UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT 5

SCHOOL OF COMPUTING
Faculty of Engineering

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SCHOOL OF COMPUTING

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SECTION/COURSE CODE	: [SECI III3] [01] – [COMPUTATIONAL MATHEMATICS]
LECTURER	: [DR. RAZANA ALWEE]
SESSION/SEMESTER	: [2020/2021] - [02]

Case Study Report submitted to School of Computing, Univerisiti Teknologi Malaysia in partial fulfilment to complete the SECI 1113 Assignment for Semester 2 session 2020/2021

Question 1

Newton forward - difference formula is used to approximate birth rate in 1980.

let x = year , y = birth rate

k	x_k	y_k	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
0	1975	30.7	0.8	-6.2	3.7	5.2
1	1985	31.5	-5.4	-2.5	8.9	
2	1995	26.1	-7.9	6.4		
3	2005	18.2	-1.5			
4	2015	16.7				

$\Delta y_0 = 31.5 - 30.7$
$= 0.8$

$\Delta^2 y_0 = -5.4 - (0.8)$
$= -6.2$

$\Delta^4 y_0 = 8.9 - 3.7$
$= 5.2$

$\Delta y_1 = 26.1 - 31.5$
$= -5.4$

$\Delta^3 y_0 = -2.5 - (-6.2)$
$= 3.7$

$x = 1980$ is between 1975 and 1985

choose $x_0 = 1975$ as reference value

Thus, $h = 1985 - 1975$
$= 10$

$r = (x - x_0)/h$
$= (1980 - 1975)/10$
$= 0.5$

Polynomial,

$$P_4(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0$$

$$P_4(1980) = 30.7 + (0.5)(0.8) + \frac{0.5(0.5-1)}{2!}(-6.2) + \frac{0.5(0.5-1)(0.5-2)}{3!}(3.7) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}(5.2)$$

$= 30.7 + 0.4 + 0.775 + 0.23125 + (-0.2031)$

$= 31.9032$ (4d.p)

$y(1980) \approx P_4(1980) = 31.9032$ (4d.p)

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Newton backward - difference formula to approximate birth rate in 2010

let x = year , y = birth rate

k	x_k	y_k	∇y_k	$\nabla^2 y_k$	$\nabla^3 y_k$	$\nabla^4 y_k$
0	1975	30.7				
1	1985	31.5	0.8			
2	1995	26.1	-5.4	-6.2		
3	2005	18.2	-7.9	-2.5	3.7	
4	2015	16.7	-1.5	6.4	8.9	5.2

$\nabla y_1 = 31.5 - 30.7$
$= 0.8$

$\nabla^2 y_2 = -5.4 - (0.8)$
$= -6.2$

$\nabla^4 y_4 = 8.9 - 3.7$
$= 5.2$

$\nabla y_2 = 26.1 - 31.5$
$= -5.4$

$\nabla^3 y_3 = -2.5 - (-6.2)$
$= 3.7$

$x = 2010$ is between 2005 and 2015

choose $x_4 = 2015$ as reference value

Thus $h = 1985 - 1975$
$= 10$

$r = (x - x_5)/h$

$r = (2010 - 2015)/10$

$r = -0.5$

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$$P_4(x) = y_4 + r\nabla y_4 + \frac{r(r+1)}{2!}\nabla^2 y_4 + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_4 + \frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 y_4$$

$$P_4(2010) = 16.7 + (-0.5)(-1.5) + \frac{(-0.5)(-0.5+1)}{2!}(6.4) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(8.9) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(5.2)$$

$P_4(2010) = 16.7 + 0.75 + (-0.8) + (-0.55625) + (-0.2031)$

$P_4(2010) = 15.8906$ (4d.p)

$y(2010) \approx P_4(2010) = 15.8906$ (4d.p)

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Question 2

a) $p(x) = a_0 + a_1 x$

$$\begin{bmatrix} S_0 & S_1 \\ S_1 & S_2 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} V_0 \\ V_1 \end{bmatrix}$$

where $S_j = \sum_{k=0}^{8} x_k^j \ , \ j = 0, 1, 2$

$V_l = \sum_{k=0}^{8} x_k^l f_k \ , \ l = 0, 1$

let x = year, y = total index

k	x_k^0	x_k^1	x_k^2	f_k	$x_k^0 f_k$	$x_k^1 f_k$
0	1	1970	3880900	6	6	11820
1	1	1975	3900625	8.6	8.6	16985
2	1	1980	3920400	13.6	13.6	26928
3	1	1985	3940225	18.7	18.7	37119.5
4	1	1990	3960100	31.6	31.6	62884
5	1	1995	3980025	53.2	53.2	106134
6	1	2000	4000000	78.8	78.8	157600
7	1	2005	4020025	100	100	200500
8	1	2010	4040100	107.1	107.1	215271
$\sum_{k=0}^{8}$	9	17910	35642400	—	417.6	835241.5

