



PROBABILITY & STATISTICAL DATA ANALYSIS

SECI2143

Assignment 3

Lecturer:

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Question 1

Assignment 3

Question 1

$$\mu = 3.433 \text{ kg}$$

$$\sigma = 495 \text{ g}$$

a) $n = 75$ $ci = 95\%$

Z ~~score~~ score at 95% confidence interval = 1.96

$$\begin{aligned}\bar{x} \pm z \frac{s}{\sqrt{n}} &= 3.433 \pm 1.96 \left(\frac{495}{\sqrt{75}} \right) \\ &= 3.433 \pm 112.029 \\ &= (-108.596, 115.462) \# \end{aligned}$$

b) $N = 75000$ $ci = 95\%$

z score = 1.96

$$\begin{aligned}\bar{x} \pm z \frac{s}{\sqrt{N}} &= 3.433 \pm 1.96 \left(\frac{495}{\sqrt{75000}} \right) \\ &= 3.433 \pm 3.543 \\ &= (-0.110, 6.976) \# \end{aligned}$$

c) Which of the preceding confidence interval is wider?

- Confidence interval with size 75 is wider. A larger sample size and lower variability will result in a tighter confidence interval with a smaller margin of error. A smaller sample size or a higher variability will result in a wider confidence interval with a larger margin of error. The level of confidence also affects the interval width. Increasing the sample size makes the confidence interval narrower. ~~Increasing~~ ^{Decreasing} the sample size makes the confidence interval wider.

Question 2

Question 2

$$n = 36, \bar{x} = 92.8, \mu = 91.4, \sigma = 3.6, \alpha = 0.05$$

Test hypothesis: $H_0: \mu = 91.4$

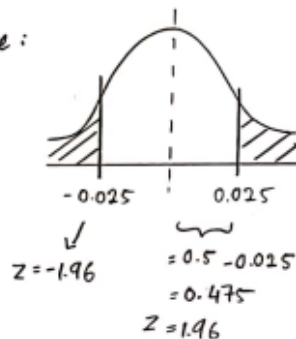
$$H_1: \mu \neq 91.4$$

Test statistic:

Since σ is known ~~and $n > 30$~~ ,
$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\therefore Z = \frac{92.8 - 91.4}{\frac{3.6}{\sqrt{36}}} = 2.33$$

Critical value:



Since it is a two tailed test, $\alpha = \frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

From Z-table, $Z = 1.96$

Since test statistic value, $Z = 2.33 > \text{critical value, } Z = 1.96$, we reject H_0 .

Thus, there is sufficient evidence that children from urban area have a mean height different from 91.4 cm.

Question 3

Assignment 3

Question 3

$$X_1 = 51 \quad X_2 = 31 \quad \alpha = 0.05$$

$$N_1 = 71 \quad N_2 = 53$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{51}{71} = 0.7183$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{31}{53} = 0.5849$$

$$= \left(\frac{51 + 31}{71 + 53} \right)$$

$$= 0.6613$$

$$H_0 :- p_1 = p_2$$

$$H_1 :- p_1 > p_2 \quad (\text{right tailed test})$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.7183 - 0.5849)}{\sqrt{(0.6613)(1-0.6613)\left(\frac{1}{71} + \frac{1}{53}\right)}}$$

$$Z = 1.55 \quad (\text{test statistics})$$

$$p\text{-value with } Z = 1.55 \text{ is } p\text{-value} = P(Z > Z_{0.05})$$

$$= P(Z > 1.55)$$

$$= 0.0606 \quad \therefore p\text{-value is } 0.0606$$

$\therefore$ Since p -value is greater than $\alpha = 0.05$, we failed to reject the null hypothesis.

There is not sufficient evidence to conclude that male drivers are more prone to go through zebra crossing compare to female drivers.

Question 4

4. Nutty Fruty : x_i

$$n_1 = 9$$

$$\bar{x}_1 = \frac{21.7 + 21.0 + 21.2 + 20.7 + 20.4 + 21.9 + 20.2 + 21.6 + 20.6}{9}$$

$$= 21.033$$

$$S_x^* = \frac{\sum (x_i - \bar{x})^2}{n_1 - 1}$$

$$= \frac{2.93984}{8}$$

$$= 0.36748$$

Biscoff : y_{aj}

$$n_2 = 13$$

$$\bar{x}_2 = \frac{21.5 + 20.5 + 20.3 + 21.6 + 21.7 + 21.3 + 23.0 + 21.3 + 18.9 + 20.0 + 20.4 + 20.8 + 20.3}{13}$$

