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Circuit Theory (SKEE 1023)

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Topics

- Basic Laws: Ohm's law; Nodes, Branches and Loops; Kirchhoff's Law; Series-parallel resistors; Voltage and Current Division.

Basic Laws

Ohm's law states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor.

The resistance R of an element denotes its ability to resist the flow of electric current, it is measured in ohms (Ω)

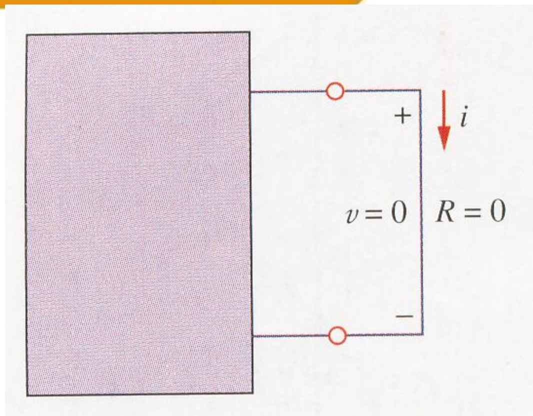
Ohm's Law

$$I = \frac{V}{R}$$

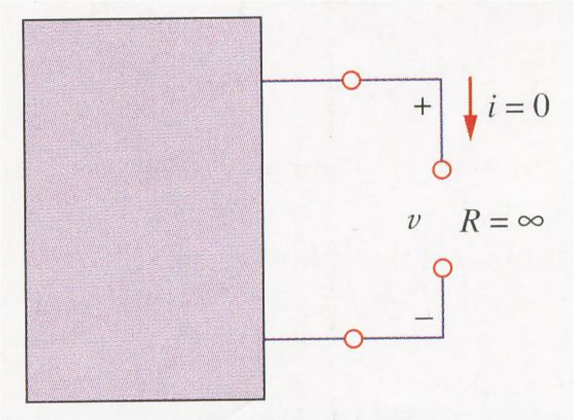
Electric current = Voltage / Resistance

Value of R can range from zero to infinity, which are the two extreme possible values of R (**0** and ∞).

Circuit Theory



short circuit



open circuit

- A short circuit is a circuit element with resistance approaching zero.
- An open circuit is a circuit element with resistance approaching infinity.
- The reciprocal of resistance, known as *conductance* and denoted by *G*;

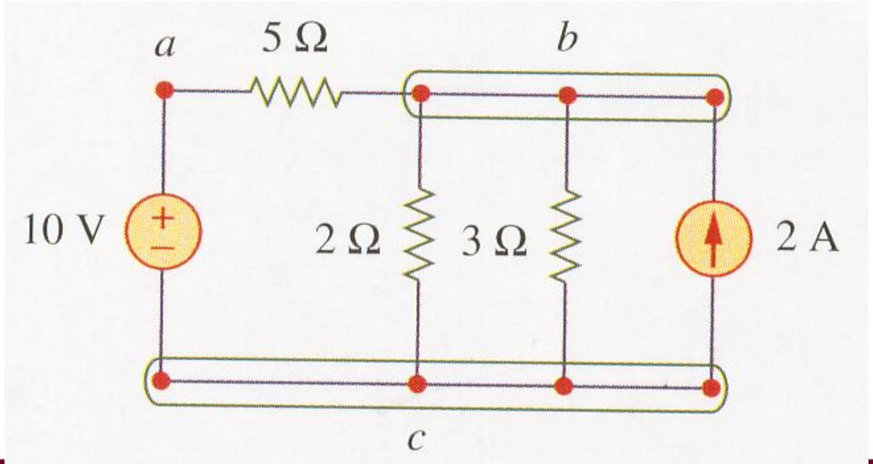
$$G = \frac{1}{R} = \frac{i}{v}$$

- Conductance is the ability of an element to conduct electric current; it is measured in mhos or siemens (S).