$x_0^0 = (1970)^0$
$= 1$
$x_1^0 = (1975)^0$
$= 1$
$x_0^1 = (1970)^1$
$= 1970$
$x_1^1 = (1975)^1$
$= 1975$
$x_0^2 = (1970)^2$
$= 3880900$

$x_0^0 f_0 = (1)(6)$
$= 6$
$x_0^1 f_0 = (1970)(6)$
$= 11820$

Matrix form :

$$\begin{bmatrix} 9 & 17910 \\ 17910 & 35642400 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 417.6 \\ 835241.5 \end{bmatrix}$$

$9a_0 + 17910 a_1 = 417.6$, $17910 a_0 + 35642400 a_1 = 835241.5$

$9a_0 = 417.6 - 17910 a_1$

$a_0 = \dfrac{417.6 - 17910 a_1}{9}$ — ①

subs ①

$17910\left(\dfrac{417.6 - 17910 a_1}{9}\right) + 35642400 a_1 = 835241.5$

$831024 - 35640900 a_1 + 35642400 a_1 = 835241.5$

$1500 a_1 = 4217.5$

$a_1 = 2.8117$ (4d.p)

when $a_1 = 2.8117$

$a_0 = \dfrac{417.6 - 17910(2.8117)}{9}$

$a_0 = -5548.883$ (4d.p)

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$\therefore$ polynomial expression : $p(x) = 2.8117x - 5548.883$

b) when $x = 1979$

$p(1979) = 2.8117(1979) - 5548.883$

$= 15.4713$ (4d.p)

$f(1979) \approx p(1979) = 15.4713$

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Question 3

k	0	1	2	3	4
x_k	1.5	2.5	4	5	6.5
y_k	0.455	1.615	3.750	5.333	7.857

$$p_4(x) = \sum_{i=0}^{4} L_i(x)(y_i), \text{ where } L_i(x) = \prod_{\substack{i=0 \\ j\neq i}}^{4} \frac{(x-x_j)}{(x_i-x_j)}$$

when $x = 3.0$

$$L_0(3.0) = \frac{(3.0-2.5)(3.0-4)(3.0-5)(3.0-6.5)}{(1.5-2.5)(1.5-4)(1.5-5)(1.5-6.5)}$$

$$L_0(3.0) = -0.08$$

$$L_1(3.0) = \frac{(3.0-1.5)(3.0-4)(3.0-5)(3.0-6.5)}{(2.5-1.5)(2.5-4)(2.5-5)(2.5-6.5)}$$

$$L_1(3.0) = 0.7$$

$$L_2(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-5)(3.0-6.5)}{(4-1.5)(4-2.5)(4-5)(4-6.5)}$$

$$L_2(3.0) = 0.56$$

$$L_3(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-6.5)}{(5-1.5)(5-2.5)(5-4)(5-6.5)}$$

$$L_3(3.0) = -0.2$$

$$L_4(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-5)}{(6.5-1.5)(6.5-2.5)(6.5-4)(6.5-5)}$$

$$L_4(3.0) = 0.02$$

$$\sum_{i=0}^{4} L_i(3.0) = -0.08 + 0.7 + 0.56 + (-0.2) + 0.02$$
$$= 1$$

Thus $p_4(3.0) = \sum_{i=0}^{4} L_i(3.0)(y_i)$

$$= (-0.08)(0.455) + 0.7(1.615) + 0.56(3.750) + (-0.2)(5.333) + 0.02(7.857)$$

$$= 2.2846 \quad (4dp)$$

$$y(3.0) \approx p_4(3.0) = 2.2846$$

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Question 4

$f(x) = 2x^3 + x^2 - 4$, $f'(x) = 6x^2 + 2x$, $f''(x) = 12x + 2$

a) $h = 0.5$, $x = 2.5$, by using central three point formula

$$f'(2.5) \approx \frac{f(2.5+0.5) - f(2.5-0.5)}{2(0.5)}$$

$$f'(2.5) \approx \frac{f(3) - f(2)}{1}$$

$$f'(2.5) \approx f(3) - f(2)$$

$$f'(2.5) \approx [2(3)^3 + 3^2 - 4] - [2(2)^3 + 2^2 - 4]$$

$$f'(2.5) \approx 43$$

$h = 0.5$, $x = 2.5$, by using central 5 point formula

$$f'(2.5) \approx \frac{1}{12(0.5)}[-25f(2.5) + 48f(2.5+0.5) - 36f(2.5+2(0.5)) + 16f(2.5+3(0.5)) - 3f(2.5+4(0.5))]$$

