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Circuit Theory
(SKEE 1023)

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Topics

- Linearity Property, Superposition, Source Transformation, Thevenin and Norton Theorem.

Linearity Property

- Linearity is the property of an element describing a linear relationship between cause and effect.
- Combination of both homogeneity (scaling) property and the additivity property.
- E.g. Resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.
- A linear circuit consists of only linear elements, linear dependent source and independent sources.

Linearity Property

Example 4.1

Solution:

Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0$$

But $v_x = 2i_1$. Equation (4.1.2) becomes

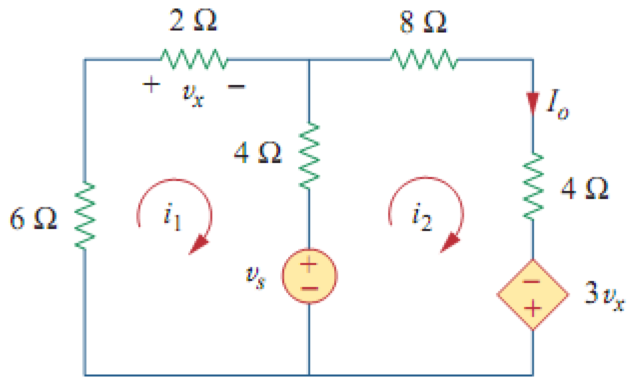
$$-10i_1 + 16i_2 - v_s = 0$$

Adding Eqs. (4.1.1) and (4.1.3) yields

$$2i_1 + 12i_2 = 0 \quad \Rightarrow \quad i_1 = -6i_2$$

Substituting this in Eq. (4.1.1), we get

$$-76i_2 + v_s = 0 \quad \Rightarrow \quad i_2 = \frac{v_s}{76}$$



When $v_s = 12$ V,

$$I_o = i_2 = \frac{12}{76}\text{ A}$$

When $v_s = 24$ V,

$$I_o = i_2 = \frac{24}{76}\text{ A}$$

showing that when the source value is doubled, I_o doubles.

Superposition Theorem

- The voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.
- Combination of both homogeneity (scaling) property and the additivity property. Applicable to linear circuit only.
- One of the method to solve the circuit with two or more independent sources.
- The number of analysis depend on how many independent sources does the circuit has.

Superposition Theorem

Steps to apply Superposition theorem/principle.

1. Choose one independent source and eliminate (turn-off) the effect of other independent sources. [*Voltage source* ⇒ short-circuit ; *Current source* ⇒ open-circuit]
2. Start the analysis using any circuit analysis methods/techniques and theorems. [KCL, KVL, nodal/mesh analysis, etc]
3. Start the analysis using any circuit analysis methods/techniques and theorems. [KCL, KVL, nodal/mesh analysis, etc]
4. Combine/add-ups all the value of each element for all separated analysis.

Superposition Theorem

- This technique does not allow to find power at any separated circuit.
- This technique has one major disadvantage; it may very likely involve more work.
- However superposition does help reduce a complex circuit to a simpler circuit.
- Combine/add-ups all the value of each element for all separated analysis.

Superposition Theorem

Problem 4.1:

Using the superposition theorem, find v_o in the circuit of Fig. P4.1. (Ans: 6 V)