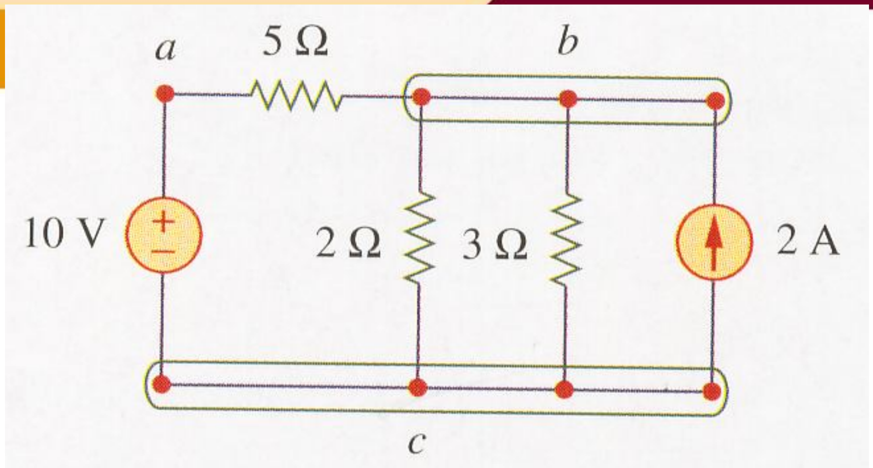
Nodes, Branches and Loops

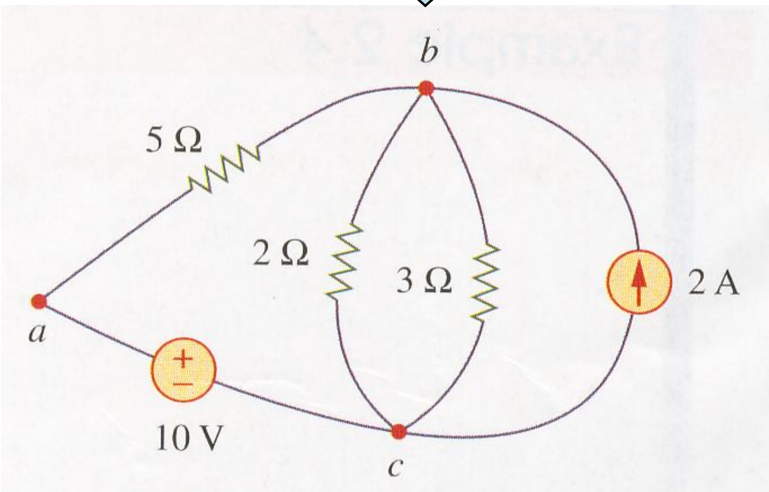
To understand some basic concepts of network topology. A network – as an interconnection of elements/devices. A circuit is a network providing one or more closed path.

- A branch represents a single element such as voltage/current source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.



M. Afendi M. Piah, PhD





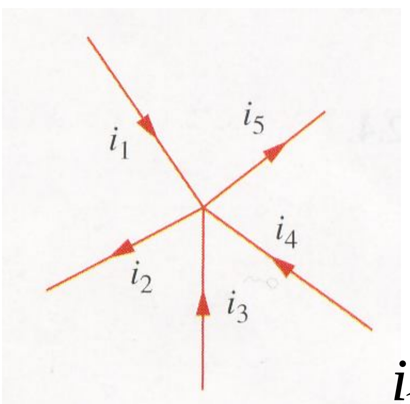
Fundamental theorem of network topology:

$$b = l + n - 1$$

b **: branches**
n **: nodes**
l **: independent loops**

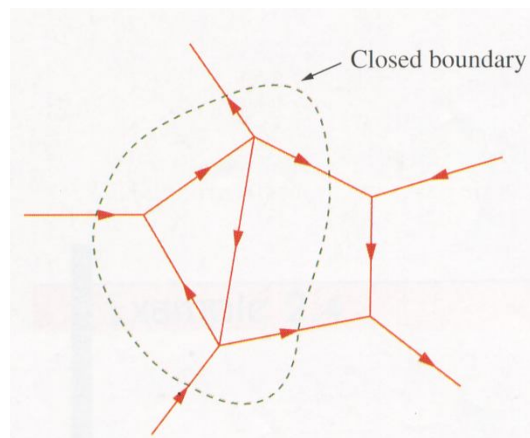
Kirchhoff's Laws

- Introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824-1887). The laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).
- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.