$$= 20.8923$$

$$S_y^* = \frac{\sum (y_i - \bar{y})^2}{n_2 - 1}$$

$$= \frac{12.0028826}{12}$$

$$= 1.0002$$

$$H_0: \mu_1 = \mu_2$$

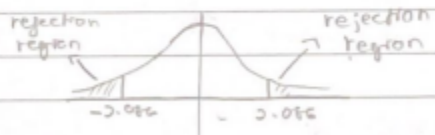
$$H_1: \mu_1 \neq \mu_2$$

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{n_1 s_x^2 + n_2 s_y^2}{n_1 + n_2 - 2} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$= \frac{21.033 - 20.8923}{\sqrt{\frac{9(0.36748) + 13(1.0002)}{20} \left(\frac{1}{13} + \frac{1}{9} \right)}}$$

$$= 0.3592$$

critical value = 2.086



$\therefore$ fail to reject $H_0: \mu_1 = \mu_2$

Question 5

5. 2-hour playing e-sport :

$$\begin{aligned}\Sigma x &= 3586 + 4470 + 3068 + 5338 + 2221 + 4791 + 4435 + 3099 + \\ &\quad 3187 + 3901 + 3868 + 3869 + 4878 + 3632 + 4518 \\ &= 58861\end{aligned}$$

$$\begin{aligned}\Sigma x^2 &= (3586)^2 + (4470)^2 + (3068)^2 + (5338)^2 + (2221)^2 + (4791)^2 + \\ &\quad (4435)^2 + (3099)^2 + (3187)^2 + (3901)^2 + (3868)^2 + (3869)^2 + (4878)^2 \\ &\quad + (3632)^2 + (4518)^2 \\ &= 240\ 610\ 699\end{aligned}$$

$$n_1 = 15$$

$$\begin{aligned}\bar{X}_1 &= \frac{\Sigma x}{n_1} = \frac{58861}{15} \\ &= 3924.0667\end{aligned}$$

$$s_1 = \sqrt{\frac{\Sigma x^2 - (\Sigma x)^2}{n - 1}}$$

$$= \sqrt{\frac{240610699 - \frac{(58861)^2}{15}}{14}}$$

$$= 829.6389$$

4-hour playing e-sport :

$$\begin{aligned}\Sigma x &= 4965 + 3918 + 1987 + 4993 + 5220 + 3653 + 3510 + 3338 + 4100 + 5792 \\ &\quad + 4547 + 3319 + 3336 + 4304 + 4057 \\ &= 61039\end{aligned}$$

$$\begin{aligned}\Sigma x^2 &= (4965)^2 + (3918)^2 + (1987)^2 + (4993)^2 + (5220)^2 + (3653)^2 + (3510)^2 + (3338)^2 + \\ &\quad (4100)^2 + (5792)^2 + (4547)^2 + (3319)^2 + (3336)^2 + (4304)^2 + (4057)^2 \\ &= 261\ 096\ 115\end{aligned}$$

$$n_2 = 15$$

$$\bar{X}_2 = \frac{\sum x}{n_2} = \frac{61039}{15} = 4069.2667$$

$$s_2 = \sqrt{\frac{\sum x^2 - (\sum x)^2}{n-1}}$$

$$= \sqrt{\frac{261096115 - (61039)^2}{15}}$$

$$= 952.8956$$

Null and alternative hypothesis:

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

$$\text{Pooled variance: } S_p^2 = \frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{(n_1-1) + (n_2-1)}$$

$$= \frac{(829.6389)^2(14) + (952.8956)^2(14)}{28}$$

$$= 798155.6381$$

Test statistic:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{3924.0667 - 4069.2667}{\sqrt{\frac{(829.6389)^2}{15} + \frac{(952.8956)^2}{15}}}$$

$$= -0.4451$$

$$df = n_1 + n_2 - 2 = 28$$

p-value = 0.6597, $\therefore$ do not reject null hypothesis as
p-value > α , not enough evidence to
conclude at 0.05 significance level

Question 6

G.	Before treatment	After treatment	(d _i)	(d _i) ²
	14	10	4	16
	12	4	8	64
	18	14	4	16
	7	6	1	1
	11	9	2	4
	9	6	3	9
	16	12	4	16
	15	12	3	9

$$H_0: \mu d \leq 3, \quad n=8, \quad df=7, \quad \alpha=0.1$$

$$H_a: \mu d > 3$$

$$\bar{d} = \frac{4+8+4+1+2+3+4+3}{8}$$

$$= 3.625$$

$$\text{critical value, } t_c = 1.4150$$

$$\text{standard deviation, } s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{29.875}{7}}$$

$$= 2.066$$

$$\text{standard error, } S.E = s/\sqrt{n}$$

$$= \frac{2.066}{\sqrt{8}} = 0.7304$$

$$\therefore t < t_c, \text{ do not reject } H_0$$

$$p\text{-value} = 0.2102$$

$$\text{test statistic, } t = \frac{(\bar{d} - \mu)}{S.E} = \frac{3.625 - 3}{0.7304}$$

$$= 0.856$$

$$\therefore p\text{-value} > \alpha = 0.1, \text{ do not reject } H_0 \text{ as not enough evidence to support}$$

Question 7

Question 7

Hypotheses:

$$H_0: p_1 = p_2 = p_3 = p_4 = 0.25$$

H_a : Proportion of male smoker lung cancer deaths is not the same for the four given tar level categories.