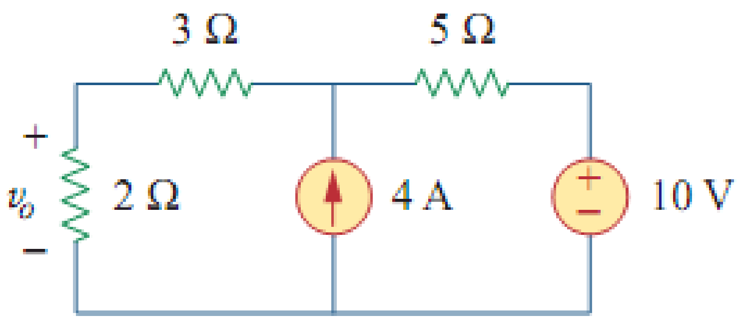
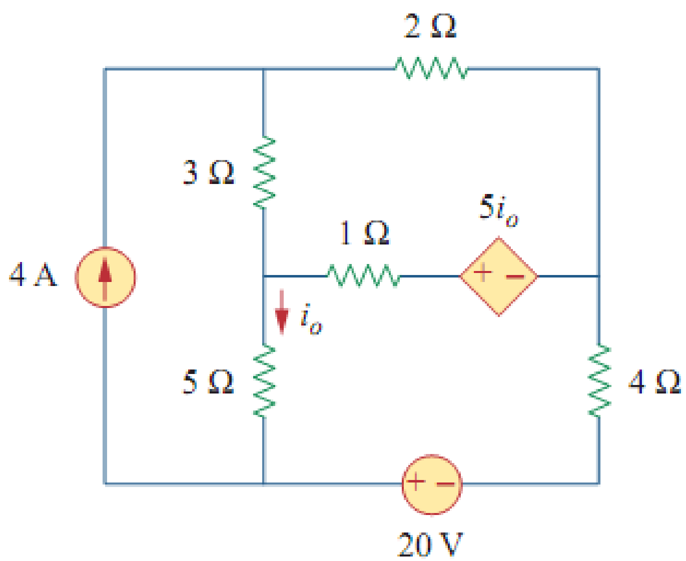


Fig. P4.1

Superposition Theorem

Example 4.2:

Find i_o in the circuit using superposition theorem.

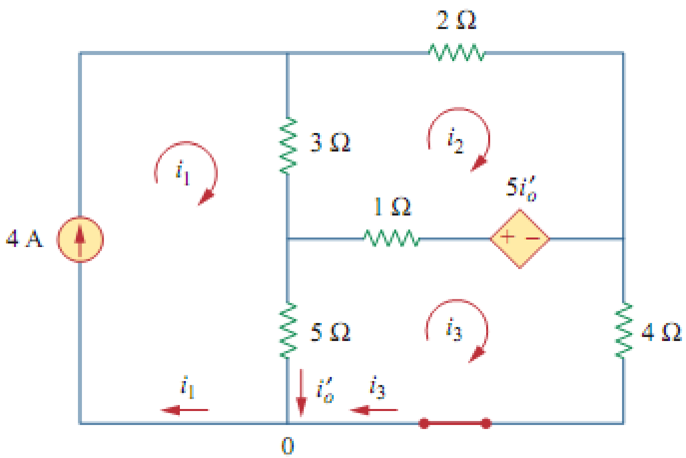


Superposition Theorem

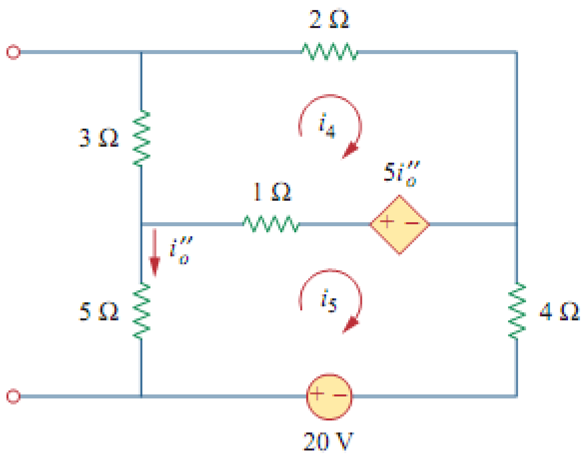
The circuit involves a dependent source, which must be left intact. The i_o is given by;

$$i_o = i_o^{'} + i_o^{''}$$

How to find i_o' ?



How to find i_o'' ?



Superposition Theorem

Problem 4.2:

Use superposition to find v_x in the circuit of Fig. P4.2. (Ans: 25 V)

