CHAPTER 7

(Part 1)

Correlation Analysis

What is Correlation?

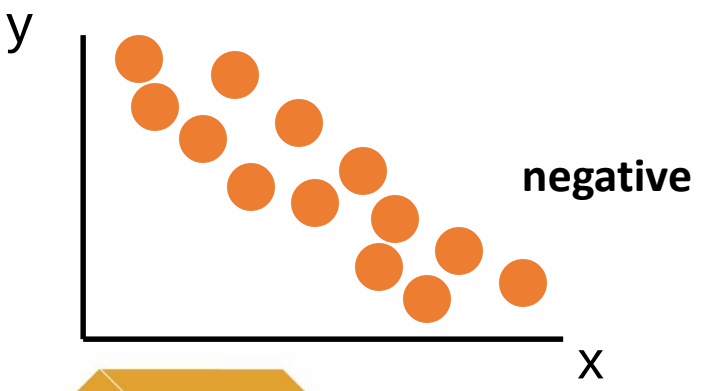
- The word Correlation is made of Co- (meaning "together"), and Relation, and it can be defined as **a measure of the statistical relationship between two comparable variables or quantities(bivariate data).**
- When two sets of data are strongly linked together we say they have a high correlation.
- Correlation is **positive** when the values increase together.
- Correlation is **negative** when one value decreases as the other increases.
- **No correlation** – the value does not tend to either increase or decrease as the other increases.

Scatter Plots

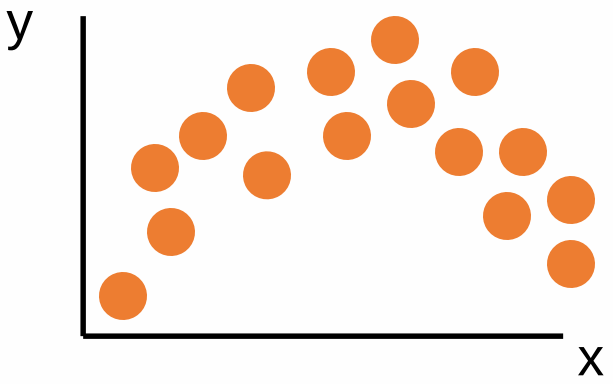
- A scatter plot (or scatter diagram) is used to show the relationship between two variables.
- One variable is on the X-axis, one on the Y-axis.
- The pattern of data is indicative of the type of relationship between two variables:
    - positive relationship
    - negative relationship
    - no relationship
    - curvilinear relationship