$$f'(2.5) \approx \frac{1}{6}[-25f(2.5) + 48f(3) - 36f(3.5) + 16f(4) - 3f(4.5)]$$

$$f'(2.5) \approx \frac{1}{6}[-25(2(2.5)^3 + (2.5)^2 - 4) + 48(2(3)^3 + 3^2 - 4) - 36(2(3.5)^3 + 3.5^2 - 4) + 16(2(4)^3 + 4^2 - 4) - 3(2(4.5)^3 + 4.5^2 - 4)]$$

$$f'(2.5) \approx \frac{1}{6}(-837.5 + 2832 - 3384 + 2240 - 595.5)$$

$$f'(2.5) \approx 42.5$$

$h = 0.5$, $x = 2.5$, by using central three point formula

$$f''(2.5) \approx \frac{f(2.5-0.5) - 2f(2.5) + f(2.5+0.5)}{0.5^2}$$

$$f''(2.5) \approx \frac{f(2) - 2f(2.5) + f(3)}{0.25}$$

$$f''(2.5) \approx \frac{[2(2)^3 + 2^2 - 4] - 2[2(2.5)^3 + (2.5)^2 - 4] + [2(3)^3 + (3)^2 - 4]}{0.25}$$

$$f''(2.5) \approx \frac{16 - 67 + 59}{0.25}$$

$$f''(2.5) \approx 32$$

$h = 0.5$, $x = 2.5$, by using central five point formula

$$f''(2.5) \approx \frac{1}{12(0.5)^2}[-f(2.5-2(0.5)) + 16f(2.5-0.5) - 30f(2.5) + 16f(2.5+0.5) - f(2.5+2(0.5))]$$

$$f''(2.5) \approx \frac{1}{3}[-f(1.5) + 16f(2) - 30f(2.5) + 16f(3) - f(3.5)]$$

$$f''(2.5) \approx \frac{1}{3}[-(2(1.5)^3 + 1.5^2 - 4) + 16(2(2)^3 + 2^2 - 4) - 30(2(2.5)^3 + 2.5^2 - 4) + 16(2(3)^3 + 3^2 - 4) - (2(3.5)^3 + 3.5^2 - 4)]$$

$$f''(2.5) \approx \frac{1}{3}(-5 + 256 - 1005 + 944 - 94)$$

$$f''(2.5) \approx 32$$

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b) $f(x) = 2x^3 + x^2 - 4$, $f'(x) = 6x^2 + 2x$, $f''(x) = 12x + 2$

$f'(2.5) = 6(2.5)^2 + 2(2.5)$
$= 42.5$

$f''(2.5) = 12(2.5) + 2$
$= 32$

c) For $f'(2.5)$ by central three point formula
true value , $\alpha = 42.5$
estimated value , $\bar{\alpha} = 43$
error , $\varepsilon = |\alpha - \bar{\alpha}|$
$\varepsilon = |42.5 - 43|$
$\varepsilon = 0.5$

For $f'(2.5)$ by central five point formula
true value , $\alpha = 42.5$
estimated value , $\bar{\alpha} = 42.5$
error , $\varepsilon = |\alpha - \bar{\alpha}|$
$\varepsilon = |42.5 - 42.5|$
$\varepsilon = 0$

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For $f''(2.5)$ by central three point formula
true value , $\alpha = 32$
estimated value , $\bar{\alpha} = 32$
error , $\varepsilon = |\alpha - \bar{\alpha}|$
$\varepsilon = |32 - 32|$
$\varepsilon = 0$

For $f''(2.5)$ by central five point formula
true value , $\alpha = 32$
estimated value , $\bar{\alpha} = 32$
error , $\varepsilon = |\alpha - \bar{\alpha}|$
$\varepsilon = |32 - 32|$
$\varepsilon = 0$

Question 5

a) $\int_0^3 \frac{2}{1+5x^2} \, dx$

$a = 0$, $b = 3$, $h = \frac{3-0}{6}$

$= 0.5$

i	x_i	$f(x_i) = \frac{2}{1+5x_i^2}$	
0	0	2	
1	0.5		0.8888
2	1.0		0.3333
3	1.5		0.1633
4	2.0		0.0952
5	2.5		0.0620
6	3.0	0.0435	
	Total	2.0435	1.5426

$$\int_0^3 \frac{2}{1+5x^2} \, dx = \frac{0.5}{2}\left[2.0435 + 2(1.5426)\right]$$

$$= 1.2822 \quad (4 d.p)$$

b) $\int_0^3 \frac{2}{1+5x^2} \, dx$

$a = 0$, $b = 3$, $h = \frac{3-0}{6}$

$= 0.5$

i	x_i	$f(x_i) = \frac{2}{1+5x_i^2}$		
0	0	2		
1	0.5		0.8888	
2	1.0			0.3333
3	1.5		0.1633	
4	2.0			0.0952
5	2.5		0.0620	
6	3.0	0.0435		
	Total	2.0435	1.1141	0.4285