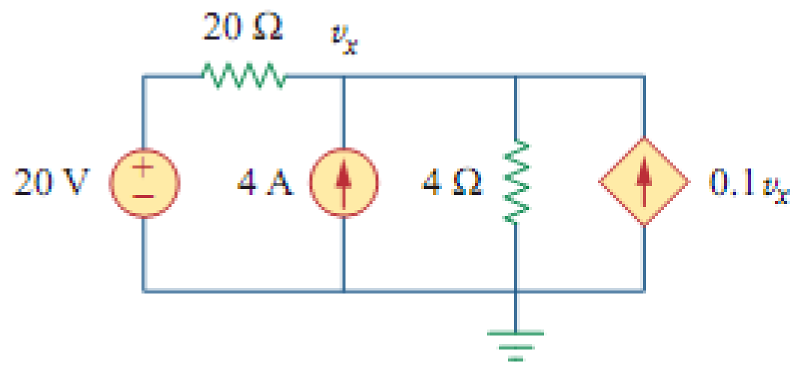
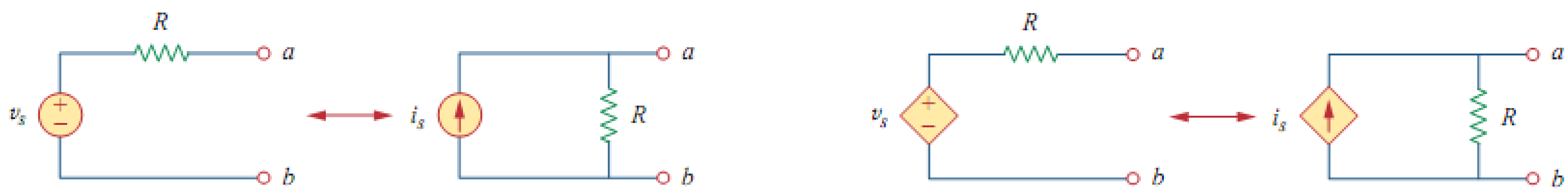


Fig. P4.2

Source Transformation

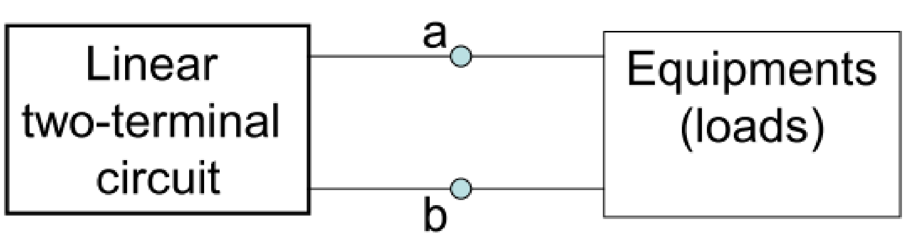
- A source transformation is the process of replacing a voltage source V_s in series with a resistor R by a current source I_s in parallel with a resistor R, or vice versa.



$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

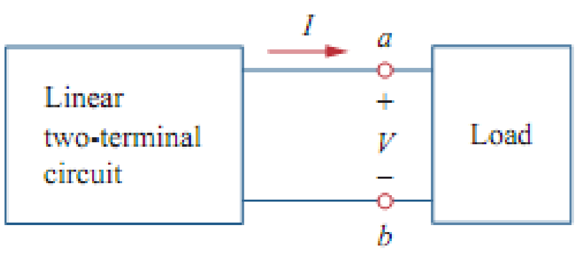
Thevenin's Theorem

- Objective : To simplify the circuit.
- Provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.
- Developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.
- When the load are varies, all the variables (voltage & current) inside the linear circuit would also varies, thus the analysis has to be done again.

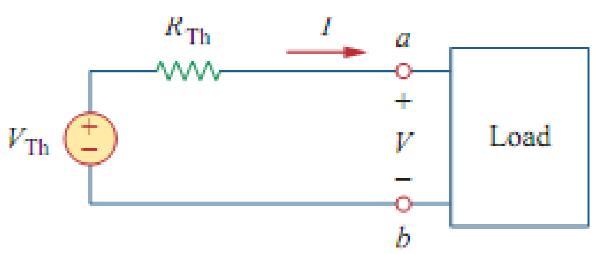


Thevenin's Theorem

- Thevenin's theorem states that a linear 2-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}.