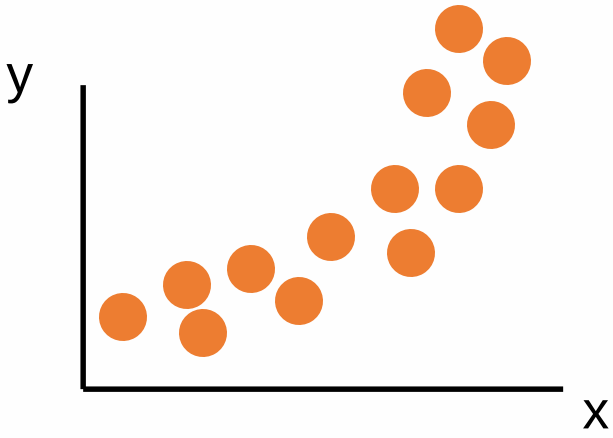
Examples – Scatter Plot

Linear relationships



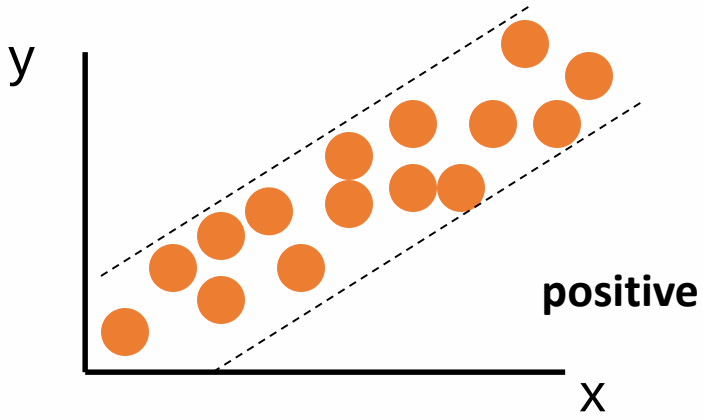
Curvilinear relationships

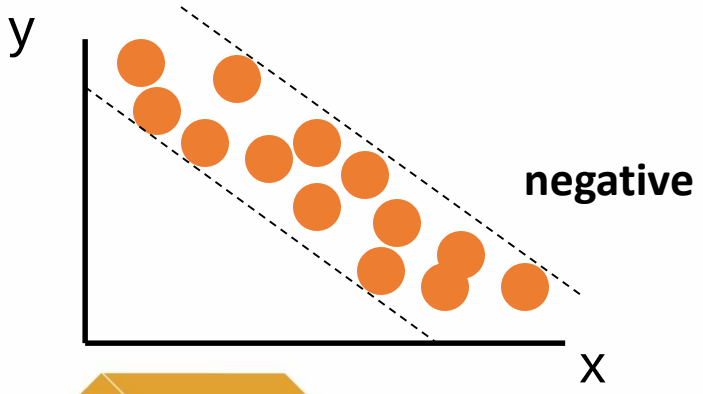




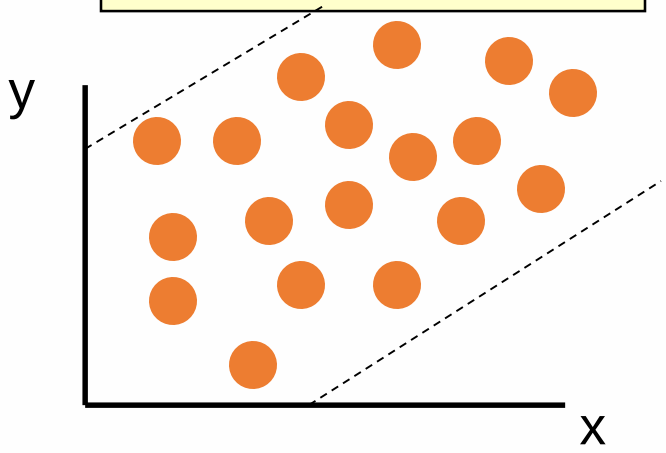
Examples – Scatter Plot

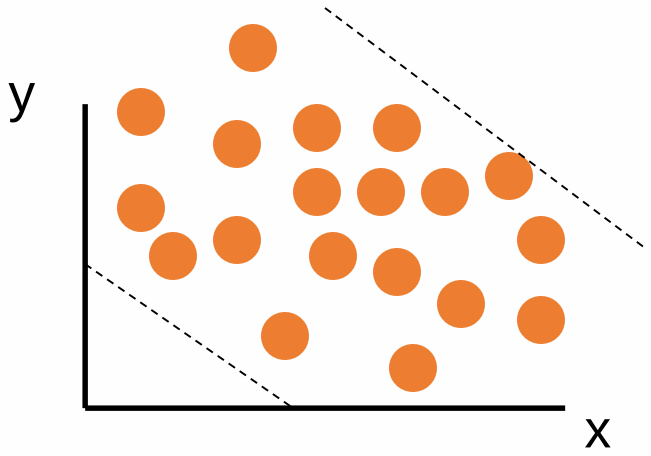
Strong relationships





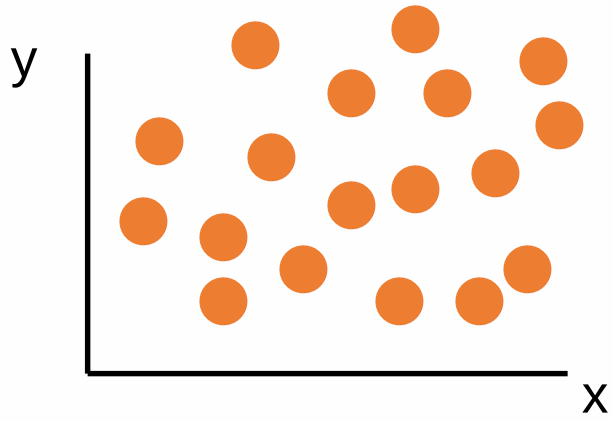
Weak relationships

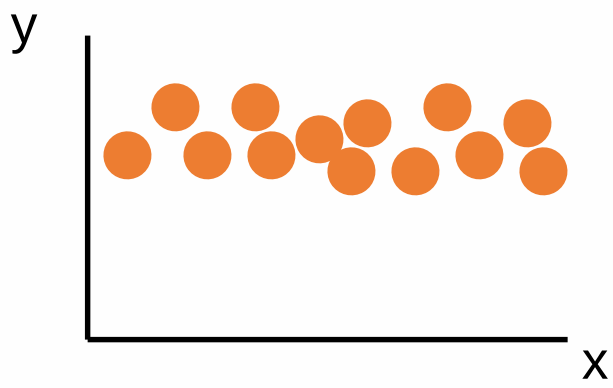




Examples – Scatter Plot

No relationship





Correlation Analysis

- Correlation analysis is used to measure strength of the association **(linear relationship**) between two variables.
    - Only concerned with strength of the relationship.
    - No causal effect is implied.
- A **correlation coefficient** is a numerical assessment of the strength of relationship between the x and y values in a set of (x,y) pairs.
- The **population correlation coefficient, ρ** (rho) measures the strength of the association between the variables.
- The **sample correlation coefficient, r** is an estimate of ρ and is used to measure the strength of the **linear relationship** in the sample observations.

Properties of r

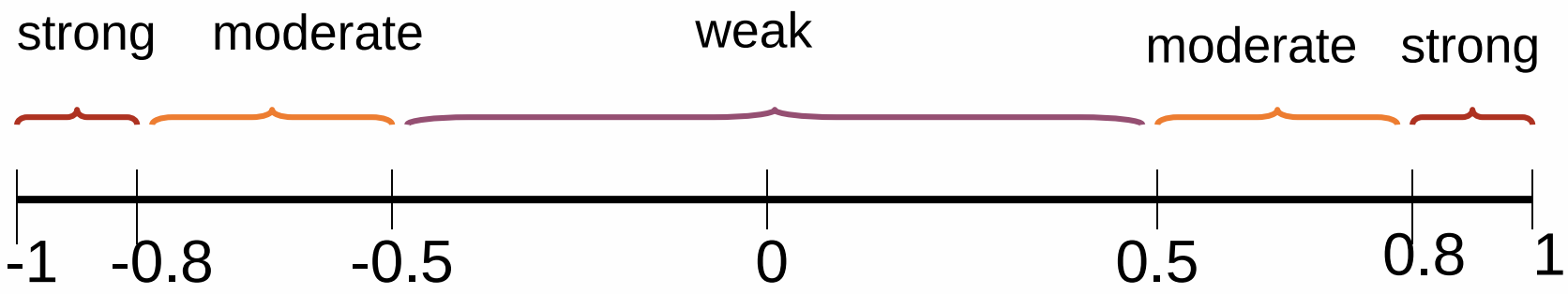
