

CHAPTER 1

SET THEORY

[Part 2: Operation on Set]

Union

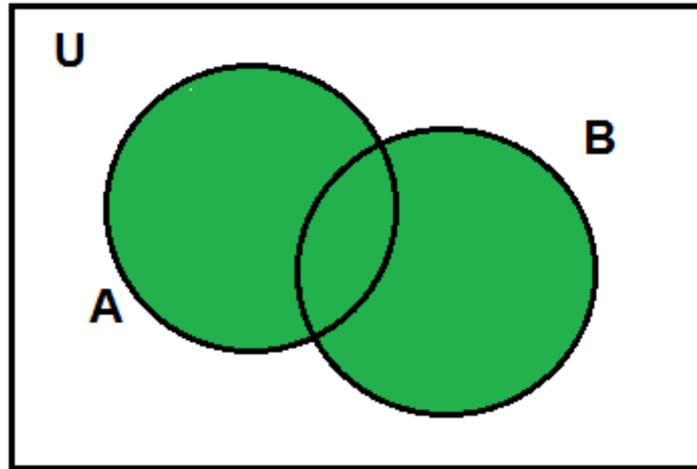
- The **union** of two sets A and B , denoted by $A \cup B$, is defined to be the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

- The union consists of all elements belonging to either A or B (or both)

Union

- Venn diagram of $A \cup B$



Example

$A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$ and $C = \{8, 9\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$$

$$B \cup C = \{2, 4, 6, 8, 9\}$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$$

Union

- If A and B are finite sets, the **cardinality** of $A \cup B$,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Intersection

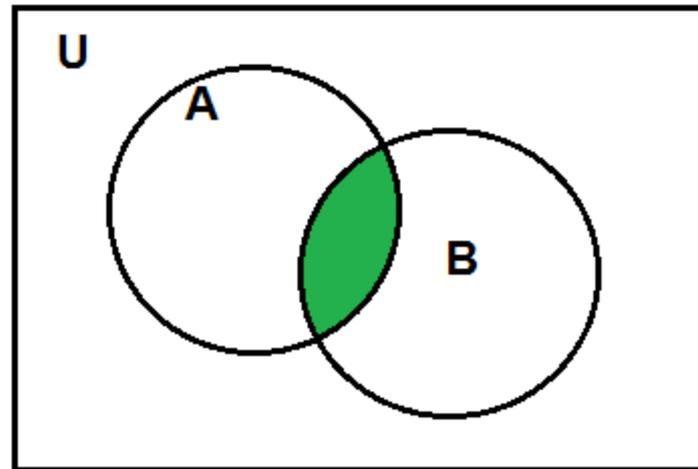
- The **intersection** of two sets A and B , denoted by $A \cap B$, is defined to be the set

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

- The **intersection** consists of all elements belonging to both A and B .

Intersection

- Venn diagram of $A \cap B$



Example

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$ and
 $C = \{1, 2, 8, 10\}$