(a)

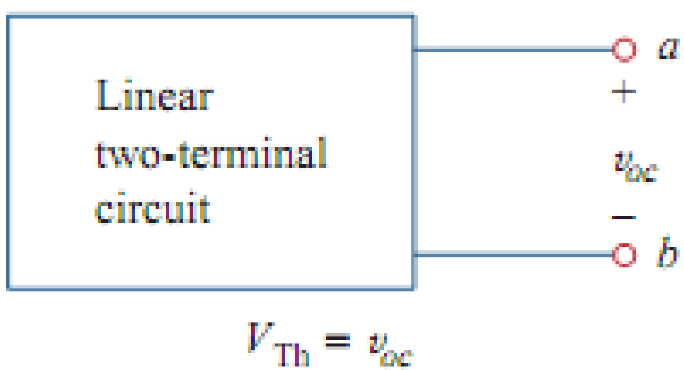


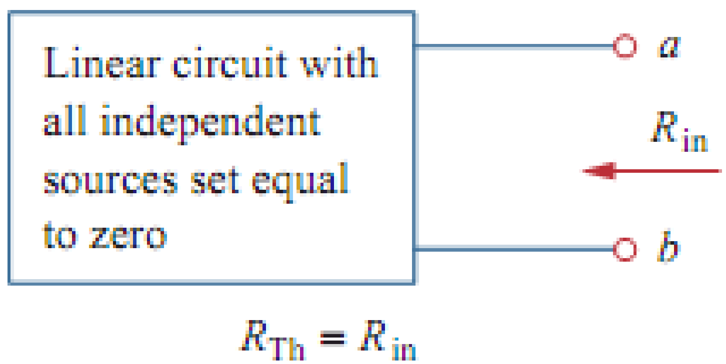
(b)

- V_{Th} is the open-circuit voltage at the terminals, and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.
- It is easy to find the open-circuit voltage (v_{oc}) and short-circuit current (i_{sc}) at terminal a-b through experimental method.
- Thus, $V_{Th} = v_{oc}$ and $R_{Th} = v_{oc}/i_{sc}$

Thevenin's Theorem

- How to find V_{Th} (Thevenin voltage) and R_{Th} (Thevenin resistance)?





Thevenin's Theorem

- A linear circuit without dependent source.

1. Find v_{oc} across the two terminal (either using mesh/nodal, etc). The Thevenin voltage (V_{Th}) is equal to v_{oc}.
2. Simply turn off all the independent sources.
3. Find the equivalent resistance (R_{eq}) between the two terminal. The Thevenin resistor (R_{Th}) is equal to the R_{eq}.

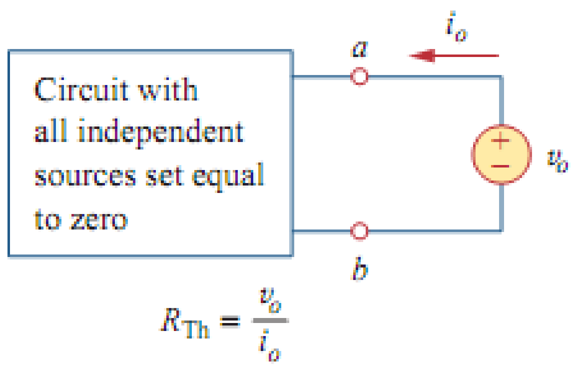
Thevenin's Theorem

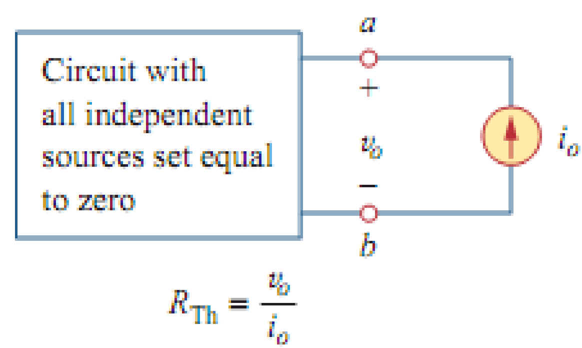
- A linear circuit with dependent source

1. Find v_{oc} across the two terminal (either using mesh/nodal, etc). The Thevenin voltage (V_{Th}) is equal to v_{oc}.
2. Simply turn off all the independent sources. The dependent sources should remain intact/unchanged.
3. Inject a voltage source (v_o) OR current source (i_o) across the two terminal. Then determine current supplied by voltage source OR voltage across the current source.
4. The value of $R_{Th} = v_o/i_o$

Thevenin's Theorem

- Finding R_{Th} when circuit has dependent sources.