- The value of r does not depend on the unit of measurement for either variable.
- The value of r does not depend on which of the two variable is considered x.
- The value of r is between -1 and 1.
- The value of r is a measure of the extent to which x and y are linearly related.
- Number represents the strength of the relationship.
- Sign (+ or -) represents the direction of the relationship (positive or negative)
    - Positive values denote positive linear correlation.
    - Negative values denote negative linear correlation.
    - A value of zero denotes no linear correlation.

Example:

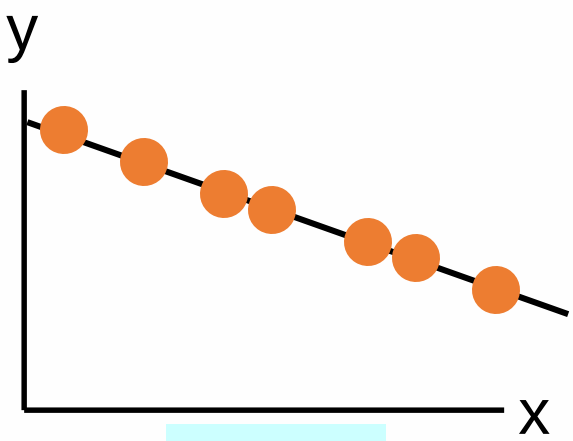
Correlation of -0.87 is considered stronger than correlation of 0.56

- The correlation coefficient $r = 1$ only when all the points in a scatterplot of the data lie exactly on a straight line that slopes upward. (Perfect positive correlation)
- Similarly, r = -1 only when all the points lie exactly on a downward-sloping line. (Perfect negative correlation)
- Coefficient usually not “perfect”.
- The closer to -1, the stronger the negative linear relationship.
- The closer to 1, the stronger the positive linear relationship.
- The closer to 0, the weaker the linear relationship.