$$\sum_{n=1}^{N} i_n = 0$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

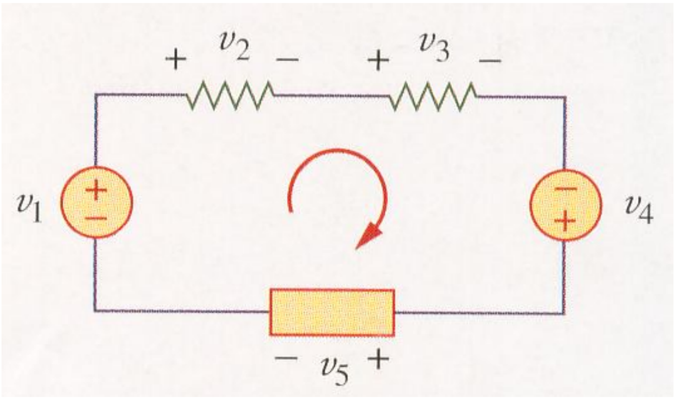


The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Kirchhoff's Laws

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\sum_{m=1}^{M} v_m = 0$$



$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

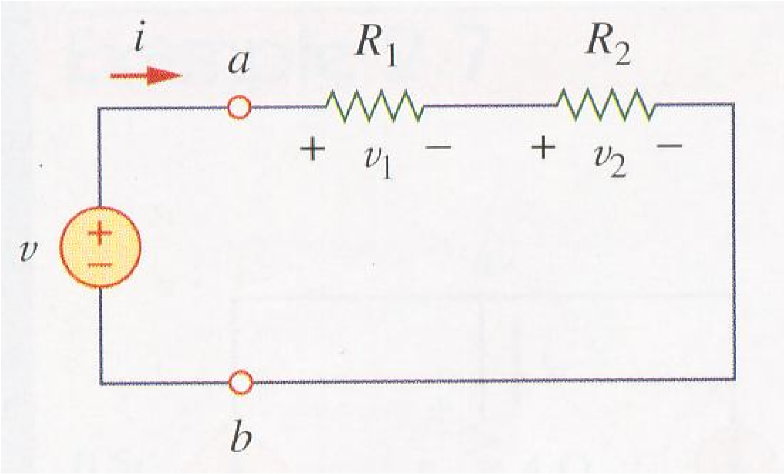
$$\therefore \quad v_2 + v_3 + v_5 = v_1 + v_4$$

Sum of voltage drops = Sum of voltage rises

Series Resistors and Voltage Division

- The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances
- For N resistors in series, then;

$$R_{eq} = R_1 + R_2 + \ldots + R_N = \sum_{n=1}^{N} R_n$$



$$v_1 = \frac{R_1}{R_1 + R_2} v \; ; \quad v_2 = \frac{R_2}{R_1 + R_2} v$$

For N resistors in series;

$$v_n = \frac{R_n}{R_1 + R_2 + \ldots + R_N} v$$

Principle of voltage division

Parallel Resistors and Current Division

- The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \; ; \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- For the circuit with *N* resistors in parallel.

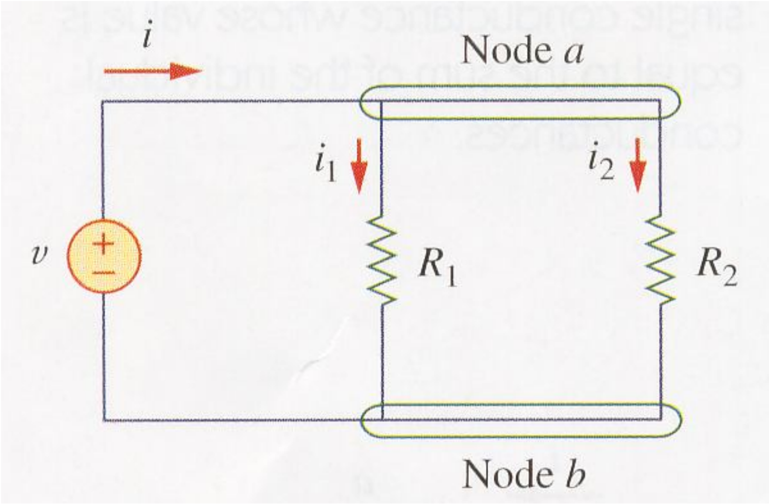
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