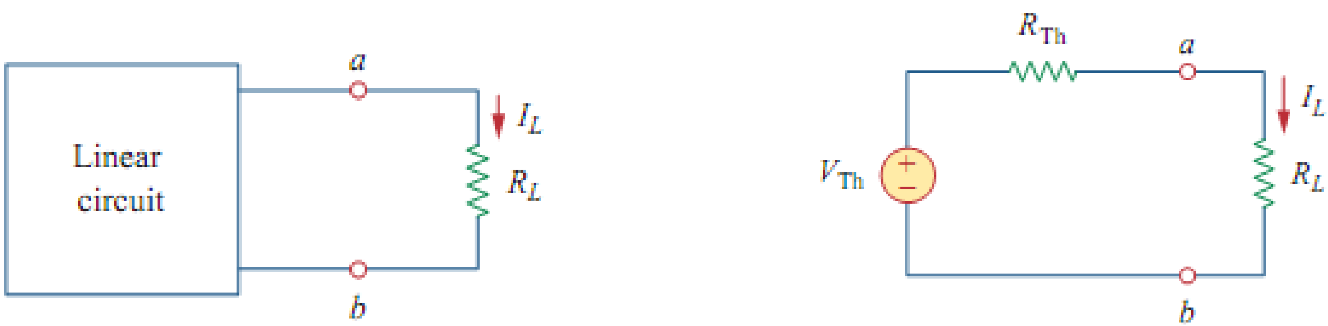




- We may assume any value of v_o and i_o . For example, we may use v_o = 1V or i_o = 1A, or even use unspecified values of v_o or i_o.

Thevenin's Theorem

- A circuit with a load: (a) original circuit, (b) Thevenin equivalent



(a) (b)

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

Thevenin's Theorem

Example 4.3:

Find the Thevenin equivlent circuit of the circuit shown in Fig. E4.3, to the left of the terminals *a*-*b*. Then find the current through R_L = 6, 16 and 36 Ω.

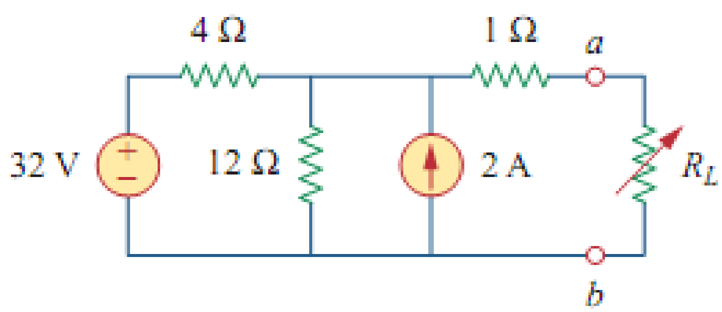


Fig. E4.3

Thevenin's Theorem

Example 4.4:

Find the Thevenin equivlent circuit of the circuit in Fig. E4.4 at terminals *a*-*b*.

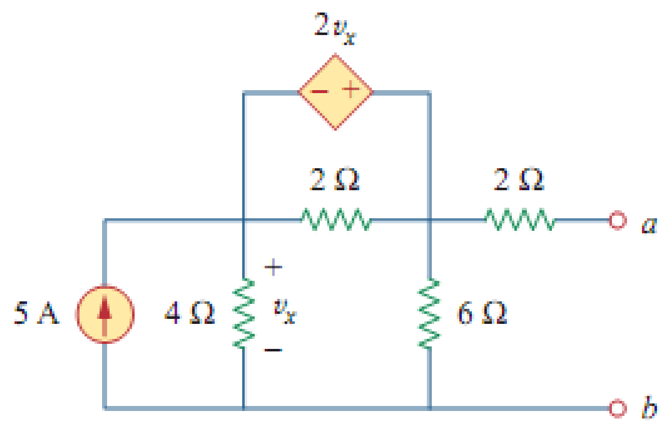
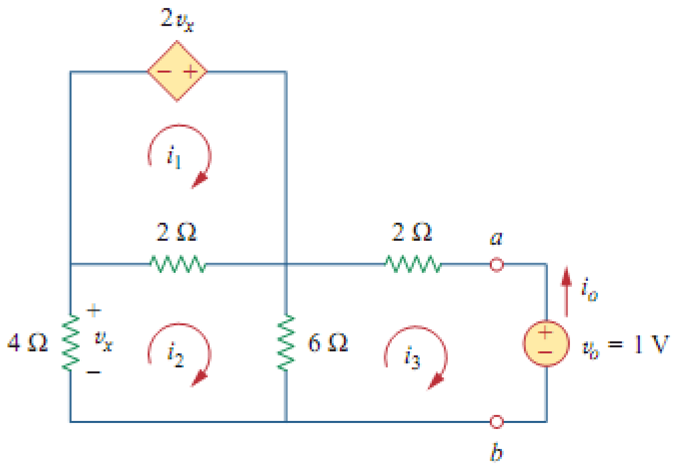