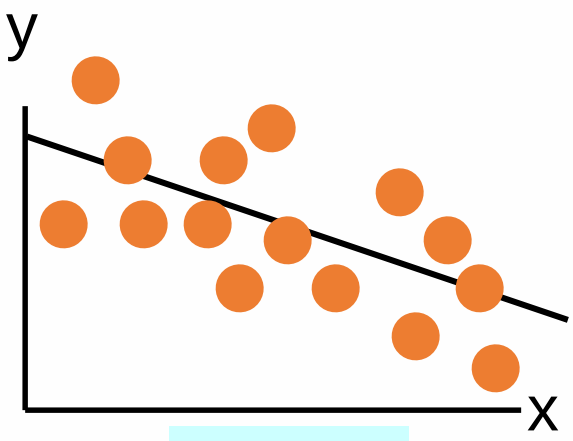
- Measure the strength of a linear relationship.



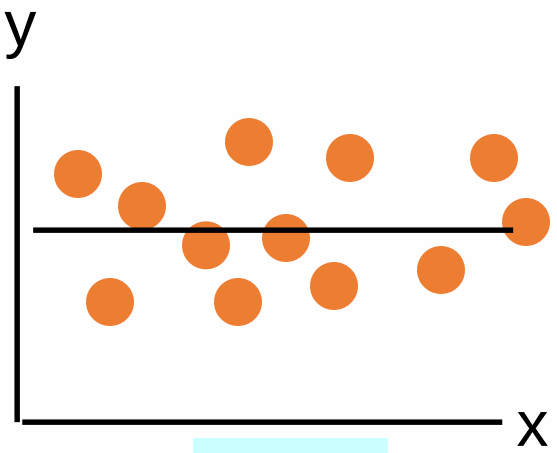
Examples: Approximate r Values



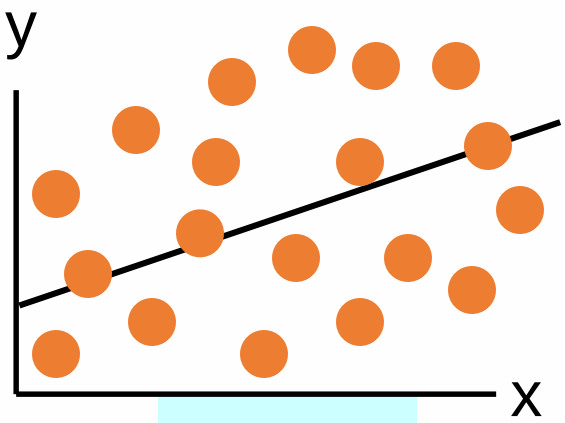
r = -1



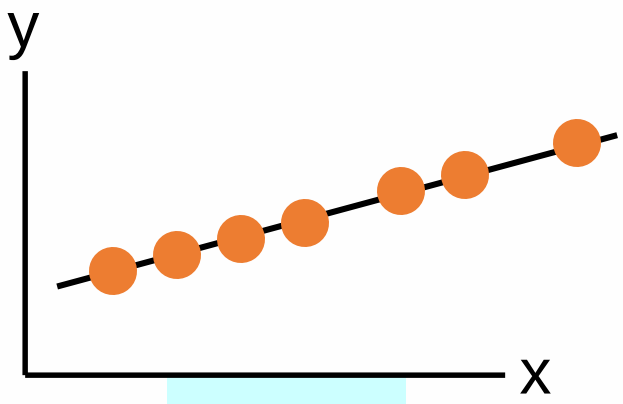
r = -.6



r = 0



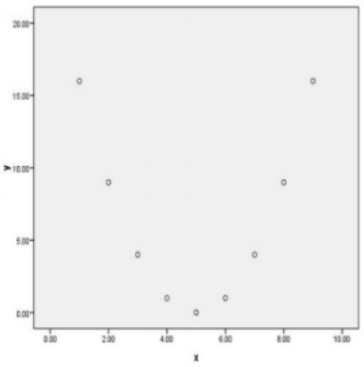
r = +.3



r = +1

No correlation

- The correlation coefficient is a measure of linear relationship and thus look at the scatterplot of the data before concluding that there is no relationship between two variables when r is close to 0.
- There could be a curvilinear relationship. For example in the following scatterplot which implies no (linear) correlation however there is a perfect quadratic relationship.



