ASSIGNMENT 4

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Faculty of Engineering

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61/67

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Question 1

a) relative error $= \left| \dfrac{\text{absolute error}}{\text{true error}} \right|$

$= \left| \dfrac{2}{60} \right|$ — estimated value

$= 0.033$

$\approx 3.3\%$

b) $f(x) = x$

$f'(x) = 1$

$h = 0.1$, $x = 2$

$f'(2) = \dfrac{f(2+0.1) - f(2)}{0.1}$

$= \dfrac{f(2.1) - f(2)}{0.1}$

$= \dfrac{2.1 - 2}{0.1}$

$= 1$

≈ 1

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$\therefore$ absolute error $= |1 - 1|$

$= 0$

i) Exact value $= 6.232 + 7.0321 + 5.43 - 3.385$

$= 15.3091$

$|\varepsilon_4| \leq 0.0005 + 0.00005 + 0.005 - 0.0005$ +

$|\varepsilon_4| \leq 0.00505$

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Therefore, the result is between $15.3091 - 0.00505$ and $15.3091 + 0.00505$

$\therefore$ result between 15.30405 and 15.31415

Both value are same of rounded to 1 decimal point. Therefore, the best approximate value is 15.3 (1 decimal point)

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Question 2

$x^6 - x - 1 = 0$, $\varepsilon = 0.001$. $[1,2]$

$a = 1$, $b = 2$, $f(x) = x^6 - x - 1$

$f(a) = f(1)$
$= (1)^6 - (1) - 1$
$= -1$

$f(b) = f(2)$
$= (2)^6 - (2) - 1$
$= 61$

$f(1) f(2) < 0$, $p \in [1,2]$

$p_1 = \frac{(1) + (2)}{2}$
$= 1.5$

$f(p_1) = f(1.5)$
$= (1.5)^6 - (1.5) - 1$
$= 8.891$

$f(1) f(1.5) < 0$, $p \in [1, 1.5]$

$p_2 = \frac{(1) + (1.5)}{2}$
$= 1.25$

$f(p_2) = f(1.25)$
$= (1.25)^6 - (1.25) - 1$
$= 1.565$

$f(1) f(1.25) < 0$, $p \in [1, 1.25]$

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i	a	b	pi	f(pi)
1	1.000	2.000	1.500	8.891
2	1.000	1.500	1.250	1.565
3	1.000	1.250	1.125	-0.098
4	1.125	1.250	1.188	0.623
5	1.125	1.188	1.157	0.242
6	1.125	1.157	1.141	0.066
7	1.125	1.141	1.133	-0.018
8	1.133	1.141	1.137	0.024
9	1.133	1.137	1.35	0.003
10	1.133	1.135	1.134	-0.007
11	1.134	1.135	1.135	0.003
12	1.134	1.135	1.135	0.003

$|p_N - p_{N-1}| < \varepsilon$

$|p_{12} - p_{11}|$

$= |1.135 - 1.135|$

$= 0.000$

$0.000 < 0.001$,

$p = p_{12} = 1.14$ (2 decimal points)

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Question 3

$f(x) = \cos(x) + 2\sin(x) + x^2$, $\varepsilon = 0.005$

a) $x_0 = 0$; $x_1 = -0.1$

$f(0) = \cos(0) + 2\sin(0) + (0)^2$
$= 1$

$f(-0.1) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$
$= 0.8053$

$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \frac{(0)(0.8053) - (-0.1)(1)}{(0.8053) - (1)}$

$= -0.5136$

$f(-0.5136)$
$= \cos(-0.5136) + 2\sin(-0.5136) + (-0.5136)^2$
$= 0.1521$

i	x_{i-1}	x_i	x_{i+1}	$f(x_{i+1})$
1	0.0000	-0.1000	-0.5136	0.1521
2	-0.1000	-0.5136	-0.6099	0.0461
3	-0.5136	-0.6099	-0.6518	0.0066
4	-0.6099	-0.6518	-0.6588	0.0004

$|f(x_5)| = 0.0004 < 0.005$, $x = x_5 = -0.6588$
$= -0.66$ (2 decimal points)

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b) $x_0 = -0.1$

$f(x) = \cos(x) + 2\sin(x) + (x)^2$

$f'(x) = -\sin(x) + 2\cos(x) + 2x$

$f(-0.1) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$
$= 0.8053$

$f'(-0.1) = -\sin(-0.1) + 2\cos(-0.1) + 2(-0.1)$
$= 1.8898$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= (-0.1) - \frac{(0.8053)}{(1.8898)}$$

$= -0.5261$

i	xi	f(xi)	f'(xi)
0	-0.1000	0.8053	1.8898
1	-0.5261	0.1372	1.1795
2	-0.6424	0.0151	0.9156
3	-0.6589	0.0003	0.8758

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$f(x_3) = 0.0003 < 0.005$, thus the root is $x_3 = -0.66$

∴ Both of the method will provide the same final result which is -0.66 for the root. However, Newton method has a shorter interation compared to Secant method to find the root.