Fig. E4.4

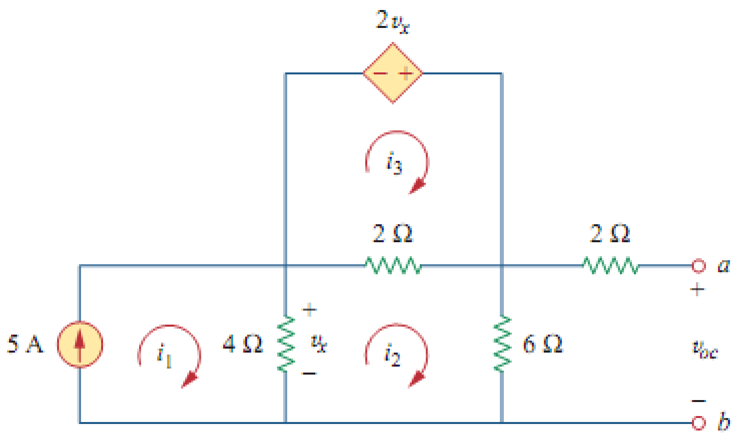
Thevenin's Theorem

Example 4.4 (cont.):

Finding R_{Th} and V_{Th} at terminals *a-b*.



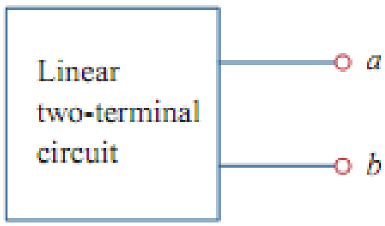
Finding R_{Th}



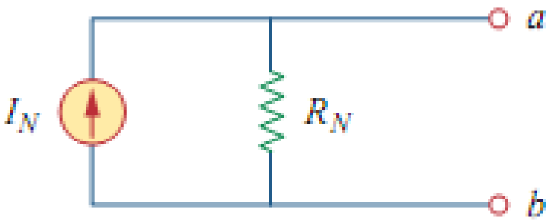
Finding V_{Th}

Norton's Theorem

- Similar with Thevenin's theorem.
- Proposed by E.L. Norton (1926), an American engineer at Bell Telephone Laboratories.
- Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source (I_N) in parallel with a resistor, R_N.



Original circuit



Norton equivalent circuit

Norton's Theorem

- I_N is the short-circuit current (i_{sc}) through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.
- Method to find R_N is same with $R_{Th} \Rightarrow R_N = R_{Th}$
- By using "Source Transformation" method, the conversion between Thevenin and Norton theorems are possible.
- Correlation between Thevenin and Norton parameters are given as follows;

$$I_N = \frac{V_{Th}}{R_{Th}}; \quad V_{Th} = v_{oc} \quad \text{and} \quad I_N = i_{sc}$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$$

Norton's Theorem

Example 4.5 :

Find the Norton equivalent circuit of the circuit of Fig. E4.5.

