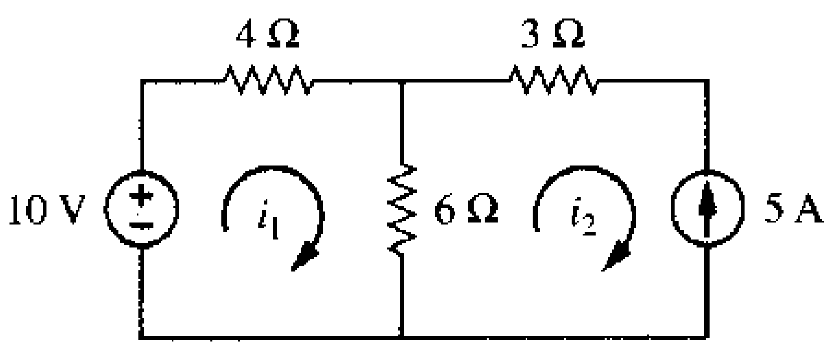
Ans: -5 A



Mesh Analysis with Current Sources

- The presence of the current sources reduces the number of equations. The number of equations (based on KVL) that need to be write is *n* – *m*, where *n*: number of meshes and *m*: number of current sources.
- Consider the following 2 possible cases:

Case 1: When current source exists only in one mesh.

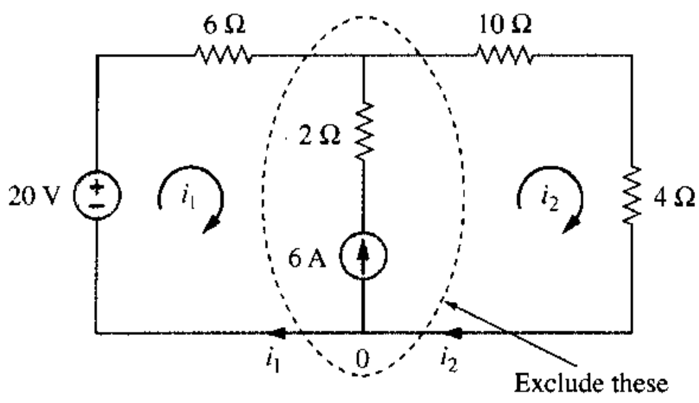


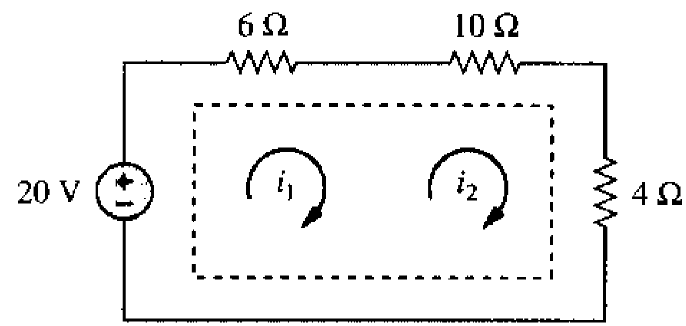
In the figure, $n - m = 2 - 1 = 1$; therefore,

KVL at mesh 1: $-10 + 4i_1 + 6(i_1 - i_2) = 0$

and $i_2 = -5$ (supportive equation)

Case 1: When current source exists between two meshes. We create a supermesh by excluding the current source and any elements connected in series with it.





(b) supermesh

KVL at supermesh gives: $-20 + 6i_1 + 10i_2 + 4i_2 = 0$

or $\quad 6i_1 + 14i_2 = 20 \qquad \Rightarrow \text{(a)}$

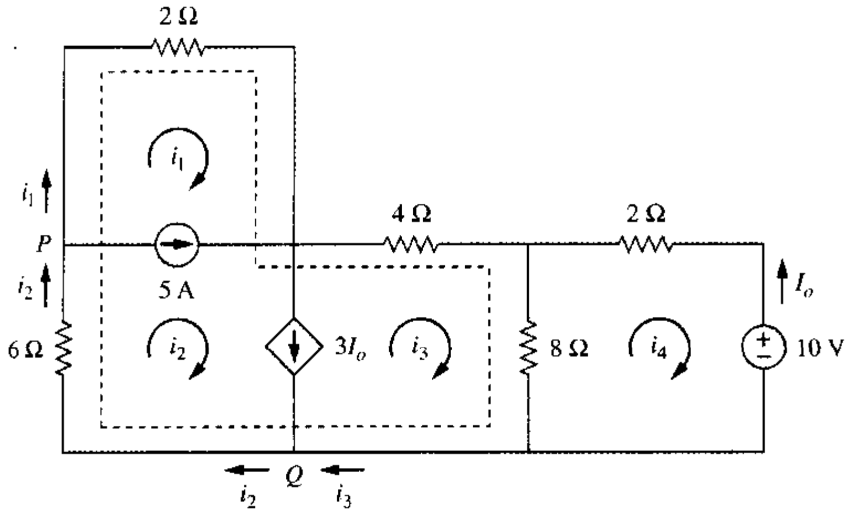
Supportive equation: $i_2 - i_1 = 6 \qquad \Rightarrow \text{(b)}$

Solving eqns (a) and (b), we get,

$i_1 = -3.2\text{A}; \quad i_2 = 2.8\text{ A}$

Example 2.7

Find i_1 to i_4 using mesh analysis in the circuit.



Solution

Number of meshes, $n = 4$; Number of Current Sources, $m = 2$

$\Rightarrow$ Number of equations need to be applied, $n - m = 4 - 2 = 2$

Meshes 1 & 2 form a supermesh. Also meshes 2 & 3 form another supermesh. The two supermeshes intersect and form a larger supermesh.

$\Rightarrow$ Applying KVL to the larger supermesh;

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or $\quad i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \qquad \Rightarrow$ (a)

$\Rightarrow$ Applying KVL in mesh 4;

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or $\quad 5i_4 - 4i_3 = -5 \qquad \Rightarrow$ (b)

Supportive equations;

$\Rightarrow \; i_2 - i_1 = 5 \qquad \Rightarrow$ (c)

$\Rightarrow \; i_2 - i_3 = 3I_0 = 3(-i_4) \qquad \Rightarrow$ (d)

From eqns (a) to (d), we get,

$$i_1 = -7.5\text{ A},\; i_2 = -2.5\text{ A},\; i_3 = 3.93\text{ A},\; i_4 = 2.143\text{ A}$$

Problem 2.7

Use mesh analysis to determine i_1, i_2 and i_3.

Ans: i_1 = 3.474 A; i_2 = 0.4737 A; i_3 = 1.1052 A