$$\text{in terms of conductance } (G);$$

$$G_{eq} = G_1 + G_2 + G_3 + \ldots + G_N$$

The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

Parallel Resistors and Current Division



$$i_1 = \frac{R_2}{R_1 + R_2} i \ ; \quad i_2 = \frac{R_1}{R_1 + R_2} i$$

Principle of current division

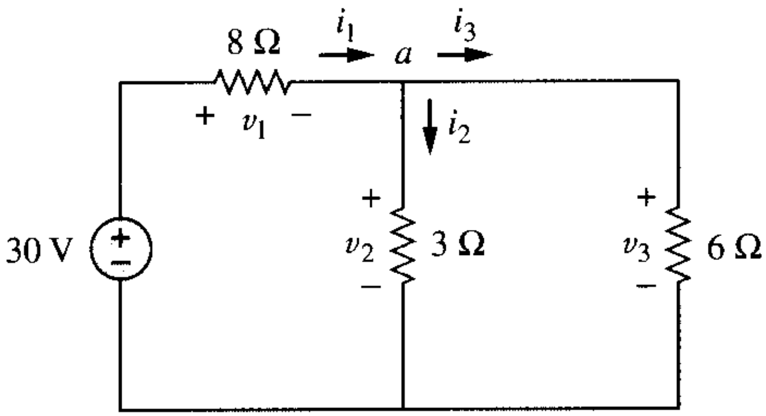
- If a current divider has *N* conductors (G_1, G_2, …G_N) in parallel with the source current *i*, the nth conductor (G_n) will have current;

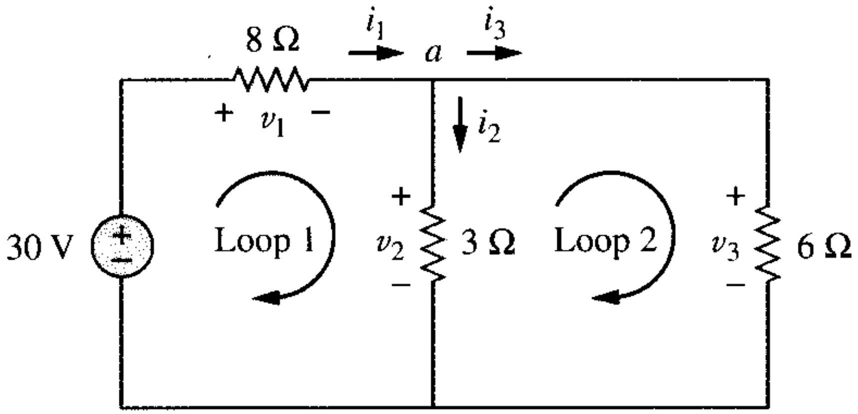
$$i_n = \frac{G_n}{G_1 + G_2 + ... + G_N} i$$