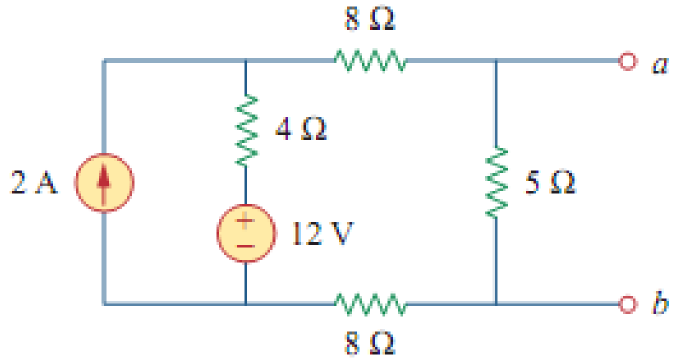
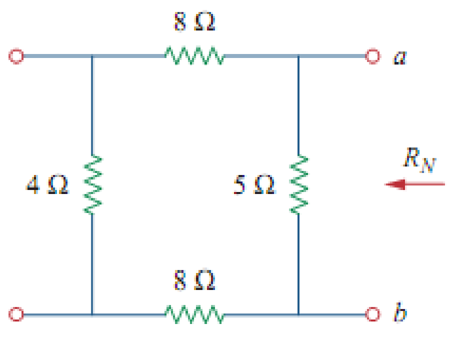


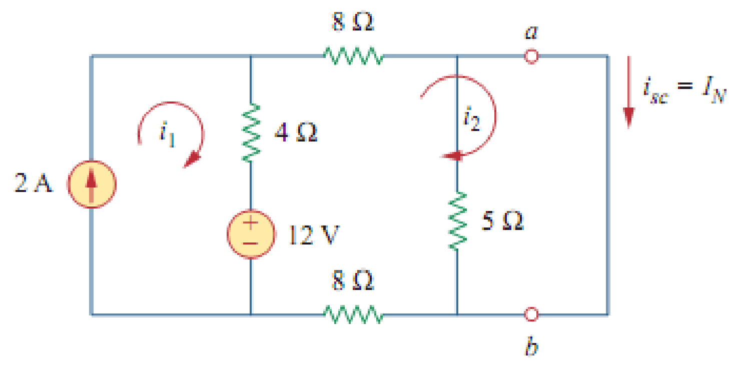
Fig. E4.5

Norton's Theorem

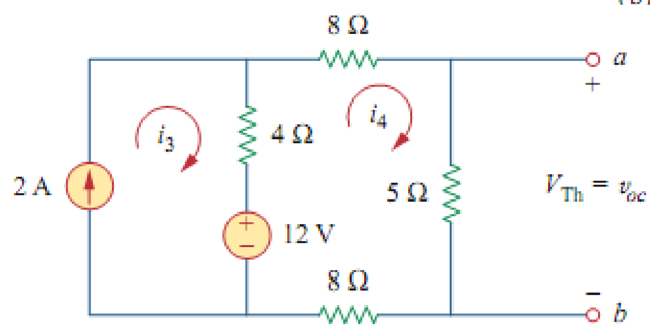
Finding (a) R_N , (b) $I_N = i_{sc}$, (c) $V_{Th} = v_{oc}$.



(a)



(b)



(c)

Norton's Theorem

Problem 4.3:

Find the Norton equivalent circuit of the circuit in Fig. P4.3 at terminals *a*-*b*.
(Ans: $R_N = 1\ \Omega$, $I_N = 10$ A)

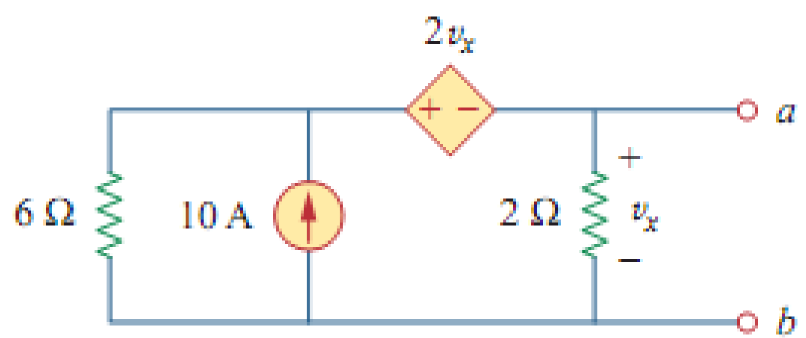
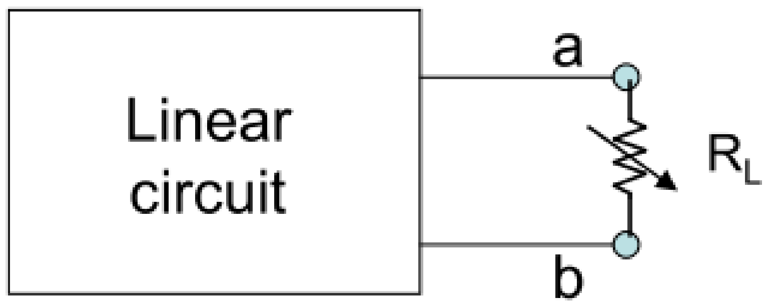