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Question 4

i) let $\frac{1}{x_n} = 1.37$

$\frac{1}{x_n} - 1.37 = 0$

let $f(x_n) = \frac{1}{x_n} - 1.37$, $f'(x_n) = -\frac{1}{x_n^2}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{(\frac{1}{x_n} - 1.37)}{(-\frac{1}{x_n^2})}$

$x_{n+1} = x_n + (x_n - 1.37 x_n^2)$

$x_{n+1} = 2x_n - 1.37 x_n^2$

$x_{n+1} = x_n(2 - 1.37 x_n)$

since $\alpha = 1.37$

$x_{n+1} = x_n(2 - \alpha x_n)$ # prove

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ii) $x_0 = 0.75$, $\alpha = 1.37$

$x_1 = 0.75[2 - (1.37)(0.75)]$

$x_1 = 0.72937500$ (8 d.p)

$x_2 = 0.72937500[2 - (1.37)(0.72937500)]$

$x_2 = 0.72992659$ (8 d.p)

$x_3 = 0.72992659[2 - (1.37)(0.72992659)]$

$= 0.72992701$ (8 d.p)

since $|x_3 - x_2| = 0.0000\ 0004$ ($< 0.0000\ 0005$) , $x = x_3 = 0.7299\ 2701$ (8 d.p)

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Question 5

$$S = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$p(\lambda) = \det(A - \lambda I)$

$$p(\lambda) = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p(\lambda) = \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix}$$

$$p(\lambda) = (1-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 1-\lambda \end{vmatrix} + 0 \begin{vmatrix} -1 & 2-\lambda \\ 0 & -1 \end{vmatrix}$$

$$p(\lambda) = (1-\lambda)\big((2-\lambda)(1-\lambda) - (-1)(-1)\big) + \big(-1(1-\lambda) - 0(-1)\big) + 0$$

$$= (1-\lambda)(2-2\lambda-\lambda+\lambda^2-1) + (-1+\lambda-0)+0$$

$$= (1-\lambda)(1-3\lambda+\lambda^2) + (-1+\lambda)$$

$$= 1-3\lambda+\lambda^2-\lambda+3\lambda^2-\lambda^3-1+\lambda$$

$$= -\lambda^3+4\lambda^2-3\lambda$$

let $-\lambda^3+4\lambda^2-3\lambda=0$

$\lambda^3-4\lambda^2+3\lambda=0$

$(\lambda-3)(\lambda-0)(\lambda-1)=0$

$\therefore \lambda=3, \quad \lambda=0, \quad \lambda=1$

let $(S-\lambda I)v=0$

when $\lambda=3$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$\left[\begin{array}{ccc|c} -2 & -1 & 0 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \xrightarrow[\substack{R_2 \to -R_2 \\ R_3 \to -R_3}]{R_1 \to -R_1} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore v_1 - v_3 = 0$, $v_2 \cancel{+} 2v_3 = 0$, v_3 is free variable, let $v_3 = a$

$v_1 - a = 0$, $v_2 - 2a = 0$

$v_1 = a$, $v_2 = 2a$

$$v = \begin{bmatrix} a \\ 2a \\ a \end{bmatrix} = a\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$, when $\lambda=3$ eigenvector is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

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when $\lambda = 0$,

$$\left(\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix} - 0\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix} = 0$$

$$\left[\begin{array}{ccc|c}1 & -1 & 0 & 0\\ -1 & 2 & -1 & 0\\ 0 & -1 & 1 & 0\end{array}\right] \xrightarrow{R_2 \rightarrow R_1 + R_2} \left[\begin{array}{ccc|c}1 & -1 & 0 & 0\\ 0 & 1 & -1 & 0\\ 0 & -1 & 1 & 0\end{array}\right]$$

$$\xrightarrow{R_3 \rightarrow R_2 + R_3} \left[\begin{array}{ccc|c}1 & -1 & 0 & 0\\ 0 & 1 & -1 & 0\\ 0 & 0 & 0 & 0\end{array}\right]$$