Expected frequencies table

O	p	$E = p \times 1194$
103	0.25	298.5
378	0.25	298.5
563	0.25	298.5
150	0.25	298.5
1194	1	1194

Calculation for test statistics

O	E	$(O-E)^2/E$
103	298.5	128.041
378	298.5	21.173
563	298.5	234.373
150	298.5	73.877
1194	1194	457.464

$$\alpha = 0.05$$

Test statistics:

$$\sum \frac{(O-E)^2}{E} = 457.464$$

Degree of freedom:

$$= 4 - 1 = 3$$

$$\chi^2_{crit} = 7.815$$

Rejection region

If $\chi^2 \geq 7.815$, reject H_0

↳ Because test statistics in rejection region, reject null hypothesis

P value: P-value = 0

↳ P-value is less than level of significance, reject null hypothesis

Thus there are no evidence to conclude proportion of male smoker lung cancer death is not the same for the four given tar level categories.

Question 8

Question 8

Hypothesis

H_0 : There is a fit between observed and expected frequencies

H_1 : There is no fit between observed and expected frequencies.

We have given observed frequencies

$\rightarrow 124, 30, 43, 11$

ratio of $\rightarrow 9:3:3:1$

$$(124 + 30 + 43 + 11) = 208$$

Test Statistic

Expected values

$$124 \rightarrow \frac{9}{16} \times 208 = 117, \quad 30 \rightarrow \frac{3}{16} \times 208 = 39$$

$$43 \rightarrow \frac{3}{16} \times 208 = 39, \quad 11 \rightarrow \frac{1}{16} \times 208 = 13$$

O	E	$(O-E)^2$	$(O-E)^2/E$	• Test statistics $\chi^2 = 3.214$ • level of significance: $\alpha = 0.01$ • Degrees of freedom $4-1 = 3$ • Critical value 11.345
124	117	49	0.419	
30	39	81	2.077	
43	39	16	0.410	
11	13	4	0.308	
208	208		2.214	

$\therefore$ Since calculated value of χ^2 is less than critical value of χ^2 , accept the null hypothesis H_0 .

Conclusion

There is a fit between expected frequencies and observed frequencies.

So ~~there~~

Question 9

Question 9

Group	Salary (Thousands of Dollars)					Total
	27-29	29-31	31-33	33-35	35 and over	
1	6	11	16	14	13	60
2	5	9	8	6	2	30
Total	11	20	24	20	15	90

$$\alpha = 0.1$$

Test hypothesis: H_0 : The variables are independent

H_1 : The variables are related (dependent)

Group	Salary (Thousands of Dollars)					Total
	27-29	29-31	31-33	33-35	35 and over	
1	Obs = 6 Exp = 7.33	Obs = 11 Exp = 13.33	Obs = 16 Exp = 16.00	Obs = 14 Exp = 13.33	Obs = 13 Exp = 10.00	60
2	Obs = 5 Exp = 3.67	Obs = 9 Exp = 6.67	Obs = 8 Exp = 8.00	Obs = 6 Exp = 6.67	Obs = 2 Exp = 5.00	30
Total	11	20	24	20	15	90

Obs = Observed, Exp = Expected

Test statistic value:

Cell, i, j	Observed Count, O_{ij}	Expected Count, E_{ij}	$(O_{ij} - E_{ij})^2 / E_{ij}$
1,1	6	$= \frac{60 \cdot 11}{90} = 7.33$	0.2413
1,2	11	$= \frac{60 \cdot 20}{90} = 13.33$	0.4073
1,3	16	$= \frac{60 \cdot 24}{90} = 16.00$	0.0000
1,4	14	$= \frac{60 \cdot 20}{90} = 13.33$	0.0337
1,5	13	$= \frac{60 \cdot 15}{90} = 10.00$	0.9000
2,1	5	$= \frac{30 \cdot 11}{90} = 3.67$	0.4820
2,2	9	$= \frac{30 \cdot 20}{90} = 6.67$	0.6139
2,3	8	$= \frac{30 \cdot 24}{90} = 8.00$	0.0000
2,4	6	$= \frac{30 \cdot 20}{90} = 6.67$	0.0673
2,5	2	$= \frac{30 \cdot 15}{90} = 5.00$	1.8000

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

$$= 4.7458$$

Critical value : $\chi^2_{k,\alpha}$

$$k = (r-1)(2-1)$$

$$= 4$$

∴ From chi square table, $\chi^2_{k=4, \alpha=0.1} = 7.779$

∴ Since test statistic value < critical value ($4.7455 < 7.779$), we do not reject H_0 at $\alpha = 0.1$. Thus, there is evidence that group assignment and the salaries are independence at $\alpha = 0.1$ significance level.