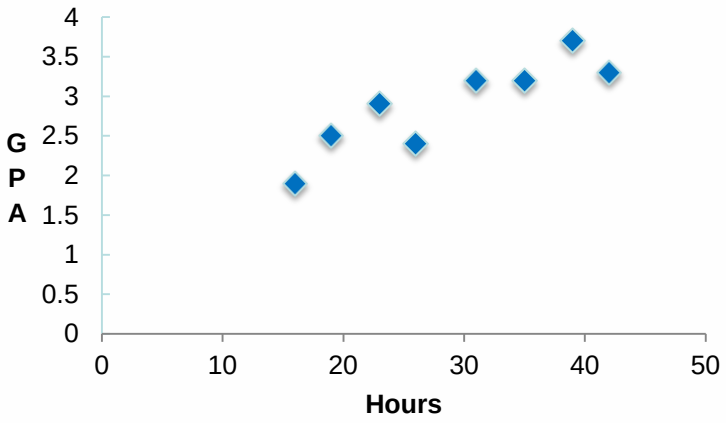
$r = 0$

Positive Correlation

- Relationship shows a **high** score on one variable is related to a **high** score on another variable. Or,
- Relationship shows a **low** score on one variable is related to a **low** score on another variable.
- Correlation coefficient is greater than 0

Example:

What is the relationship between the number of hours spent per week studying and GPA?



$r = 0.884$

Students	Study Hours	GPA
S1	42	3.3
S2	23	2.9
S3	31	3.2
S4	35	3.2
S5	16	1.9
S6	26	2.4
S7	39	3.7
S8	19	2.5

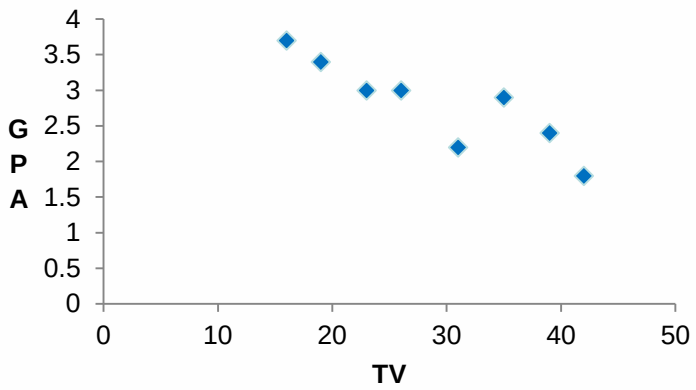
It can be seen that the GPA increases as the hours increases. A scatter plot and correlation analysis of the data indicates that there is positive relationship between the number of hours spent per week studying and GPA.

Negative Correlation

- Relationship shows that a high score on one variable is related to a low score on the second variable.
- Correlation coefficient is less than 0

Example:

- What is the relationship between the number of hours spent per week watching TV and GPA?



r = - 0.892

Students	TV	GPA
S1	42	1.8
S2	23	3.0
S3	31	2.2
S4	35	2.9
S5	16	3.7
S6	26	3.0
S7	39	2.4
S8	19	3.4

It can be seen that the GPA decreases as the hours increases. A scatter plot and correlation analysis of the data indicates that there is negative relationship between the number of hours spent per week watching TV and GPA.

Correlation Types

- The two most popular correlation coefficients are:
  - Pearson's product-moment correlation coefficient.
  - Spearman's rho rank correlation coefficient
- When calculating a correlation coefficient for **ordinal data**, select Spearman's rho technique.
- For **interval** or **ratio-type data**, use Pearson's technique.

Pearson's product-moment correlation coefficient

Formula to calculate the sample correlation coefficient, r:

$$r = \frac{\sum xy - \left(\sum x \sum y\right)/n}{\sqrt{\left[\left(\sum x^2\right) - \left(\sum x\right)^2/n\right]\left[\left(\sum y^2\right) - \left(\sum y\right)^2/n\right]}}$$

where:

r = Sample correlation coefficient

n = Sample size

x = Value of the independent variable

y = Value of the dependent variable

- Assumes normality in both variables (bivariate normally distributed).
- There needs to be a **linear relationship** between the two variables.
- Two variables should be measured at the **interval** or **ratio level.**
- It is sensitive to outliers (can have a very large effect on the line of best fit and the Pearson correlation coefficient, leading to very difficult conclusions regarding the data).