Circuit Theory

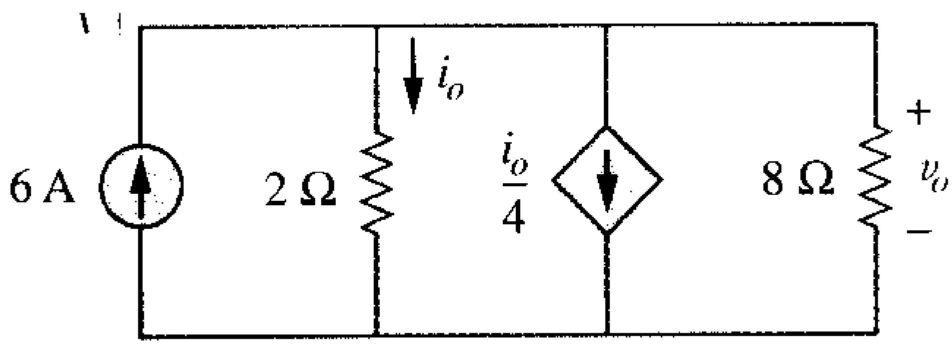
Examples and Problems

1. Find currents and voltages in the circuit below.



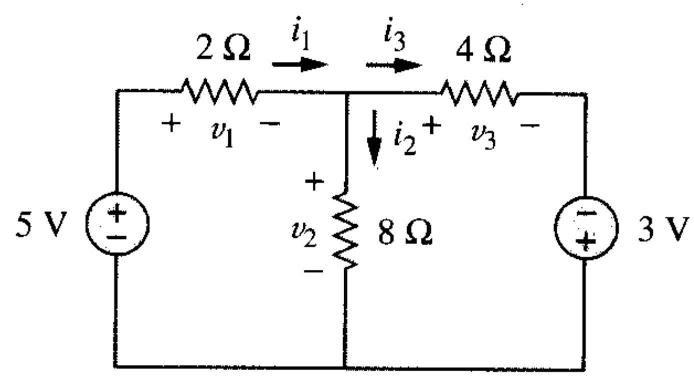


2. Find v_o and i_o.

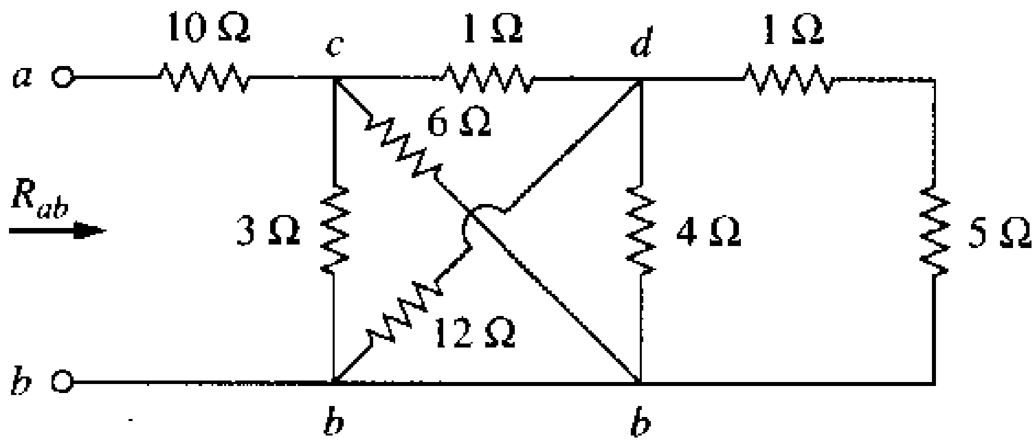


Circuit Theory

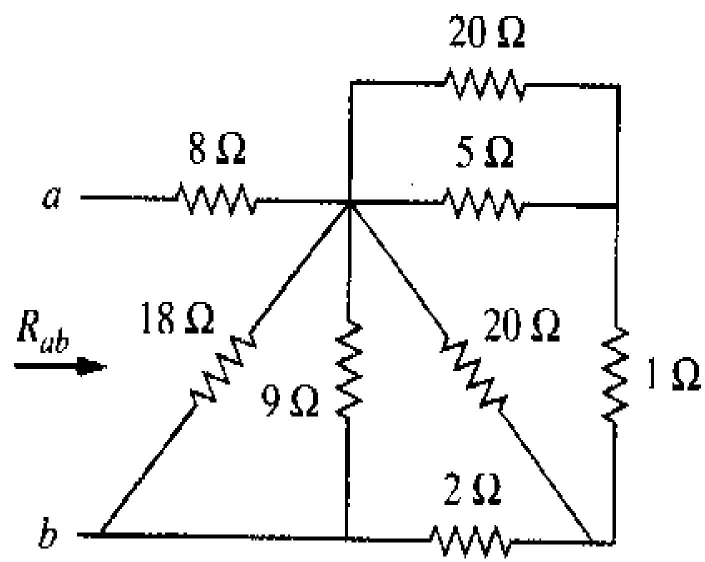
3. Find currents and voltages in the circuit below.