$\therefore$ $V_1 - V_2 = 0$, $V_2 - V_3 = 0$

since V_3 is free variable , let $V_3 = a$

$V_2 - a = 0$, $V_1 - a = 0$

$V_2 = a$ $V_1 = a$

$$V = \begin{bmatrix}a\\ a\\ a\end{bmatrix} = a\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$$

$\therefore$ for $\lambda = 0$, eigenvector is $\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$

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when $\lambda = 1$

$$\left(\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix} - 1\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix} = 0$$

$$\left[\begin{array}{ccc|c}0 & -1 & 0 & 0\\ -1 & 1 & -1 & 0\\ 0 & -1 & 0 & 0\end{array}\right] \xrightarrow{R_1 \rightarrow R_1 - R_3} \left[\begin{array}{ccc|c}0 & 0 & 0 & 0\\ -1 & 1 & -1 & 0\\ 0 & -1 & 0 & 0\end{array}\right]$$

$$\xrightarrow[R_3 \rightarrow -R_3]{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c}-1 & 1 & -1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{array}\right]$$

$$\xrightarrow[R_2 \leftrightarrow R_3]{R_1 \rightarrow -R_1} \left[\begin{array}{ccc|c}1 & -1 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right]$$

$\therefore$ $V_1 - V_2 + V_3 = 0$, $V_2 = 0$, V_3 is free variable

let $V_3 = a$

$V_1 - 0 + a = 0$

$V_1 = -a$

$$V = \begin{bmatrix}-a\\ 0\\ a\end{bmatrix} = a\begin{bmatrix}-1\\ 0\\ 1\end{bmatrix}$$

$\therefore$ for $\lambda = 1$, eigenvector is $\begin{bmatrix}-1\\ 0\\ 1\end{bmatrix}$

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Question 6

a) $B = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix}$

Row 1 : $|\lambda - (-18)| < |20| + |-6|$

$|\lambda + 18| < 26$

$-44 < \lambda < 8$

$\lambda \in (-44, 8)$

Row 2 : $|\lambda - 14| < |-12| + |-3|$

$|\lambda - 14| < 15$

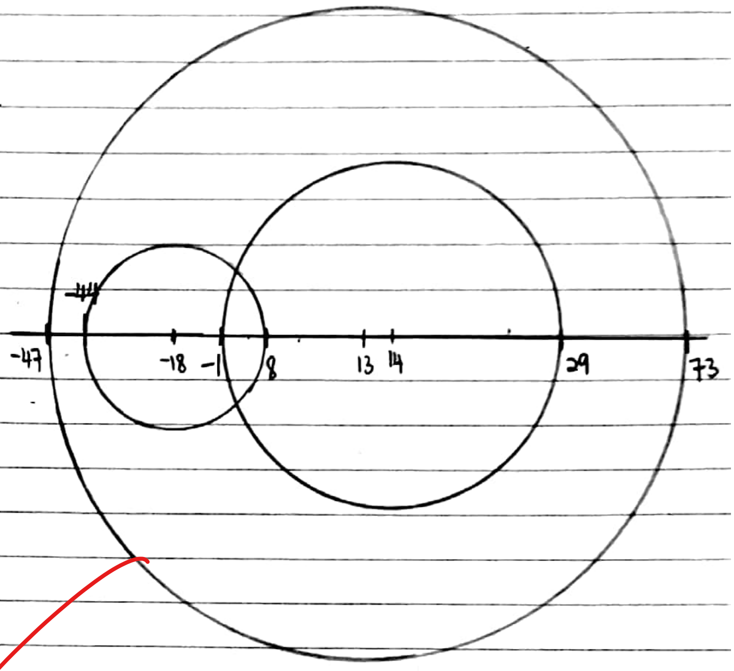
$-1 < \lambda < 29$

$\lambda \in (-1, 29)$

Row 3 : $|\lambda - 13| < |30| + |-30|$

$|\lambda - 13| < 60$

$-47 < \lambda < 73$



$\therefore \lambda \in (-47, 73)$ containing all eigenvalues of B.

b) $v_0 = [0, 0, 1]^T$, $\varepsilon = 0.05$

$v = Bv^0$

$= \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix}$ $m_1 = 13$

$v_1 = v/m_1$

$= \begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix} \left(\frac{1}{13}\right)$