$$A \cap B = \{2, 4, 6\}$$

$$A \cap C = \{1, 2\}$$

$$C \cap B = \{2, 8, 10\}$$

$$A \cap B \cap C = \{2\}$$

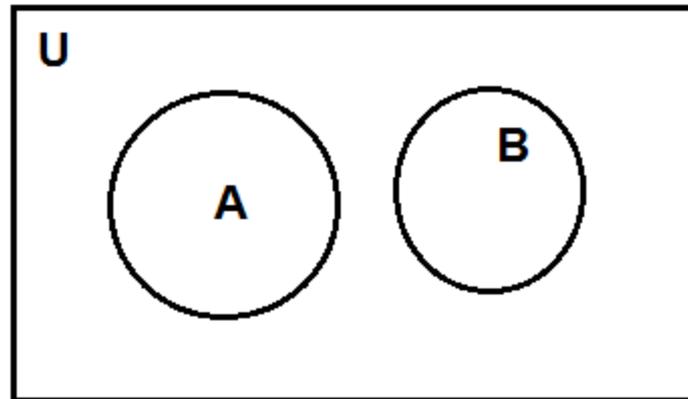
Disjoint

- Two sets A and B are said to be **disjoint** if,

$$A \cap B = \emptyset$$

Disjoint

- Venn diagram, $A \cap B = \emptyset$



Example

$$A = \{1, 3, 5, 7, 9, 11\}$$

$$B = \{2, 4, 6, 8, 10\}$$

$$A \cap B = \emptyset$$

Difference

- The set

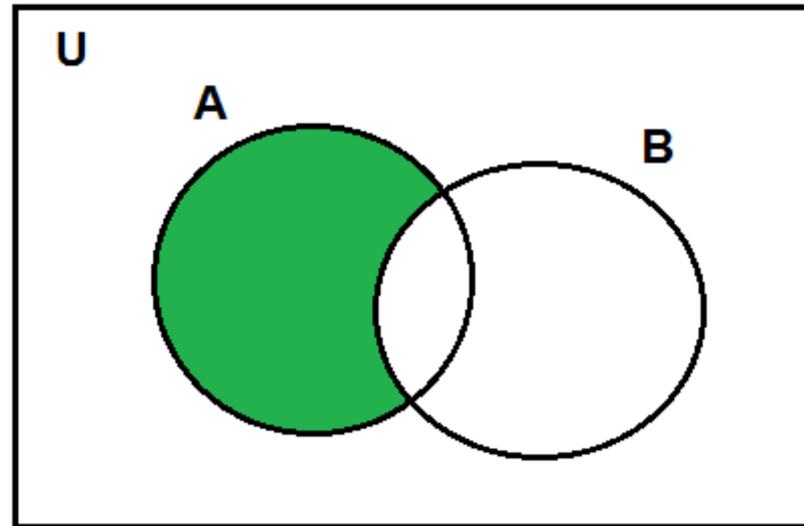
$$A-B = \{x \mid x \in A \text{ and } x \notin B\}$$

is called the **difference**.

- The difference $A - B$ consists of all elements in A that are not in B .

Difference

- Venn diagram of $A-B$



Example

$$A = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$$

$$B = \{ 2, 4, 6, 8 \}$$

$$A - B = \{ 1, 3, 5, 7 \}$$

Complement

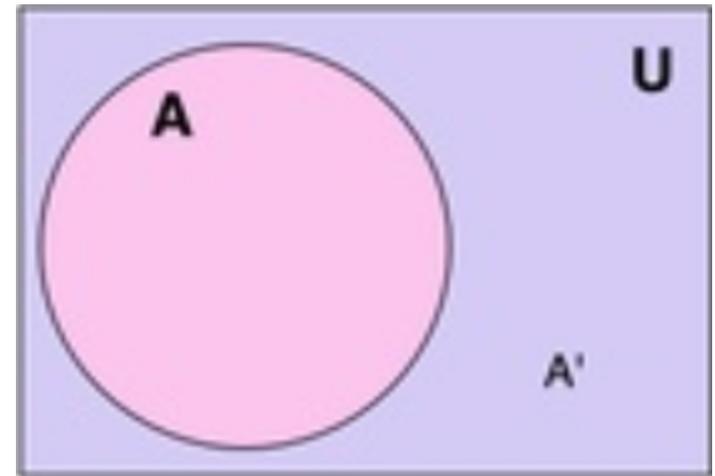
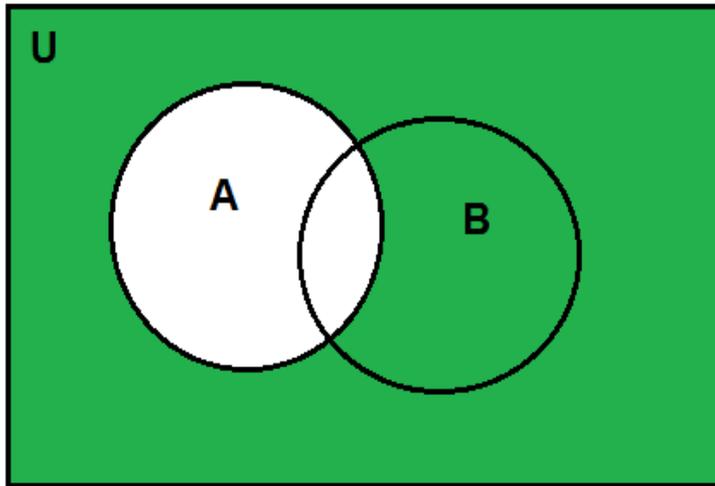
- The complement of a set A with respect to a universal set U , denoted by A' is defined to be

$$A' = \{x \in U \mid x \notin A\}$$

$$A' = U - A$$

Complement

Venn diagram of A'



Example

Let U be a universal set,

$$U = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

$$A = \{ 2, 4, 6 \}$$

$$A' = U - A = \{ 1, 3, 5, 7 \}$$

Exercise

- Let,

$$U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m \}$$

$$A = \{ a, c, f, m \}$$

$$B = \{ b, c, g, h, m \}$$

- Find:

$$A \cup B, A \cap B, |A \cup B|, A - B \text{ dan } A'.$$

Exercise

Let the universe be the set $U = \{1, 2, 3, 4, \dots, 10\}$.

Let $A = \{1, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and
 $C = \{2, 4, 6, 8\}$.