4. Calculate the equivalent resistance R_{ab}.



5. Find R_{ab}.



6. Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Answer: $v_1 = 5$ V, $i_1 = 416.7$ mA, $p_1 = 2.083$ W, $v_2 = 10$ V, $i_2 = 250$ mA, $p_2 = 2.5$ W.

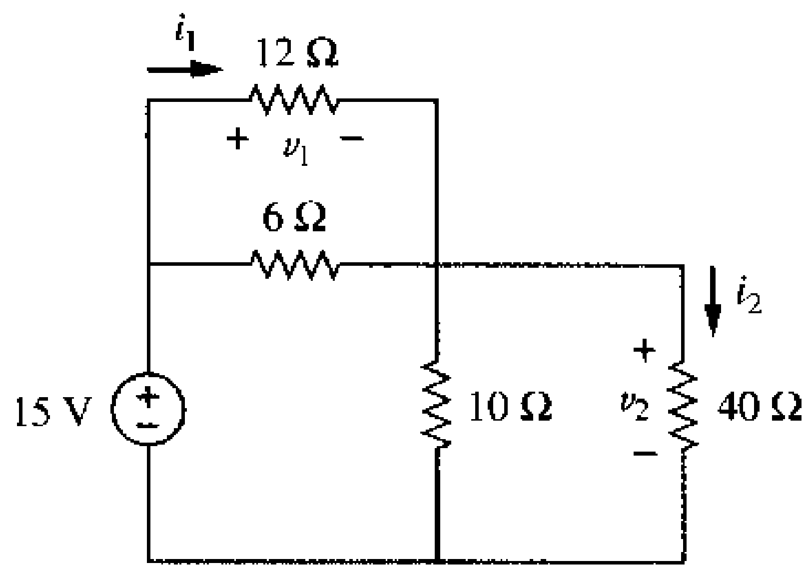


Fig 2.43

7. For the circuit shown in Fig. 2.45, find: (a) v_1 and v_2, (b) the power dissipated in the 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.

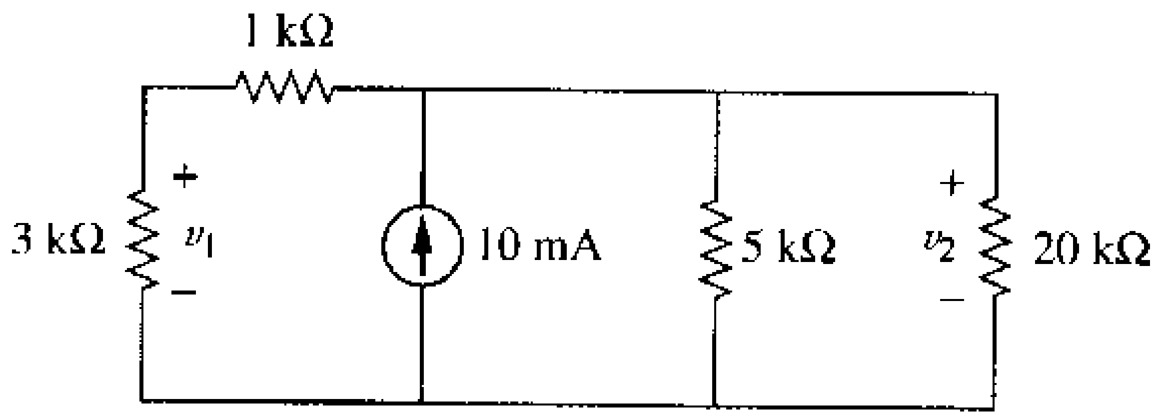
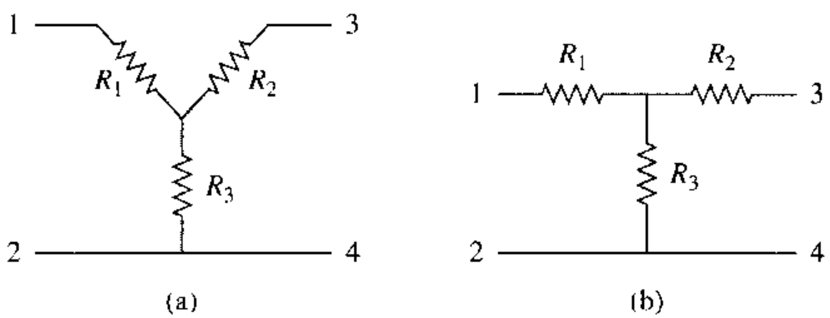