Fig. P4.3

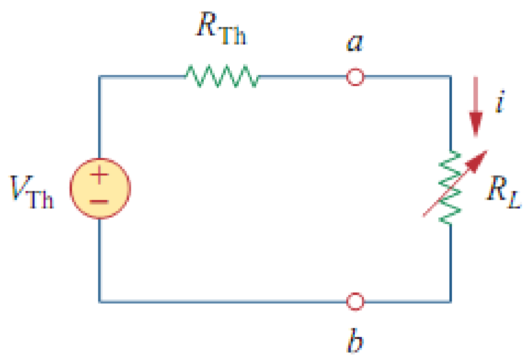
Maximum Power Transfer

- A circuit is purposely designed to suit the demand of the loads (*in terms of voltage/current/power*).
- If the load (resistive load) is variable, then what is the maximum power that can be transferred to the load?



Maximum Power Transfer

- Thevenin equivalent is useful in finding the maximum power that the linear circuit can deliver to a load.

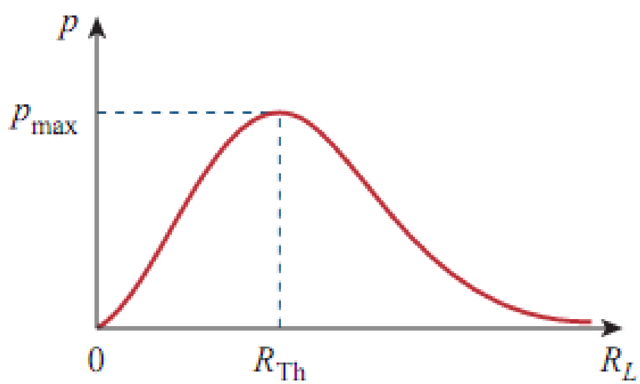


- Power absorbed by the load;

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

Maximum Power Transfer

- By varying the load resistance R_L , the power delivered to the load varies as sketched in figure below;



- Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load (***$R_L = R_{Th}$***)

Maximum Power Transfer

Example 4.6 :

Find the value of R_L for maximum power transfer in the circuit of Fig. E4.6. Find maximum power. (Ans: $R_L = 9\ \Omega$, 13.44 W)

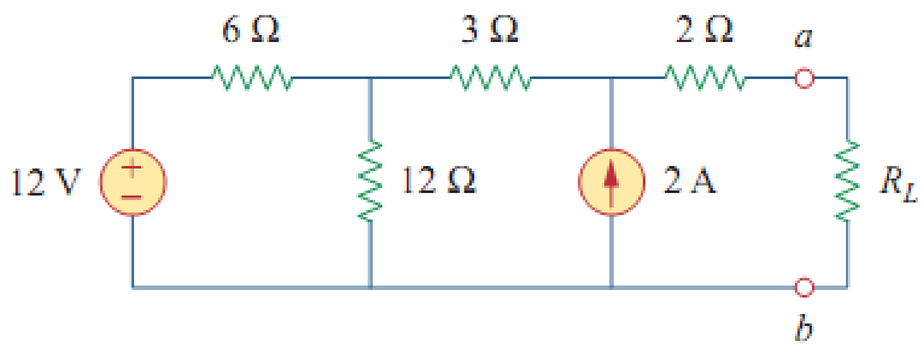


Fig. E4.6