List the elements of each set:

a) U'

b) $B' \cap (C - A)$

c) $B - A$

d) $(A \cup B)(C - B)$

Properties of Sets

- Commutative laws

$$A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

Properties of Sets

- Associative laws

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Properties of Sets

- Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Properties of Sets

- Absorption laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

Properties of Sets

- Idempotent laws

$$A \cap A = A$$

$$A \cup A = A$$

Properties of Sets

- Complement laws

$$(A')' = A$$

$$A \cap A' = \emptyset$$

$$A \cup A' = U$$

$$\emptyset' = U$$

$$U' = \emptyset$$

Properties of Sets

- De Morgan's laws

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

Properties of Sets

- Properties of universal set

$$A \cup U = U$$

$$A \cap U = A$$

Properties of Sets

- Properties of empty set

$$A \cup \emptyset = A$$

$$A \cap \emptyset = \emptyset$$

Example

- Let A , B and C denote the subsets of a set S and let C' denote a complement of C in S .
- If $A \cap C = B \cap C$ and $A \cap C' = B \cap C'$, then prove that $A = B$

Example

$$\begin{aligned} A &= A \cap S \\ &= A \cap (C \cup C') \\ &= (A \cap C) \cup (A \cap C') && \text{by distributivity} \\ &= (B \cap C) \cup (B \cap C') && \text{by the given conditions} \\ &= B \cap (C \cup C') && \text{by distributivity} \\ &= B \cap S \\ &= B \end{aligned}$$

Simplification using the Set Law

Simplify the set

$$(((A \cup B) \cap C)' \cup B')' =$$

$$= ((A \cup B) \cap C)'' \cap B''$$

[DeMorgan]

$$= ((A \cup B) \cap C) \cap B$$

[Double Complement]

$$= (A \cup B) \cap (C \cap B)$$

[Associativity of \cap]

$$= (A \cup B) \cap (B \cap C)$$

[Commutativity of \cap]

$$= ((A \cup B) \cap B) \cap C$$

[Associativity of \cap]

$$= B \cap C$$

[Absorption]

Exercise

- Let A , B and C be sets.
- Show that

$$(A \cup (B \cap C))' = A' \cap (B' \cup C')$$

Exercise

- Let A , B and C be sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$
- Prove that $B = C$

Cartesian Product

- Let A and B be sets.
- An **ordered pair** of elements $a \in A$ dan $b \in B$ written (a, b) is a listing of the elements a and b in a specific order.
- The ordered pair (a, b) specifies that a is the first element and b is the second element.

Cartesian Product

- An ordered pair (a, b) is considered distinct from ordered pair (b, a) , unless $a=b$.
- Example $(1, 2) \neq (2, 1)$

Cartesian Product

- The Cartesian product of two sets A and B , written $A \times B$ is the set,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

- For any set A ,

$$A \times \emptyset = \emptyset \times A = \emptyset$$

Example

$$A = \{a, b\}, B = \{1, 2\}.$$

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$$

$$B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$$

Cartesian Product

- if $A \neq B$, then $A \times B \neq B \times A$.
- if $|A| = m$ and $|B| = n$, then $|A \times B| = mn$.

Example

- $A = \{1, 3\}$, $B = \{2, 4, 6\}$.

$$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$$

$$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$$

$$A \neq B, A \times B \neq B \times A$$

$$|A| = 2, |B| = 3, |A \times B| = 2 \cdot 3 = 6.$$

Cartesian Product

- The Cartesian product of sets A_1, A_2, \dots, A_n is defined to be the set of all n -tuples (a_1, a_2, \dots, a_n) where $a_i \in A_i$ for $i=1, \dots, n$;
- It is denoted $A_1 \times A_2 \times \dots \times A_n$
 $|A_1 \times A_2 \times \dots \times A_n| = |A_1| \cdot |A_2| \cdot \dots \cdot |A_n|$

Example

- $A = \{a, b\}$, $B = \{1, 2\}$, $C = \{x, y\}$

$$A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), \\ (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$$

- $|A \times B \times C| = 2 \cdot 2 \cdot 2 = 8$

Exercise

- Let $A = \{w, x\}$, $B = \{1, 2\}$ and $C = \{KB, SD, PS\}$.
- Find $|A \times B|$, $|B \times C|$, $|A \times C|$, $|A \times B \times C|$, $|B \times C \times A|$, $|A \times B \times A \times C|$
- Determine the following set,
 - a) $A \times B$, $B \times C$, $A \times C$
 - b) $A \times B \times C$
 - c) $B \times C \times A$
 - d) $A \times B \times A \times C$

Exercise

- Let $X = \{1, 2\}$, $Y = \{a\}$ and $Z = \{b, d\}$.
- List the elements of each set.
 - a) $X \times Y$
 - b) $Y \times X$
 - c) $X \times Y \times Z$
 - d) $X \times Y \times Y$
 - e) $X \times X \times X$
 - f) $Y \times X \times Y \times Z$