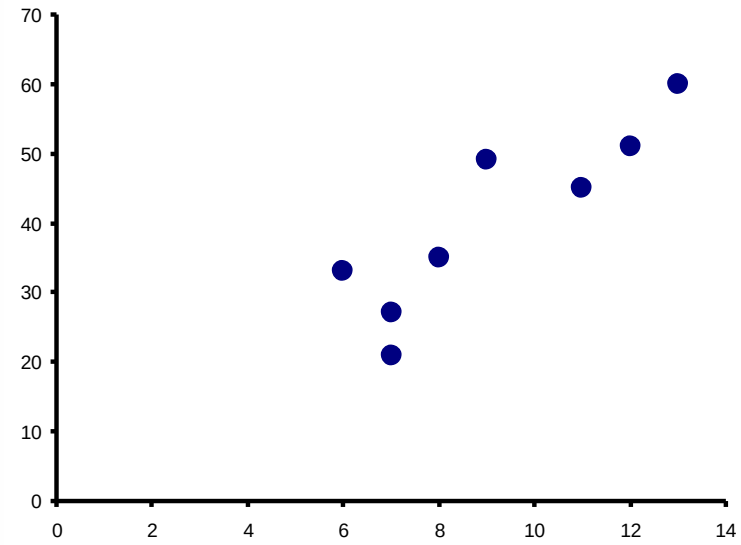
Example:

Tree Height	Trunk Diameter			
y	x	xy	y^2	x^2
35	8	280	1225	64
49	9	441	2401	81
27	7	189	729	49
33	6	198	1089	36
60	13	780	3600	169
21	7	147	441	49
45	11	495	2025	121
51	12	612	2601	144
Σ=321	**Σ=73**	**Σ=3142**	**Σ=14111**	**Σ=713**

$$r = \frac{\sum xy - (\sum x \sum y)/n}{\sqrt{[(\sum x^2) - (\sum x)^2/n][(\sum y^2) - (\sum y)^2/n]}}$$

$$= \frac{(3142) - (73)(321)/8}{\sqrt{[(713) - (73)^2/8][(14111) - (321)^2/8]}}$$

$$= 0.886$$



r = 0.886 → relatively strong positive linear association between x and y

Spearman's rho rank correlation coefficient

- Denote by r_s for sample data (with r used to denote that it is a correlation coefficient, and the subscript s to denote that it is named after the statistician Spearman.)
- The linear correlation coefficient between the ranks of data on variable x and y

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)}$$

where

$d_i = y_i - x_i$ (difference in ranks)

n = sample size

- It is less sensitive to bias due to outliers
- It is applied to ordinal variables.

Example:

- As an example, let us consider a musical (solo vocal) talent contest where 10 competitors are evaluated by two judges, A and B.
- Usually judges award numerical scores for each contestant after his/her performance.
- Spearman Rho Rank Correlation Coefficient can indicate if judges agree to each other's views as far as talent of the contestants are concerned (though they might award different numerical scores) – in other words if the judges are unanimous.
- Suppose that scores of the judges (out of 10 were as follows):

Contestant No.	1	2	3	4	5	6	7	8	9	10
Score by Judge A	5	9	3	8	6	7	4	8	4	6
Score by Judge B	7	8	6	7	8	5	10	6	5	8

- In order to compute Spearman Rank Correlation Coefficient, it is necessary that the data be ranked.
- Ranks are assigned separately for two judges either starting from the highest or from the lowest score. Here, the highest score given by Judge A is 9.
- If we begin from the highest score, we assign rank 1 to contestant 2 corresponding the score of 9.
- The second highest score is 8 but two competitors have been awarded the score of 8. In this case both the competitors are assigned a common rank which is the arithmetic mean of ranks 2 and 3 => $(\frac{2+3}{2} = 2.5)$
- In this way, score of Judge A can be converted into ranks.
- Similarly, ranks are assigned to the scores awarded by Judge B and then difference between ranks for each contestant are used to evaluate r_s.
- For the example, ranks are as follows:

Contestant No.	1	2	3	4	5	6	7	8	9	10
Ranks of scores by Judge A	7	1	10	2.5	5.5	4	8.5	2.5	8.5	5.5
Ranks of scores by Judge B	5.5	3	7.5	5.5	3	9.5	1	7.5	9.5	3

Contestant No.	Ranks of scores by Judge A	Ranks of scores by Judge B	d_i	d_i^2
1	7	5.5	1.5	2.25
2	1	3	-2	4
3	10	7.5	2.5	6.25
4	2.5	5.5	-3	9
5	5.5	3	2.5	6.25
6	4	9.5	-5.5	30.25
7	8.5	1	7.5	56.25
8	2.5	7.5	-5	25
9	8.5	9.5	-1	1
10	5.5	3	2.5	6.25