$$\int_0^3 \frac{2}{1+5x^2} \, dx = \frac{0.5}{3}\left[2.0435 + 4(1.1141) + 2(0.4285)\right]$$

$$= 1.22615 \quad (5 d.p)$$

c) $\int_0^3 \frac{2}{1+5x^2}\,dx$

$a=0$, $b=3$, $h=0.25$, $N=\frac{3-0}{0.25}$

$=12$

i	x_i	$f(x_i)=\frac{2}{1+5x_i^2}$		
0	0	2		
1	0.25		1.5238	
2	0.50		0.8888	
3	0.75			0.5246
4	1.00		0.3333	
5	1.25		0.2270	
6	1.50			0.1633
7	1.75		0.1226	
8	2.00		0.0952	
9	2.25			0.0760
10	2.50		0.0620	
11	2.75		0.0515	
12	3.00	0.0435		
	Total	2.0435	3.3042	0.7639

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$$\int_0^3 \frac{2}{1+5x^2}\,dx = \frac{3(0.25)}{8}\left[2.0435 + 3(3.3042) + 2(0.7639)\right]$$

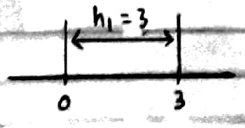
$$= 1.2641 \quad (4\,d.p)$$

d) $\int_0^3 \frac{2}{1+5x^2}\,dx$, $a=0$, $b=3$

i	$h_i=\frac{b-a}{2^{i-1}}$	$R_{i,1}$	$R_{i,2}$	$R_{i,3}$
1	3	3.0653		
2	1.5	1.7776	1.3484	
3	0.75	1.3393	1.1932	1.1829

$h_1 = 3-0$

$h_1 = 3$



$f_0(0) = \frac{2}{1+5(0)^2}$

$= 2$

$f_1(3) = \frac{2}{1+5(3)^2}$

$= 0.0435 \quad (4d.p)$

$R_{1,1} = \frac{h_1}{2}[f_0+f_1]$

$= \frac{3}{2}[2+0.0435]$

$= 3.0653 \quad (4d.p)$

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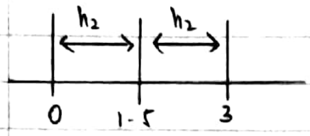
$h_2 = \frac{1}{2} h_1$

$h_2 = \frac{1}{2}(3)$

$h_2 = 1.5$

$f_1(1.5) = \dfrac{2}{1+5(1.5)^2}$

$= 0.1633 \text{ (4d.p)}$



$R_{2,1} = \frac{1}{2}[R_{1,1} + h_1(f_1)]$

$R_{2,1} = \frac{1}{2}[3.0653 + 3(0.1633)]$

$R_{2,1} = 1.7776$

$R_{2,2} = \dfrac{4R_{2,1} - R_{1,1}}{4-1}$

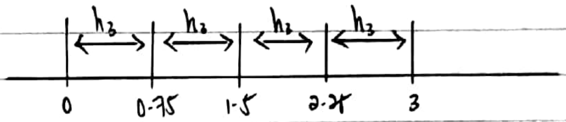
$R_{2,2} = \dfrac{4(1.7776) - 3.0653}{3}$

$= 1.3484 \text{ (4d.p)}$

$h_3 = \frac{1}{2} h_2$

$h_3 = \frac{1}{2}(1.5)$

$h_3 = 0.75$



$f_1(0.75) = \dfrac{2}{1+5(0.75)^2}$

$= 0.5246 \text{ (4d.p)}$

$f_3(2.25) = \dfrac{2}{1+5(2.25)^2}$

$= 0.0760 \text{ (4d.p)}$

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$R_{3,1} = \frac{1}{2}[R_{2,1} + h_2(f_1 + f_3)]$

$R_{3,1} = \frac{1}{2}[1.7776 + 1.5(0.5246 + 0.0760)]$

$R_{3,1} = 1.3393 \text{ (4d.p)}$

$R_{3,2} = \dfrac{4R_{3,1} - R_{2,1}}{4-1}$

$R_{3,2} = \dfrac{4(1.3393) - 1.7776}{3}$

$R_{3,2} = 1.1932$

$R_{3,3} = \dfrac{4^2 R_{3,2} - R_{2,2}}{4^2 - 1}$

$R_{3,3} = \dfrac{16(1.1932) - 1.3484}{15}$

$R_{3,3} = 1.1829 \text{ (4d.p)}$

$\therefore \displaystyle\int_0^3 \frac{2}{1+5x^2}\,dx = 1.1829 \text{ (4d.p)}$

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