Fig. 2.45

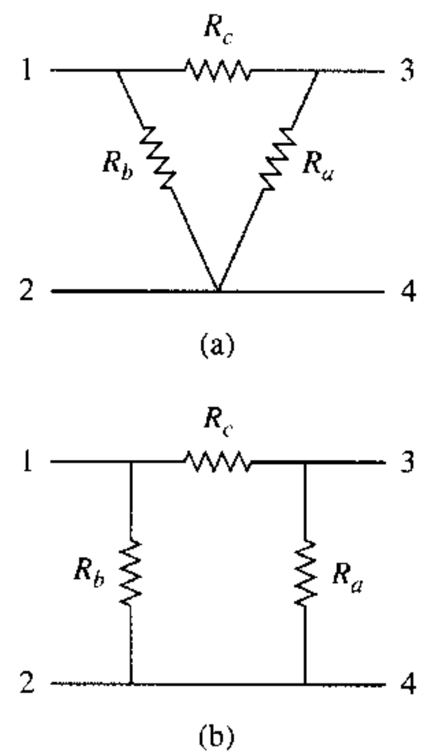
Answer: (a) 15 V, 20 V, (b) 75 mW, 20 mW, (c) 200 mW.

Wye-Delta Transformations

- Situations often arise in circuit analysis when the resistors are neither in parallel nor in series.
- Two types of network, i.e. wye and delta.



Two forms of the same network: (a) Y, (b) T.



Two forms of the same network: (a) Δ, (b) Π.

Delta to Wye (star) Conversion

$$R_{12}(Y) = R_1 + R_3 \quad \text{and} \quad R_{12}(\Delta) = R_b // (R_a + R_c)$$

Setting $R_{12}(Y) = R_{12}(\Delta)$, gives;

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

Similarly;

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

Therefore;

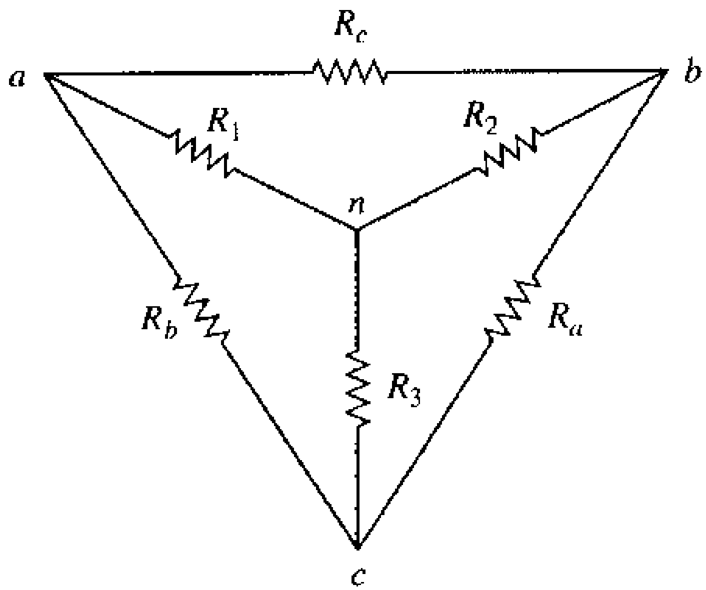
$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$$

Finally, for delta to Y (star) conversion;

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Wye (star) to Delta Conversion

By using the equations of R_1, R_2 & R_3;