$= \begin{bmatrix} -0.4615 \\ -0.2308 \\ 1.0000 \end{bmatrix}$

$\|v_1 - v_0\| = \sqrt{(-0.4615-0)^2 + (-0.2308-0)^2 + (1.0000-1)^2}$

$= 0.5160 \ (> 0.05)$

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k	$(v^k)^T$			$(Av^k)^T$			m_{k+1}
0	0	0	1	-6	-3	13	13
1	-0.4615	-0.2308	1	-2.309	-0.6932	6.0790	6.0790
2	-0.3798	-0.1140	1	-1.4436	-0.0384	5.0260	5.0260
3	-0.2872	-0.0076	1	-0.9824	0.3400	4.6120	4.6120
4	-0.2130	0.0737	1	-1.6920	0.5878	4.3990	4.3990
5	-0.1573	0.1336	1	-0.4966	0.7580	4.2730	4.2730
6	-0.1162	0.1774	1	-0.3604	0.8780	4.1920	4.1920
7	-0.0860	0.2094	1				

$$\|v^7 - v^6\| = \sqrt{(-0.0860-(-0.1162))^2 + (0.2094-0.1774)^2 + (1-1)^2}$$

$$= 0.0440 \quad (< 0.05)$$

$$\lambda_1 \approx m_7 = 4.1920 \approx 4$$

$$v_1 \approx v_7 = \begin{bmatrix} -0.0860 \\ 0.2094 \\ 1 \end{bmatrix}$$

$$Bv_1 = \lambda v_1$$

$$\begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.0860 \\ 0.2094 \\ 1 \end{bmatrix} = 4 \begin{bmatrix} -0.0860 \\ 0.2094 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -0.2640 \\ 0.9636 \\ 4.1380 \end{bmatrix} = \begin{bmatrix} -0.344 \\ 0.8376 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -0.2640 \\ 0.9636 \\ 4.1380 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$$

c) let $A = B_{shifted}$, $\lambda_1 = 4$

$$A = B - 4I$$

$$A = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}$$

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when $v^0 = [0, 0, 1]^T$, $\varepsilon < 0.05$

$v = Bv^0$

$$v = \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v = \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix} \qquad m_1 = 9$$

$$v_1 = v / m_1$$

$$= \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix} \left(\frac{1}{9}\right)$$

$$= \begin{bmatrix} -0.6667 \\ -0.3333 \\ 1.0000 \end{bmatrix}$$

$$\|v^1 - v^0\| = \sqrt{(-0.6667 - 0)^2 + (-0.3333 - 0)^2 + (1.0000 - 1)^2}$$

$$= 0.7454 \quad (> 0.05)$$

k	$(v^k)^T$			$(Av^k)^T$			m_{k+1}
0	0	0	1	-6	-3	9	9
1	-0.6667	-0.3333	1	2.0014	1.6674	-1.002	2.0014
2	1	0.8331	-0.5006	-2.3344	-2.1672	0.5016	-2.3344
3	1	0.9284	-0.2149	-2.1426	-2.0713	0.2139	-2.1426
4	1	0.9667	-0.0998	-2.0672	-2.0336	0.1008	-2.0672
5	1	0.9837	-0.0488	-2.0332	-2.0166	0.0498	-2.0332
6	1	0.9918	-0.0245				

since $\|v^6 - v^5\| = \sqrt{(1-1)^2 + (0.9918 - 0.9837)^2 + (-0.0245 - (-0.0488))^2}$

$$= 0.0256 \quad (< 0.05)$$

$\therefore \lambda_{shifted} = m^6$, $v = [1 \quad 0.9918 \quad -0.0245]^T$

$$= -2.0332$$

$$\approx -2$$

$$\lambda = \lambda_{shifted} + \lambda_1$$

$$\lambda = -2 + 4$$

$$\lambda = 2$$

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since $\lambda = 2.0$ is the extreme eigenvalue and has the same sign as largest magnitude eigenvalue of the original matrix, $\lambda_1 = 4.0$, then all eigenvalue of B are positive and $\lambda = 2.0$ is the eigenvalue of matrix A.

$$|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$$

$$4 \geq |\lambda_2| \geq 2$$

$$\sum_{i=1}^{4} \lambda_i = \sum_{i=1}^{4} a_{ii}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = a_{11} + a_{22} + a_{33}$$

$$4 + \lambda_2 + 2 = -18 + 14 + 13$$

$$\lambda_2 + 6 = 9$$

$$\lambda_2 = 3$$

$$\therefore \lambda_1 = 4, \quad \lambda_2 = 3, \quad \lambda_3 = 2$$

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