$\Sigma d_i^2 = 146.5$

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)} = 1 - \frac{6(146.5)}{10(100-1)} = 0.112$$

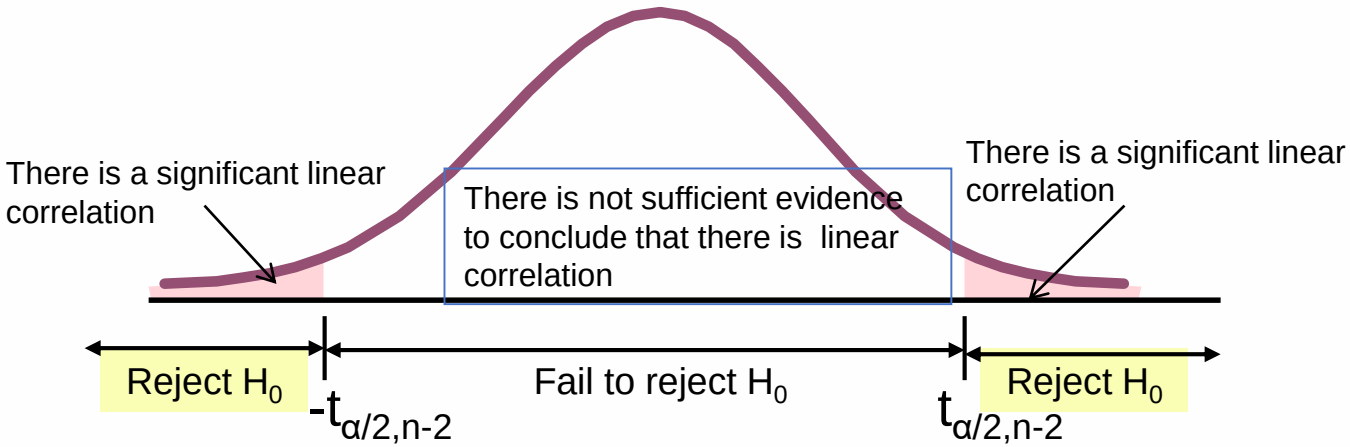
- Hypotheses:

$H_0: \rho = 0$	(no linear correlation)
$H_A: \rho \neq 0$	(linear correlation exists)

- Test statistic:

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

- Select the significance level, α
- Find the critical value of *t* with *n*-2 degrees of freedom from *t* distribution table (Table A-3:Triola Table)
- If the test statistic in the critical region, reject H_0. Otherwise fail to reject H_0



Example:

Is there evidence of a linear relationship between tree height and trunk diameter at the .05 level of significance?

i) State the hypothesis statement:

H_0: ρ = 0 (No linear correlation)

H_1: ρ ≠ 0 (linear correlation exists)

ii) Find the critical value: $\alpha = 0.05, d.f = 8 - 2 = 6;\ t_{\alpha/2=0.025,6} = \pm 2.4469$

iii) Calculate the test statistic:

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{.886}{\sqrt{\frac{1-.886^2}{8-2}}} = 4.68$$

iv) State the conclusion:

Since $t = 4.68 > t_{+0.025,6} = 2.4469$, reject H_0.

There is sufficient evidence of a linear relationship between tree height and trunk diameter at the 5% level of significance.

Exercise #1

The data represent x = score on a measure of test anxiety and y = exam score for a sample of n = 9 students:

x	23	14	14	0	17	20	20	15	21
y	43	59	48	77	50	52	46	51	51

Higher values for x indicate higher levels of anxiety.

a) Construct a scatter plot, and comment on the features of the plot.

b) Does there appear to be a linear relationship between the two variables? How would you characterize the relationship?

c) Compute the value of the correlation coefficient. Is the value of r consistent with your answer to part (b)?

d) Is it reasonable to conclude that test anxiety caused poor exam performance? Explain.

Exercise #2

The data represent x = the amount of catalyst added to accelerate a chemical reaction and y = the resulting reaction time:

x	1	2	3	4	5
y	49	46	41	34	25

a) Calculate r. Does the value of r suggest a strong linear relationship?

b) Construct a scatter plot. From the plot, does the word linear really provide the most effective description of the relationship between x and y? Explain.