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2}$$

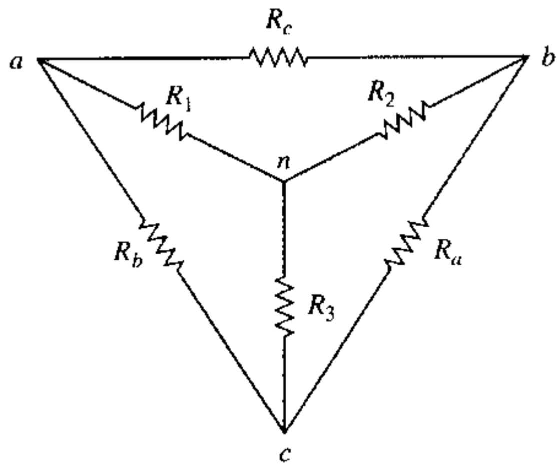
$$= \frac{R_aR_bR_c}{R_a + R_b + R_c}$$

Therefore;

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be *balanced* when.

$$R_1 = R_2 = R_3 = R_Y \text{ , and } \quad R_a = R_b = R_c = R_\Delta$$

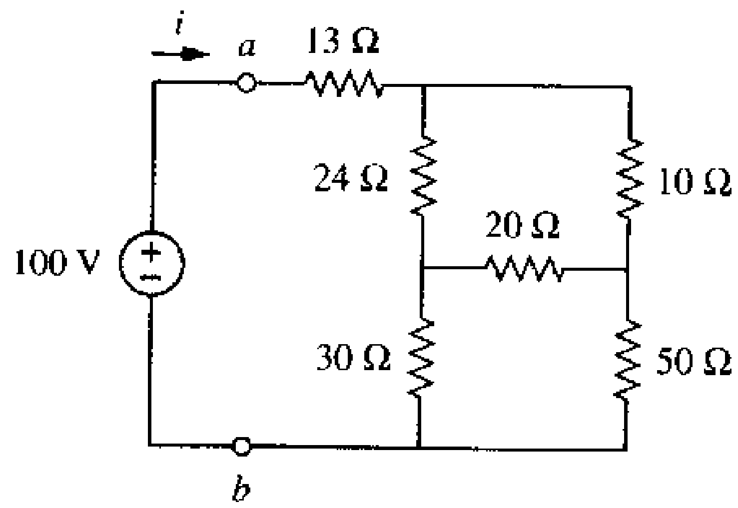
Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3}, \quad \text{or} \quad R_\Delta = 3R_Y$$

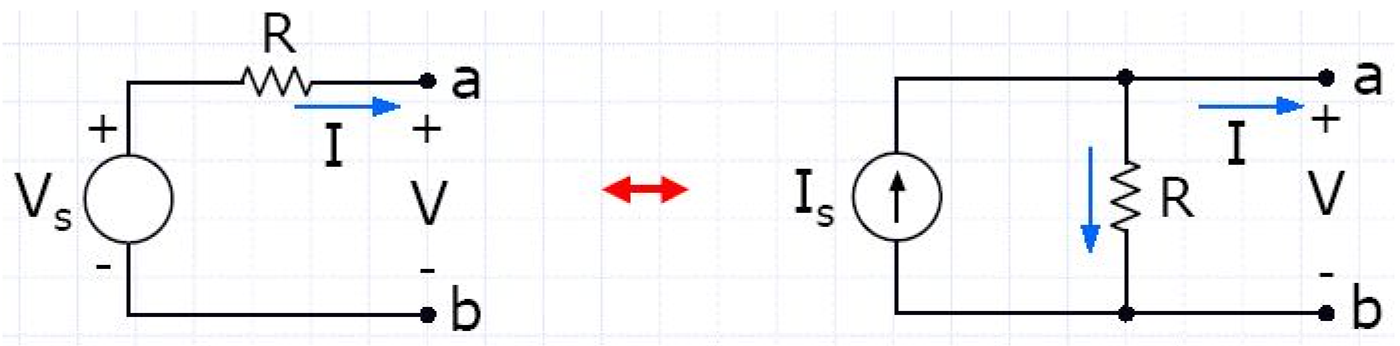
Problem

For the bridge network shown in the figure, find R_{ab} and i.

Ans: 40Ω, 2.5A



Source Transformations



- If the two networks are equivalent with respect to terminals ab, then V and I must be identical for both networks. Thus,

$$V_S = I_S R\,, \text{ or } \quad I_S = \frac{V_S}{R}$$

Problem

By using the method of source transformations, find v_x.